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RESEARCH ARTICLE

A Process Calculus for Energy-Aware Multicast Communications of Mobile Ad-Hoc Networks†

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ABSTRACT

Energy conservation is a critical issue in mobile ad-hoc networks both for nodes and network lifetime, as only batteries power nodes. In this paper we present the E-BUM calculus, a Energy-aware calculus for Broadcast, Unicast and Multicast communications of mobile ad-hoc networks. In order to reason about cost-effective ad-hoc routing protocols, our calculus captures the possibility for a node to control the transmission radius of its communications. We show how to use the E-BUM calculus in order to prove some useful connectivity properties of MANETS, to control network topology and to reason about the problem of reducing interference. In particular, we formalize the notions of sender- and receiver-centered interference and provide efficient proof techniques for verifying the absence of interference between a specific set of nodes.

KEYWORDS

MANETS; process algebra; energy conservation; topology control; interference

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1. INTRODUCTION

A Mobile Ad-hoc Network (MANET) is a self-configuring network of mobile devices connected by wireless links. Each device in a MANET is free to move independently in any direction, and will therefore change its links to other devices frequently. Each node must forward traffic unrelated to its own usage, and then be a router. The primary challenge in building a MANET is equipping each device to continuously maintain the information required to properly route traffic. The devices communicate with each other via radio transceivers through the protocol IEEE 802.11 (WiFi) [21]. This type of communication has a physical scope, because a radio transmission spans over a limited area. Moreover, nodes are primarily powered by a weak battery and thus energy conservation is among the foremost critical issues for network lifetime.

Energy efficiency is an important design criteria, since mobile nodes may be powered by batteries with limited capacity. Power failure of a node not only affects the node itself but also its ability to forward packets on behalf of others and thus the overall network lifetime. For this reason, many research efforts have been devoted to develop energy-aware routing protocols.

Energy efficient routing protocols use broadcast to transmit unicast and multicast data packets between nodes. The use of unicast and multicast has many benefits including power and bandwidth saving, and lower error rates. Indeed, since radio signals are likely to overlap with others in a geographical area, a straightforward broadcasting by flooding is usually very costly and results in serious redundancy, contention, and collisions. For this reason, modern ad-hoc routing protocols indicates the real addresses of transmitted packets to reduce the number of control packets (see, for instance, [1, 5, 16]). In addition, power aware protocols reduce the total energy consumption by adjusting each node's transmission power (e.g., radius) just enough to reach up the intended recipients only (see, e.g., [22]).

The main goal of *topology control* is to reduce node power consumption in order to extend the lifetime of the network. This can be considered a trade-off between power saving and network connectivity: choosing a low transmission power for a node will reduce its power consumption, but it will also possibly reduce its

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connectivity with the other nodes in the network. One of the main approaches to reducing energy consumption consists in minimizing interference between the network nodes. In the context of topology control, interference is usually confined by constructing sparse topologies or topologies with low node degrees, without providing rigorous motivations or proofs.

In this paper we present the E-BUM calculus, a calculus for Energy-aware Broadcast, Unicast and Multicast communications of mobile ad-hoc networks. It allows us to model the ability of a node to broadcast a message to any other node within its physical transmission range, and to move in and out of the transmission range of other nodes in the network. The connectivity of a node is represented by a location and a transmission radius. Broadcast communications are limited to the transmission cell of the sender, while unicast and multicast communications are modelled by specifying, for each output action, the addresses of the intended recipients of the message. Moreover, arbitrary and unexpected connections and disconnections of nodes as well as the possibility for a node to dynamically adjust its transmission power are modelled by enabling nodes to modify the corresponding transmission radius.

We show how to use the E-BUM calculus in order to prove some useful connectivity properties of MANETs which can be exploited to control power/energy consumption and also to reduce interference.

Based on the E-BUM model, we formally introduce two different definitions of interference: a sender-centered definition which measures the number of nodes potentially disturbed by the sender of a message, and a receiver-centered definition which gives a measure of the number of nodes potentially disturbing a given receiver. These two definitions are based on the notion of observability that pertains to the semantics of our calculus: what we observe of a transmission is its ability to reach the set of its intended recipients. Efficient proof techniques for verifying the absence of interference between a specific set of nodes are also proposed.

Plan of the paper. After discussing related work in the following section, we introduce the E-BUM model and its observation semantics in Section 3. An equivalent LTS semantics and a bisimulation-based proof technique is also introduced, providing an efficient method to check whether two networks are observational equivalent with respect to the set of their intended recipients. Some useful power-aware connectivity properties for MANETs are studied in Section 4. Section 5 formalizes both a sender- and a receiver-centered notion of interference and provides a bisimulation-based proof technique for verifying the absence of interference for a specific set of nodes. Section 6 concludes the paper.

2. RELATED WORK

Many researchers have proposed algebraic models for wireless ad-hoc networks. The E-BUM calculus presented in this paper is an extension of CMN (Calculus of Mobile Ad-Hoc Networks) [10] and it allows us to model unicast and multicast communications as well as the ability for a node to control its transmission power. Related to our model is also the ω-calculus [18], a conservative extension of the π-calculus, which deals with unicast and multicast communications by allowing two nodes to share a private channel (hidden to the external network). We believe that our model provides a more realistic representation of the dynamics of mobile ad-hoc networks: a message sent to a specific group of recipients is not hidden to the rest of the network but all the nodes within the transmission cell of the sender will be able to receive the message anyway. The other important feature of our calculus is that it allows us model the possibility for a node to control its power consumption by adjusting the transmission power of its communications. The E-BUM calculus can then be used to compare different protocols and communications strategies in order to evaluate the best solution to save energy without loosing connectivity. We are not aware of other process calculi designed to study the problem of energy conservation in ad-hoc networks.

As mentioned above, reducing interference is one of the main goals of topology control besides direct energy conservation by restriction of transmission power. Most of the proposed topology control algorithms try to reduce interference implicitly as a consequence of sparseness or low degree of the resulting topology graph. An explicit concept of interference, based on the current network traffic, has been proposed in [11], while a definition that is independent on the network traffic has been presented in [3]. This definition measures the number of nodes which are affected by a communication over a given link. In contrast the definition presented in [7] considers interference at the intended receiver of a message. We deal with both kinds of interference and address the problem of verifying the absence of interference between a specific set of nodes.

3. THE CALCULUS

We introduce the E-BUM calculus, an extension of CMN (Calculus of Mobile Ad-hoc Networks) [10], that models mobile ad-hoc networks as a collection of nodes, running in parallel, and using channels to broadcast messages. Our calculus extends CMN to support multicast and unicast communications. Moreover, it allows one to model the arbitrary and unexpected connections and disconnections of nodes in a network as well as the possibility for a node to administrate energy consumption by choosing the optimal transmission radius to communicate with the desired recipients.

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Networks	
M,N ::= **0**	Empty network
$\mid M_1 \mid M_2$	Parallel composition
$\mid n[P]_l$	Node (or device)
Processes	
P,Q,R ::= **0**	Inactive process
$\mid c(\tilde{x}).P$	Input
$\mid \bar{c}_{L,r}\langle\tilde{w}\rangle.P$	Output
$\mid [w_1 = w_2]P, Q$	Matching
$\mid A\langle\tilde{w}\rangle$	Recursion

Table I. Syntax

$$
\begin{array}{l}
n[\mathbf{0}]_l \equiv \mathbf{0} \\
n[[v = v]P, Q]_l \equiv n[P]_l \\
n[[v_1 = v_2]P, Q]_l \equiv n[Q]_l \quad \text{if } v_1 \neq v_2 \\
n[A\langle\tilde{v}\rangle]_l \equiv n[P\{\tilde{v}/\tilde{x}\}]_l \quad \text{if } A(\tilde{x}) \stackrel{\text{def}}{=} P \wedge |\tilde{x}| = |\tilde{v}| \\
M|N \equiv N|M \\
(M|N)|M' \equiv M|(N|M') \\
M|\mathbf{0} \equiv M
\end{array}
$$

Table II. Structural Congruence

Syntax. We use letters c and d for *channels*; m and n for *nodes*; l, k and h for *locations*; r for *transmission radii*; x, y and z for *variables*. *Closed values* contain nodes, locations, transmission radii and any basic value (booleans, integers, ...). *Values* include also variables. We use u and v for closed values and w for (open) values. We denote by $\tilde{v}$, $\tilde{w}$ tuples of values.

The syntax of E-BUM is shown in Table I. This is defined in a two-level structure: the lower one for processes, the upper one for networks. Networks are collections of nodes (which represent devices), running in parallel, using channels to communicate messages. As usual, **0** denotes the empty network and $M_1|M_2$ represents the parallel composition of two networks. Processes are sequential and live within the nodes. Process **0** denotes the inactive process. Process $c(\tilde{x}).P$ can receive a tuple $\tilde{w}$ of (closed) values via channel c and continue as $P\{\tilde{w}/\tilde{x}\}$, i.e., as P with $\tilde{w}$ substituted for $\tilde{x}$ (where $|\tilde{x}| = |\tilde{w}|$). Process $\bar{c}_{L,r}\langle\tilde{w}\rangle.P$ can send a tuple of (closed) values $\tilde{w}$ via channel c and continue as P. The tag L is used to maintain the set of locations of the intended recipients: $L = \infty$ represents a broadcast transmission, while a finite set of locations L denotes a multicast communication (unicast if L is a singleton). The tag r represents the power of the transmission: we assume that the choice of the transmission power may depend on precise strategies which are implemented in the communication protocol; hence it is reasonable considering the transmission radius of a communication as an information given by the process running in the sender node. In the following we assume that the tag r of a transmission never exceeds the maximum transmission radius of the sender node. Syntactically, the tags L and r associated to the channel c in an output action may be variables, but they must be instantiated when the output prefix is ready to fire. Process $[w_1 = w_2]P, Q$ behaves as P if $w_1 = w_2$, and as Q otherwise. We write $A\langle\tilde{w}\rangle$ to denote a process defined via a (possibly recursive) definition $A(\tilde{x}) \stackrel{\text{def}}{=} P$, with $|\tilde{x}| = |\tilde{w}|$, where $\tilde{x}$ contains all channels and variables that appear free in P.

Nodes cannot be created or destroyed. We write $n[P]_l$ for a node named n (this is the logic location of the device in the network), located at l (this is the physical location of the node), and executing a process P. We associate to each node identifier n a pair $\langle r_n, \delta_n\rangle$ where r_n represents the maximum transmission radius for n, while δ_n denotes the maximum distance that n can cover in a computational step. We say that $n[P]_l$ is *unpowered* when $r_n = 0$; we say that $n[P]_l$ is *stationary* when $\delta_n = 0$. The possibility that nodes communicate with each other is verified by looking at the physical locations and the transmission radius of the sender, in other words if a node broadcasts a message, this information will be received only by the nodes that lie in the area delimited by the transmission radius of the sender. In the definition of the operational semantics we then assume the possibility of comparing locations so to determine whether a node lies or not within the transmission cell of another node. We do so by means of a function $d(\cdot,\cdot)$ which takes two locations and returns their distance.

Throughout, we assume that processes are closed (i.e., they have no free variables) and identify processes and networks up to α-conversion. We denote by $\prod_{i\in I} M_i$ the parallel composition of networks M_i, for $i \in I$. We write c_l for $c_{\{l\}}$, $\bar{c}_{L,r}\langle w\rangle$ for $\bar{c}_{L,r}\langle w\rangle.\mathbf{0}$, **0** for $n[\mathbf{0}]_l$ and $[w_1 = w_2]P$ for $[w_1 = w_2]P, \mathbf{0}$. Moreover, we assume that in any network each node identifier is unique.

Reduction Semantics. The dynamics of the calculus is specified by the *reduction relation* over networks ($\longrightarrow$), described in Table III. As usual, it relies on an auxiliary relation, called structural congruence ($\equiv$), which is the least contextual equivalence relation satisfying the rules defined in Table II.

Rule (R-Bcast) models the transmission of a tuple $\tilde{v}$ through a channel $c_{L,r}$. The set L associated to channel c indicates the locations of the intended recipients. Indeed, nodes communicate using radio frequencies that enable only broadcast messages (monopolizing channels is not permitted). However, a node may decide to communicate with a specific node (or group of nodes), this is the reason why we decided to associate to each output action a set of transmission recipients. The cardinality of this set indicates the kind of communication that is used: if $L = \infty$ then the recipient set is the whole network and a broadcast transmission is performed, while if L is a finite set (resp., a singleton) then a multicast (resp., a unicast)

$$\text{(R-Bcast)}\ \frac{-}{n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | \prod_{i\in I} n_i[c(\tilde{x}_i).P_i]_{l_i} \longrightarrow n[P]_l | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i}}$$

$$\text{with } 0 < r \leq r_n,\ \forall i \in I. d(l, l_i) \leq r,\ |\tilde{x}| = |\tilde{v}|$$

$$\text{(R-Move)}\ \frac{-}{n[P]_l \longrightarrow n[P]_k}\ \text{ with } 0 < d(l,k) \leq \delta_n$$

$$\text{(R-Par)}\ \frac{M \longrightarrow M'}{M|N \longrightarrow M'|N} \qquad \text{(R-Struct)}\ \frac{M \equiv N\ N \longrightarrow N'\ N' \equiv M'}{M \longrightarrow M'}$$

Table III. Reduction Semantics

communication is realized. A radius r is also associated to the channel c, indicating the transmission radius required for that communication which may depend on the power consumption strategy adopted by the surrounding protocol. In our calculus transmission is a *non-blocking action*: transmission proceeds even if there are no nodes listening for messages. The messages transmitted will be received only by those nodes which lie in the transmission area of the sender. It may occur that some recipients within the range of the transmitter do not receive the message. This may be due to several reasons that concern the instability and dynamism of the network. In terms of observation this corresponds to a local activity of the network which an observer is not party to.

Rule (R-Move) models arbitrary and unpredictable movements of mobile nodes. As said above, δ_n denotes the maximum distance that node n can cover in a computational step. Movements are atomic actions, e.g., while moving, a node cannot do anything else.

The remaining rules are standard in process calculi.

We denote by $\longrightarrow^*$ the reflexive and transitive closure of $\longrightarrow$.

Behavioral Semantics. The central actions of our calculus are transmission and reception of messages. However, only the transmission of messages can be observed. An observer cannot be sure whether a recipient actually receives a given value. Instead, if a node receives a message, then surely someone must have sent it. Following [15], we use the term *barb* as a synonymous of observable.

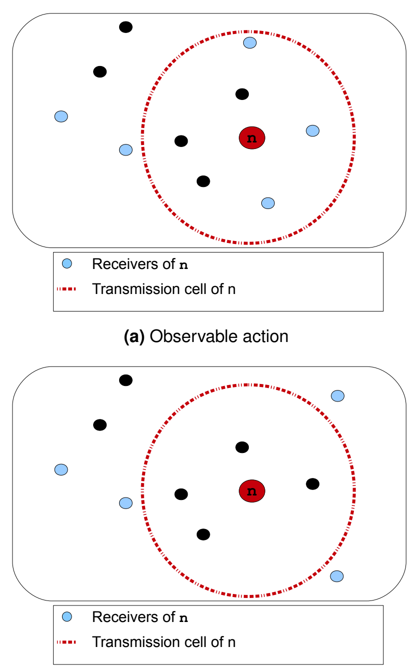
In our definition of barb a transmission is observable only if at least one location in the set of the intended recipients is able to receive the message.

Definition 3.1 (Barb)
We write $M \downarrow_{c@K}$ if M is of the form $n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | M'$ and the set $K \subseteq \{k \in L : d(l,k) \leq r\}$ is not empty. We write $M \Downarrow_{c@K}$ if $M \longrightarrow^* M' \downarrow_{c@K}$.

Notice that, if $M \equiv n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | M'$ and $M \downarrow_{c@K}$ then at least one of the recipients in L is actually able to receive the message.

The concept of observable action is illustrated in Figure 1. Consider a node n (the red node in the picture) broadcasting a message which is destined to a specific set L of recipients. The black circles in the picture represent the network nodes not included in L, while the light blue circles represent the nodes in L, i.e., the intended recipients of the message. Figure 1(a) depicts the situation in which at least one of the nodes in L lies in the transmission area of the sender, while Figure 1(b) illustrates the case of a non-observable action, where none of the nodes in L is able to receive the message.

To define our observation equivalence we will ask for the largest relation which satisfies the following properties.



(a) Observable action

(b) Non-observable action

Figure 1. Observability

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$$\text{(Output)}\ \frac{-}{\bar{c}_{L,r}\langle\tilde{v}\rangle.P \xrightarrow{\bar{c}_{L,r}\tilde{v}} P} \qquad \text{(Input)}\ \frac{-}{c(\tilde{x}).P \xrightarrow{c\tilde{v}} P\{\tilde{v}/\tilde{x}\}}$$

$$\text{(Then)}\ \frac{P \xrightarrow{\eta} P'}{[\tilde{v} = \tilde{v}]P, Q \xrightarrow{\eta} P'} \qquad \text{(Else)}\ \frac{Q \xrightarrow{\eta} Q'}{[\tilde{v}_1 = \tilde{v}_2]P, Q \xrightarrow{\eta} Q'}\ \ \tilde{v}_1 \neq \tilde{v}_2$$

$$\text{(Rec)}\ \frac{P\{\tilde{v}/\tilde{x}\} \xrightarrow{\eta} P'}{A\langle\tilde{v}\rangle \xrightarrow{\eta} P'}\ \ A(\tilde{x}) \stackrel{\text{def}}{=} P$$

Table IV. LTS rules for Processes

Definition 3.2
Let $\mathcal{R}$ be a relation over networks:

- *Barb preservation.* $\mathcal{R}$ is *barb preserving* if $M \,\mathcal{R}\, N$ and $M \downarrow_{c@K}$ implies $N \Downarrow_{c@K}$.
- *Reduction closure.* $\mathcal{R}$ is *reduction closed* if $M \,\mathcal{R}\, N$ and $M \longrightarrow M'$ implies that there exists N' such that $N \longrightarrow^* N'$ and $M' \,\mathcal{R}\, N'$.
- *Contextuality.* $\mathcal{R}$ is *contextual* if $M \,\mathcal{R}\, N$ implies $\mathcal{C}[M] \,\mathcal{R}\, \mathcal{C}[N]$ for any context $\mathcal{C}[\cdot]$, where a context is a network term with a hole $[\cdot]$ defined by the grammar: $\mathcal{C}[\cdot] ::= [\cdot] \mid [\cdot]|M \mid M|[\cdot]$.

Definition 3.3 (Reduction barbed congruence)
Reduction barbed congruence, written $\cong$, is the largest symmetric relation over networks, which is reduction closed, barb preserving, and contextual.

Two networks are related by $\cong$ if they exhibit the same behaviour relative to the corresponding sets of intended recipients. Hereafter we develop a bisimulation-based proof technique for $\cong$. It provides an efficient method to check whether two networks are related by $\cong$.

Bisimulation-based Proof Technique. We develop a proof technique for the relation $\cong$. More precisely, we define a LTS semantics for E-BUM terms, which is built upon two sets of rules: one for processes and one for networks.

Table IV presents the LTS rules for processes. Transitions are of the form $P \xrightarrow{\eta} P'$, where η ranges over input and output actions of the form $c\tilde{v}$ and $\bar{c}_{L,r}\tilde{v}$, respectively. Rules for processes are simple and they not need deeper explanations.

Table V contains the LTS rules for networks. Transitions are of the form $M \xrightarrow{\gamma} M'$, where the grammar for γ is:

$$\gamma ::= c?\tilde{v}@l \mid c_L!\tilde{v}[l,r] \mid c!\tilde{v}@K \triangleleft R \mid \tau.$$

Rule (Snd) models the sending, with transmission radius r, of the tuple $\tilde{v}$ of values via channel c to a specific set L of recipients, while rule (Rcv) models the reception of $\tilde{v}$ at l via channel c. Rule (Bcast) models the broadcast message propagation: all the nodes lying within the transmission cell of the transmitter may receive the message, regardless of the fact that they are in L. Rule (Obs) models the observability of a transmission: every output action may be detected (and hence *observed*) by any node located within the transmission cell of the sender. We are interested in observing the output actions reaching at least one of the intended recipients. The action $c!\tilde{v}@K \triangleleft R$ represents the transmission of the tuple $\tilde{v}$ of messages via c to the set K of recipients in L, located within the transmission cell of the transmitter. R is the set of all the nodes able to receive the message, regardless of the fact that they are in L. When $K \neq \emptyset$ this is an observable action corresponding to the barb $\downarrow_{c@K}$. Rule (Lose) models message loss. Rule (Move) models the migration of a mobile node from a location l to a new location k, where δ_n represents the maximum distance that node n can cover in a single computational step. Rule (Par) is standard.

The following lemma shows the relationships between the LTS rules and the reduction semantics.

Lemma 3.4
Let M be a network.

1. If $M \xrightarrow{c?\tilde{v}@l} M'$, then there exist n, P and M_1 such that $M \equiv (n[c(\tilde{x}).P]_l | M_1)$ and $M' \equiv (n[P\{\tilde{v}/\tilde{x}\}]_l | M_1)$.
2. If $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$, then there exist n, P, M_1, I (possibly empty), and n_i, P_i, l_i with $i \in I$ and $d(l, l_i) \leq r$, such that: $M \equiv (n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | \prod_{i\in I} n_i[c(\tilde{x}_i).P_i]_{l_i} | M_1)$ and $M' \equiv (n[P]_l | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i} | M_1)$.

Proof
By induction on the shape of the transition rules in Table V.
Case 1: $M \xrightarrow{c?\tilde{v}@l} M'$.

(Rcv) Let $M \xrightarrow{c?\tilde{v}@l} M'$, then there exist n, Q and r such that $Q \xrightarrow{c\tilde{v}} Q'$, $M \equiv n[Q]_l$ and $M' \equiv n[Q']_l$. Since $Q \xrightarrow{c\tilde{v}} Q'$ then, by structural congruence, there must be P such that $n[Q]_l \equiv n[c(\tilde{x}).P]_l$ and $n[Q']_l \equiv n[P\{\tilde{v}/\tilde{x}\}]_l$. Hence, if we suppose that M_1 is the empty network then the lemma is proved since $M \equiv n[c(\tilde{x}).P]_l | M_1$ and $M' \equiv n[P\{\tilde{v}/\tilde{x}\}]_l | M_1$.

$$\text{(Snd)}\ \frac{P \xrightarrow{\bar{c}_{L,r}\tilde{v}} P'}{n[P]_l \xrightarrow{c_L!\tilde{v}[l,r]} n[P']_l}\ 0 < r \leq r_n \qquad \text{(Rcv)}\ \frac{P \xrightarrow{c\tilde{v}} P'}{n[P]_l \xrightarrow{c?\tilde{v}@l} n[P']_l}$$

$$\text{(Bcast)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M' \quad N \xrightarrow{c?\tilde{v}@l'} N'}{M|N \xrightarrow{c_L!\tilde{v}[l,r]} M'|N'}\ d(l,l') \leq r$$

$$\text{(Obs)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M'}{M \xrightarrow{c!\tilde{v}@K\triangleleft R} M'}\ R \subseteq \{k : d(l,k) \leq r\} \wedge K = L \cap R \neq \emptyset$$

$$\text{(Lose)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M'}{M \xrightarrow{\tau} M'} \qquad \text{(Move)}\ \frac{-}{n[P]_l \xrightarrow{\tau} n[P]_k}\ 0 < d(l,k) \leq \delta_n \qquad \text{(Par)}\ \frac{M \xrightarrow{\gamma} M'}{M|N \xrightarrow{\gamma} M'|N}$$

Table V. LTS rules for Networks

(Par) Let $M|N \xrightarrow{c?\tilde{v}@l} M'|N$ because $M \xrightarrow{c?\tilde{v}@l} M'$. By induction hypothesis, $M \equiv (n[c(\tilde{x}).P]_l|M'_1)$ and $M' \equiv (n[P\{\tilde{v}/\tilde{x}\}]_l|M'_1)$. Hence, by applying the rule (Struct Par Assoc) of structural congruence we can write $M|N \equiv (n[c(\tilde{x}).P]_l|(M'_1|N))$ and $M'|N \equiv (n[P\{\tilde{v}/\tilde{x}\}]_l|(M'_1|N))$. The lemma is proved with $M_1 = (M'_1|N)$.

Case 2: $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$.

(Snd) Let $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$, then there exist n and Q such that $Q \xrightarrow{\bar{c}_{L,r}\tilde{v}} Q'$, $M \equiv n[Q]_l$ and $M' \equiv n[Q']_l$. Since $Q \xrightarrow{\bar{c}_{L,r}\tilde{v}} Q'$ then, by structural congruence, there must exist P such that $n[Q]_l \equiv n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l$ and $n[Q']_l \equiv n[P]_l$. Hence, if we suppose that M_1 is the empty network then the lemma is proved because $M \equiv (n[\bar{c}_{L,r}\langle\tilde{v}\rangle.Q]_l|M_1)$ and $M' \equiv (n[Q]_l|M_1)$.

(Bcast) Let $M|N \xrightarrow{c_L!\tilde{v}[l,r]} M'|N'$ because $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$ and $N \xrightarrow{c?\tilde{v}@l'} N'$, with $d(l,l') \leq r$. By induction hypothesis, there exist n, P, M'_1, I (possibly empty), and n_i, P_i, l_i, with $i \in I$ and $d(l,l_i) \leq r$, such that:
$M \equiv (n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l|\prod_{i\in I} n_i[c(\tilde{x}_i).P_i]_{l_i},|M'_1)$
and
$M' \equiv (n[P]_l|\prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i}|M'_1)$.
Moreover, there exist n', Q, and N_1 such that $N \equiv (n'[c(\tilde{x}).Q]_{l'}|N_1)$ and $N' \equiv (n'[Q\{\tilde{v}/\tilde{x}\}]_{l'}|N_1)$. Hence $M|N$ and $M'|N'$ have the required form.

(Par) The proof of this case is analogous to the case (Par) above.

□

Lemma 3.5
Let M be a network. If $M \xrightarrow{\gamma} M'$ and $M \equiv N$ then there exists N' such that $N \xrightarrow{\gamma} N'$ and $M' \equiv N'$.

Proof
Straightforward by induction on the depth of the inference $M \xrightarrow{\gamma} M'$. □

Theorem 3.6 (Harmony Theorem)
Let M be a network.

1. $M \downarrow_{c@K}$ if and only if $M \xrightarrow{c!\tilde{v}@K\triangleleft R}$ for some tuple of values $\tilde{v}$ and set of locations R.
2. If $M \xrightarrow{\tau} M'$ then $M \longrightarrow M'$.
3. If $M \longrightarrow M'$ then $M \xrightarrow{\tau}\equiv M'$.

Proof

1. The first statement follows from the definition of barb, Lemma 3.4 and the LTS rules for networks.
Suppose that $M \downarrow_{c@K}$, then by definition of barb there exists n, l, L, r, P and M' (possibly empty): $M \equiv n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l \mid M'$ and $K \subseteq \{k \in L : d(l,k) \leq r\}$. If we consider the set $R \subseteq \{k : d(l,k) \leq r\}$, such that $K = R \cap L$, by applying rule (Snd) to the node n, and then the rule (Bcast) and (Obs) to the network M we get $M \xrightarrow{c!\tilde{v}@K\triangleleft R}$, as required.
Suppose now that $M \xrightarrow{c!\tilde{v}@K\triangleleft R}$, i.e., $M \xrightarrow{c_L!\tilde{v}[l,r]}$ for some location l, some set L of recipients and some transmission radius r. By Lemma 3.4 there exist n, P, M_1, I (possibly empty), and n_i, P_i, l_i with $i \in I$ and $d(l,l_i) \leq r$, such that: $M \equiv (n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l|\prod_{i\in I} n_i[c(\tilde{x}_i).P_i]_{l_i}|M_1)$
and, since by the rule (Obs) $K = R \cap L \neq \emptyset$, we get, by definition of barb, $M \downarrow_{c@K}$, as required.
2. The second statement is proved by induction on the derivation $M \xrightarrow{\tau} M'$.
Suppose that the τ-action has been generated by an application of the rule (Lose). In this case we have $M \xrightarrow{\tau} M'$ because $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$. Then, by an application of Lemma 3.4 it holds that for some n, P, M_1 and I (possibly empty), n_i, P_i, l_i such that $d(l,l_i) \leq r$ for $i \in I$,

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$M \equiv (n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | \prod_{i\in I} n_i[c(\tilde{x}).P_i]_{l_i} | M_1)$ and $M' \equiv (n[P]_l | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i} | M_1)$. By applying the rules (R-Bcast) and (R-Par) we get

$$n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | \prod_{i\in I} n_i[c(\tilde{x}).P_i]_{l_i} | M_1 \rightarrow n[P]_l | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i} | M_1$$

and, by applying (R-Struct), we obtain $M \longrightarrow M'$, as required.

Suppose now that the τ-action has been generated by an application of rule (Move) with $M = n[P]_l$, $M' = n[P]_k$ and $d(k,l) \leq \delta_n$. Then, by an application of rule (R-Move) we get $n[P]_l \longrightarrow n[P]_k$, i.e., $M \rightarrow M'$.

The other cases follow straightforwardly from the congruence rules and the reduction relation.

3. The third statement is proved by induction on the derivation $M \rightarrow M'$. Suppose that the derivation $M \rightarrow M'$ has been generated by an application of rule (R-Bcast), i.e., $M = n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l | \prod_{i\in I} n_i[c(\tilde{x}_i).P_i]_{l_i}$, $M' = n[P]_l | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x}_i\}]_{l_i}$ and $M \rightarrow M'$ with $0 < r \leq r_n$, $\forall i \in I. d(l,l_i) \leq r$, $|\tilde{x}| = |\tilde{v}|$. Then, by applying rules (Snd), (Rcv) and $|I| - 1$ times rule (Bcast) we obtain $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$, and by applying rule (Lose) we get $M \xrightarrow{\tau} M'$ as required.

Suppose now that the derivation $M \rightarrow M'$ has been generated by an application of rule (R-Struct), i.e., $M \rightarrow M'$ because $M \equiv N$, $N \rightarrow N'$ and $N' \equiv M'$. By induction hypothesis $N \xrightarrow{\tau}\equiv N'$, then there exists N'' such that $N \xrightarrow{\tau} N''$ and $N'' \equiv N'$. Hence, by Lemma 3.5 there exists M'' such that $M \xrightarrow{\tau}\equiv M''$ and $M'' \equiv N''$. By transitivity of $\equiv$ it follows that $M'' \equiv M'$, then $M \xrightarrow{\tau}\equiv M'$ as required.

The cases when the reduction $M \longrightarrow M'$ is derived by rules (R-Move) and (R-Par) are straightforward.

□

Based on the LTS semantics, we define a labelled bisimilarity that is a complete characterisation of our *reduction barbed congruence*. It is built upon the following actions:

$$\alpha \ ::= \ c?\tilde{v}@l \ | \ c!\tilde{v}@K \triangleleft R \ | \ \tau.$$

Since we are interested in *weak behavioural equivalences*, that abstract over τ-actions, we introduce the notion of *weak action*. We denote by $\Rightarrow$ the reflexive and transitive closure of $\xrightarrow{\tau}$; we use $\xRightarrow{c?\tilde{v}@l}$ to denote $\Rightarrow\xrightarrow{c?\tilde{v}@l}\Rightarrow$; we use $\xRightarrow{c!\tilde{v}@K\triangleleft R}$ for $\Rightarrow\xrightarrow{c!\tilde{v}@K\triangleleft R}\Rightarrow$; finally $\xRightarrow{\hat{\alpha}}$ denotes $\Rightarrow$ if $\alpha = \tau$ and $\xRightarrow{\alpha}$ otherwise.

Definition 3.7 (Labelled Bisimilarity)
A binary relation $\mathcal{R}$ over networks is a *simulation* if $M\mathcal{R}N$ implies:

- If $M \xrightarrow{\alpha} M'$, $\alpha \neq c?\tilde{v}@l$, then there exists N' such that $N \xRightarrow{\hat{\alpha}} N'$ and $M'\mathcal{R}N'$;
- If $M \xrightarrow{c?\tilde{v}@l} M'$ then there exists N' such that either $N \xRightarrow{c?\tilde{v}@l} N'$ and $M'\mathcal{R}N'$ or $N \Rightarrow N'$ and $M'\mathcal{R}N'$.

We say that N *simulates* M if there is some simulation $\mathcal{R}$ such that $M\mathcal{R}N$. A relation $\mathcal{R}$ is a *bisimulation* if both $\mathcal{R}$ and its converse are simulations. *Labelled bisimilarity*, written $\approx$, is the largest bisimulation over networks. We say that M and N are bisimilar, written $M \approx N$, if there exists some bisimulation $\mathcal{R}$ such that $M\mathcal{R}N$.

It is easy to prove that labelled bisimilarity is an equivalence relation: reflexivity and symmetry are trivial, while transitivity follows from definition of $\xRightarrow{\hat{\alpha}}$.

The next lemma shows that labelled bisimilarity is closed under contexts.

Lemma 3.8 ($\approx$ is contextual)
Let M and N be two networks such that $M \approx N$. Then $M|O \approx N|O$, for all networks O.

Proof
It is sufficient to prove that the relation

$$\mathcal{S} = \{(M|O, N|O) \mid M \approx N \text{ and } O \text{ is a network }\}$$

is a bisimulation. To prove it we do a case analysis on the transition $M|O \xrightarrow{\alpha} \hat{M}$. The interesting cases are those where the transition is due to an interaction between M and O, and this happens by an application of rule (Bcast). Let $M|O \xrightarrow{c!\tilde{v}@K\triangleleft R} \hat{M}$ because $M|O \xrightarrow{c_L!\tilde{v}[l,r]} \hat{M}$ for some L, l and r with $K \neq \emptyset$ and $K \subseteq L \cap R$. Suppose that $M|O \xrightarrow{c_L!\tilde{v}[l,r]} \hat{M}$ follows by an application of rule (Bcast). Two cases are critical:

1. $M|O \xrightarrow{c_L!\tilde{v}[l,r]} \hat{M}$ because $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$ and $O \xrightarrow{c?\tilde{v}@l'} O'$ with $d(l,l') \leq r$ and $\hat{M} = M'|O'$;
2. $M|O \xrightarrow{c_L!\tilde{v}[l,r]} \hat{M}$ because $M \xrightarrow{c?\tilde{v}@l'} M'$ and $O \xrightarrow{c_L!\tilde{v}[l,r]} O'$, with $d(l,l') \leq r$ and $\hat{M} = M'|O'$.

Case 1. By applying rule (Obs) we have that $M \xrightarrow{c!\tilde{v}@K\triangleleft R} M'$, and since by hypothesis $M \approx N$, $N \xRightarrow{c!\tilde{v}@K\triangleleft R} N'$ and $M' \approx N'$. Hence $N \Longrightarrow\xrightarrow{c!\tilde{v}@K\triangleleft R}\Longrightarrow N'$ with $N' \approx M'$. That means that there exists l'', r'' and L'' such that $N \Longrightarrow N'' \xrightarrow{c_{L''}!\tilde{v}[l'',r'']} N''' \Longrightarrow N'$ with $\{k : d(l'',k) \leq r''\} \supseteq R$ and $K = R \cap L''$. But since $l' \in R$, by hypothesis, $d(l',l'') \leq r''$ and, by an application of the rules (Par) and (Bcast):

$N \,|\, O \Longrightarrow N'' \mid O \xrightarrow{c_{L''}!\tilde{v}[l'',r'']} N'''|O'... \Longrightarrow N' \,|\, O'$.

Finally, by applying rule (Obs) we can turn again the transition $N'' \mid O \xrightarrow{c_{L''}!\tilde{v}[l'',r'']} N''' \mid O'$ into $N'' \mid O \xrightarrow{c!\tilde{v}@K\triangleleft R} N''' \mid O'$. This implies

$N \mid O \xRightarrow{c!\tilde{v}@K\triangleleft R} N' \mid O'$, with $(M' \mid O', N' \mid O') \in \mathcal{R}$ as required.

Case 2. $M|O \xrightarrow{c_L!\tilde{v}[l,r]} \hat{M}$ because $M \xrightarrow{c?\tilde{v}@l'} M'$ and $O \xrightarrow{c_L!\tilde{v}[l,r]} O'$, with $d(l,l') \leq r$ and $\hat{M} = M'|O'$. As $M \approx N$ then there exists N' such that:

- $N \xRightarrow{c?\tilde{v}@l'} N'$, with $M' \approx N'$; in this case
$$N|O \Rightarrow\xrightarrow{c_L!\tilde{v}[l,r]}\Rightarrow N'|O'$$
and, by an application of rule (Obs), also $N|O \xRightarrow{c!\tilde{v}@K\triangleleft R} N'|O'$, with $(M'|O', N'|O') \in \mathcal{S}$, as required.
- or $N \Longrightarrow N'$, with $M' \approx N'$; in this case by applying rule (Par) we obtain
$$N|O \xrightarrow{c_L!\tilde{v}[l,r]} N|O' \Longrightarrow N' \mid O'$$
and, by applying rule (Obs) also $N|O \xrightarrow{c!\tilde{v}@K\triangleleft R} N|O' \Longrightarrow N' \mid O'$, with $(M'|O', N'|O') \in \mathcal{S}$, as required.

The cases where there is no interaction between M and O are straightforward. □

We can now demonstrate that our labelled bisimilarity is a valid proof method for *reduction barbed congruence*.

Theorem 3.9 (Soundness)
Let M and N be two arbitrary networks such that $M \approx N$. Then $M \cong N$.

Proof

We have to prove that the relation $\approx$ is:

1. reduction closed
2. barb preserving
3. contextual

1. *Reduction Closure*. If $M \approx N$ and $M \rightarrow M'$, by the Theorem 3.6 $\exists \hat{M} \equiv M'$ such that $M \xrightarrow{\tau} \hat{M}$, and, by Lemma 3.5, $M' \approx \hat{M}$. Since $M \approx N, \exists N'$ such that $N \Longrightarrow N'$ and $\hat{M} \approx N'$. Again, by the Theorem 3.6 $N \rightarrow^* N'$ and, by transitivity of the relation $\approx$, $M' \approx N'$.
2. *Barb preservation*. Suppose $M \downarrow_{c@K}$. By Theorem 3.6 it means $M \xrightarrow{c!\tilde{v}@K\triangleleft R}$ for some set $R \supseteq K$. Since $M \approx N$, it follows that $N \xRightarrow{c!\tilde{v}@K\triangleleft R}$ too and, by the definition of weak actions, $N \Longrightarrow \hat{N} \xrightarrow{c!\tilde{v}@K\triangleleft R}$. Again, by Theorem 3.6 we get $N \rightarrow^* \hat{N} \downarrow_{c@K}$, that means $N \Downarrow_{c@K}$, as required.
3. *Contextuality*. It follows straightforwardly from Lemma 3.8.

□

In order to prove completeness, we use the following proposition which easily follows from the definition of reduction barbed congruence.

Proposition 3.10
If $M \cong N$ then

- $M \Downarrow_{c@K}$ if and only if $N \Downarrow_{c@K}$;
- $M \Rightarrow M'$ implies that there is N' such that $N \Rightarrow N'$ and $M' \cong N'$.

Theorem 3.11 (**Completeness**)
Let M and N be two arbitrary networks, such that $M \cong N$. Then $M \approx N$

Proof
We prove that the relation $\mathcal{R} = \{(M,N) \mid M \cong N\}$ is a bisimulation. The result will follow by co-induction.

- Suppose that $M\mathcal{R}N$ and $M \xrightarrow{\tau} M'$. By Theorem 3.6, $M \rightarrow M'$. Then, by reduction closure, there exists N' such that $N \rightarrow^* N'$, hence $N \Longrightarrow N'$.
- Suppose that $M\mathcal{R}N$ and $M \xrightarrow{c!\tilde{v}@K\triangleleft R} M'$, with $R = \{k_1, ..., k_n\}$ and $K \subseteq R$. As the action $c!\tilde{v}@K \triangleleft R$ can only be generated by an application of rule (Obs), it follows that $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$ for some l, L, r such that $d(l,k) \leq r \; \forall k \in R$ and $K = L \cap R$. Let us build a context which mimics the effect of the action $c!v@K \triangleleft R$ and also allows us to subsequently compare the residuals of the two systems under consideration. Our context has the form
$\mathcal{C}[\cdot] \stackrel{\text{def}}{=} [\cdot] \mid \prod_{i=1}^{n}(m_i[c(\tilde{x}_i).$
$[\tilde{x}_i = \tilde{v}]\bar{\mathtt{f}}^{(i)}_{k_i,r_i}\langle\tilde{x}_i\rangle]_{k_i} \mid n_i[\mathtt{f}^{(i)}(\tilde{y}_i).\bar{\mathtt{ok}}^{(i)}_{k_i,r_i}\langle\tilde{y}_i\rangle]_{k_i})$
with $r_{m_i}, r_{n_i} > 0$, $r_i \leq r_{n_i}$ and $r_i \leq r_{m_i}$, names m_i, n_i for $1 \leq i \leq n$ and channels names $\mathtt{f}^{(i)}$, $\mathtt{ok}^{(i)}$ for $1 \leq i \leq n$ fresh. Intuitively, the existence of the barbs on the fresh channels $\mathtt{f}^{(i)}$ indicates that the output action has not yet happened, whereas the presence of the barbs on channels $\mathtt{ok}^{(i)}$, together with the absence of the barbs on channels $\mathtt{f}^{(i)}$ ensures that the action has been performed.
As $\cong$ is preserved by network contexts, $M \cong N$ implies $\mathcal{C}[M] \cong \mathcal{C}[N]$. As $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$ it follows that
$\mathcal{C}[M] \Longrightarrow M' | \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i}|n_i[\bar{\mathtt{ok}}^{(i)}_{k_i,r_i}\langle\tilde{v}\rangle]_{k_i}) = \hat{M}$, with $\hat{M} \not\Downarrow_{\mathtt{f}^{(i)}@k_i}$ and $\hat{M} \Downarrow_{\mathtt{ok}^{(i)}@k_i}$, for $1 \leq i \leq n$.
The reduction sequence must be matched by a corresponding reduction sequence $\mathcal{C}[N] \Longrightarrow \hat{N}$ with $\hat{M} \cong \hat{N}$, $\hat{N} \not\Downarrow_{\mathtt{f}^{(i)}@k_i}$ and $\hat{N} \Downarrow_{\mathtt{ok}^{(i)}@k_i}$ for $1 \leq i \leq n$. The constrains on the barbs allow us to deduce the structure of the above reduction sequence
$\mathcal{C}[N] \Longrightarrow N' | \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i}|n_i[\bar{\mathtt{ok}}^{(i)}_{k_i,r_i}\langle\tilde{v}\rangle]_{k_i}) = \hat{N}$.
By barb preservation we also know that, since $M \downarrow_{c@K}$, then $N \Downarrow_{c@K}$. This implies $N \xRightarrow{c!\tilde{v}@K\triangleleft R} N'$.
As $\hat{M} \cong \hat{N}$ and the fact that $\cong$ is closed under contexts, we have that $\hat{M} \mid \prod_{i=1}^{n} p_i[\mathtt{ok}^{(i)}(\tilde{y}_i)]_{k_i} \cong$

$N' \mid \prod_{i=1}^{n} p_i[\mathtt{ok}^{(i)}(\tilde{y}_i)]_{k_i}$. By applying rule (R-Bcast) n times we get
$\hat{M} \mid \prod_{i=1}^{n} p_i[\mathtt{ok}^{(i)}(\tilde{y}_i)]_{k_i} \rightarrow^* M' \mid \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i} \mid n_i[\mathbf{0}]_{k_i} \mid p_i[\mathbf{0}]_{k_i})$
and, analogously,
$\hat{N} \mid \prod_{i=1}^{n} p_i[\mathtt{ok}^{(i)}(\tilde{y}_i)]_{k_i} \rightarrow^* N' \mid \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i} \mid n_i[\mathbf{0}]_{k_i} \mid p_i[\mathbf{0}]_{k_i})$.
By structural congruence, $M' \mid \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i} \mid n_i[\mathbf{0}]_{k_i} \mid p_i[\mathbf{0}]_{k_i}) \equiv M' \mid \mathbf{0} \equiv M'$ and $N' \mid \prod_{i=1}^{n}(m_i[\mathbf{0}]_{k_i} \mid n_i[\mathbf{0}]_{k_i} \mid p_i[\mathbf{0}]_{k_i}) \equiv N' \mid \mathbf{0} \equiv N'$. As a consequence, it follows that $M' \cong N'$ as required.

- Suppose that $M \mathcal{R} N$ and $M \xrightarrow{c?\tilde{v}@l} M'$.
The reception of a message cannot be directly observed. So we have to build a context which let the action be observable.
A context associated to the action $M \xrightarrow{c?\tilde{v}@l} M'$ could be:
$\mathcal{C}[\cdot] \stackrel{\text{def}}{=} [\cdot] | n[\bar{c}_{l,r}\langle\tilde{v}\rangle.\bar{\mathtt{f}}_{l,r}\langle\tilde{v}\rangle.\bar{\mathtt{ok}}_{l,r}\langle\tilde{v}\rangle]_k$
with $\mathtt{f}$ and $\mathtt{ok}$ fresh channels, $r \leq r_n$ and $d(l,k) \leq r$. As $\cong$ is preserved by network contexts, $\mathcal{C}[M] \cong \mathcal{C}[N]$. As $M \xrightarrow{c?\tilde{v}@l} M'$ it follows that
$\mathcal{C}[M] \Longrightarrow M' | n[\bar{\mathtt{ok}}_{l,r}\langle\tilde{v}\rangle]_k = \hat{M}$
with $\hat{M} \not\Downarrow_{\mathtt{f}@l}$ and $\hat{M} \Downarrow_{\mathtt{ok}@l}$. The reduction sequence must be matched by a corresponding reduction sequence $\mathcal{C}[N]$, so we have $\mathcal{C}[N] \Longrightarrow \hat{N}$ and $\hat{M} \cong \hat{N}$, with $\hat{N} \not\Downarrow_{\mathtt{f}@l}$ and $\hat{N} \Downarrow_{\mathtt{ok}@l}$. The constrains on the barb ensure that the action $c?\tilde{v}@l$ has been performed, so there exists N' such that $N \xRightarrow{c?\tilde{v}@l} N'$, or $N \Longrightarrow N'$, in the case rule (Lose) is applied to the node N.
As $\hat{M} \cong \hat{N}$ and $\cong$ is closed under contexts, it follows that $\hat{M} \mid p[\mathtt{ok}(\tilde{y})]_k \cong \hat{N} \mid p[\mathtt{ok}(\tilde{y})]_k$. By rule (R-Bcast) we have $\hat{M} \mid p[\mathtt{ok}(\tilde{y})]_k \rightarrow M' \mid n[\mathbf{0}]_k \mid p[\mathbf{0}]_k$ and, analogously, $\hat{N} \mid p[\mathtt{ok}(\tilde{y})]_k \rightarrow N' \mid n[\mathbf{0}]_k \mid p[\mathbf{0}]_k$. By structural congruence, $M' \mid n[\mathbf{0}]_k \mid p[\mathbf{0}]_k \equiv M' \mid \mathbf{0} \equiv M'$ and $N' \mid n[\mathbf{0}]_k \mid p[\mathbf{0}]_k \equiv N' \mid \mathbf{0} \equiv N'$, hence $M' \cong N'$, as required.
We have proved that $\cong \subseteq \approx$.

□

4. CONNECTIVITY PROPERTIES

In this section we use the E-BUM calculus to define and prove some useful connectivity properties of mobile ad-hoc networks. We use a running example, depicted in Figure 2, describing the case of an emergency due to an earthquake. The hospital (H) sends three ambulances (A1, A2, A3) to the emergency area. An ad-hoc network with a router near the epicentre (e) of the earthquake is installed to manage the communication between the ambulances.

In the following we assume that for each process P executed by a network node, it is possible to identify the set of all the intended receivers that may appear in a output action performed by P. We denote by $\mathtt{rcv}(P)$ the minimum set of locations ensuring that for each output action $\bar{c}_{L,r}\langle\tilde{v}\rangle$ performed by P it holds that $L \subseteq \mathtt{rcv}(P)$. Indeed, the tag L associated to an output action occurring in P can be either a variable or a set of locations, then we are not able to statically calculate $\mathtt{rcv}(P)$. However, since an ad-hoc network is usually designed to guarantee the communication within a specific area, we can reasonably assume that the underling protocol will always multicast messages to recipients located within the interested area and we can abstractly represent them by a finite set of locations.

4.1. Silent nodes cannot be observed

This property states that if a node sends no messages, i.e., it does not interact with the network, then an external observer cannot be aware of it. As an example, consider the interactions between the hospital and the ambulances as depicted in Figure 3. Suppose that A1 and A2 communicate with the hospital to prepare the acceptance of a patient, while A3 have no patients to be accepted and then it does not send any message to the hospital. The hospital broadcasts emergency messages to update the network about the general situation. An observer listening the communication between the network nodes cannot be

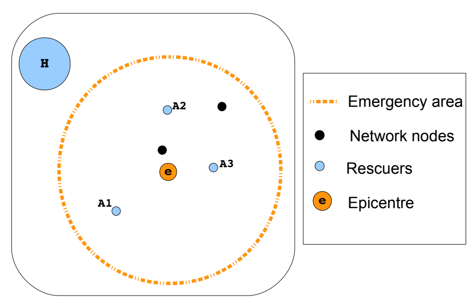

Figure 2. A mobile ad-hoc network in an earthquake area

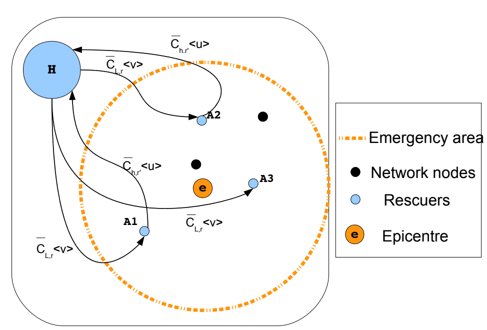

Figure 3. Message exchange between H and the ambulances

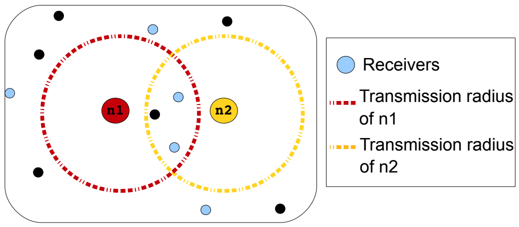

Figure 4. Example of simulation of stationary nodes

aware of the presence of A3, because it does not receive any message from that node.

Theorem 4.1 (Silent nodes are not observable)
Let P be a process which does not contain any output action. Then $n[P]_l \approx \mathbf{0}$ for any n and l.

Proof
It follows from the definition of bisimulation in which both τ-actions and input actions can be matched by weak τ-actions. □

4.2. Simulation of stationary nodes

The tag L associated to each output action allows us to express a property of simulation for stationary devices at different locations. Indeed, two stationary nodes, placed at different locations (with therefore different neighbours), but communicating with the same set of intended recipients, result to be observational equivalent (see Figure 4).

Theorem 4.2 (Simulation of stationary nodes)
Let $n[P]_{l_n}$ and $m[P]_{l_m}$ be two stationary nodes with $\delta_n = \delta_m = 0$. Assume $\mathtt{rcv}(P) = L$, $r \leq r_n$ and $r \leq r_m$ for all r associated to the output actions of P, $R = \{k \mid d(l,k) \leq r_n\}$ and $R' = \{k \mid d(l',k) \leq r_m\}$. It holds that:

1. If $R' \subseteq R$, then $n[P]_l$ simulates $m[P]_{l'}$;
2. if $R = R'$, then $n[P]_l \approx m[P]_{l'}$.

Proof

1. We prove that the relation

   $$\mathcal{S} = \{(m[P]_{l_m}, n[P]_{l_n}) \mid R' \subseteq R,\ \mathtt{rcv}(P) \subseteq L\}$$

   is a simulation.
   Suppose that $m[P]_{l_m} \xrightarrow{c!\tilde{v}@K \triangleleft \hat{R}} m[P']_{l_m}$ because $m[P]_{l_m} \xrightarrow{c_{L'}!\tilde{v}[l_m,r]} m[P']_{l_m}$ for some $L' \subseteq L$, $\hat{R} \subseteq R'$ and $K = \hat{R} \cap L'$. Hence $P \xrightarrow{\bar{c}_{L',r}\tilde{v}} P'$. Since, by hypothesis $r \leq r_n$, by rule (Snd), $n[P]_{l_n} \xrightarrow{c_{L'}!\tilde{v}[l_n,r]} n[P']_{l_n}$. Since $R' \subseteq R$, we have that $\hat{R} \subseteq R$, and hence, by rule (Obs), $n[P]_{l_n} \xrightarrow{c!\tilde{v}@K \triangleleft \hat{R}} n[P']_{l_n}$. As $\mathtt{rcv}(P') \subseteq L$, $(m[P']_{l_m}, n[P']_{l_n}) \in \mathcal{S}$ as required.
   The other cases are straightforward.
2. If $R = R'$ then $R \subseteq R'$ and $R' \subseteq R$; so, by applying the same reasoning used to prove the first item of this theorem, we can demonstrate that the relation

   $$\mathcal{S} = \{(n[P]_{l_n}, m[P]_{l_m}) \mid R = R',\ \mathtt{rcv}(P) \subseteq L\}$$

   is a bisimulation.

□

This property is useful, e.g., to minimize the number of routers within a network while ensuring the correct communication between a given set of locations. Consider, for instance, the case in which we want to determine the lowest number of routers to be installed in a specific area. If we detect that two different routers result to exhibit the same behaviour then one of them can be turn off, thus allowing us to save both power and physical resources. Figure 5 shows an example of optimal routers allocation, by turning off the router `r3`, which is not necessary since it is simulated by `r1`.

4.3. Range repeaters

Range repeaters are devices which regenerate a network signal in order to extend the range of the existing network infrastructure. Here we generalize the definition of repeater given in [10] and introduce a notion of *complete* range repeater. In the following we consider range repeaters with both one and two channels.

Definition 4.3 (Range repeater with two channels)
Let a and b be two channels, l be a location, r be a transmission radius and L be a set of locations. A *repeater with two channels* a and b relative to L with transmission radius r is a stationary device, denoted $rr[a \hookrightarrow_{L,r} b]_l$, where $a \hookrightarrow_{L,r} b$ is a process whose general recursive definition is:

$$a \hookrightarrow_{L,r} b \stackrel{\text{def}}{=} a(x).\bar{b}_{L,r}\langle x\rangle.a \hookrightarrow_{L,r} b.$$

A range repeater with two channels receives values through the input channel a and retransmits them through the output channel b to the set of intended recipients L.

A range repeater with one channel operates analogously, but input and output channels coincide.

Definition 4.4 (Range repeater with one channel)
Let c be a channel, l be a location, r be a transmission radius and L be a set of locations. A *range repeater with one channel* c relative to L with transmission radius r is a stationary device, denoted $rr[c \hookrightarrow_{L,r} c]_{l,r}$ where

$$c \hookrightarrow_{L,r} c \stackrel{\text{def}}{=} c(x).\bar{c}_{L,r}\langle x\rangle.c \hookrightarrow_{L,r} c.$$

Range repeaters are usually exploited to enlarge the transmission cell of a stationary node and, if such a node always communicates with the same set of devices, each time through the same channel, by using a range repeater we can simulate the presence of the sender in the location of the repeater.

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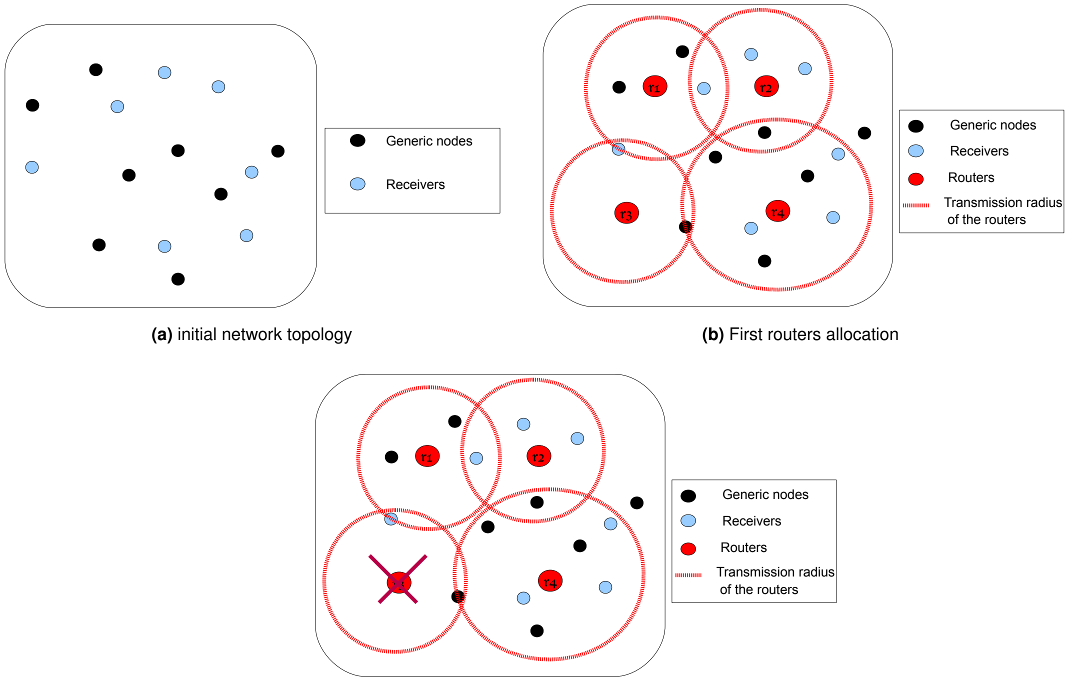


(a) initial network topology

(b) First routers allocation

(c) Optimal routers allocation

Figure 5. Example of optimized routers allocation

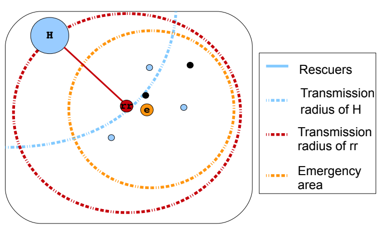


Figure 6. A range repeater in the earthquake area

In our running example, if we consider the distance between the hospital and the earthquake area, we may have that this is too large to guarantee the correct communication with the ambulances running up in the emergency area. It could be necessary to employ a range repeater powerful enough to cover all the area and, at the same time, to be reachable by the central server of the hospital (see Figure 6). If the earthquake epicenter is too distant from the hospital we can install a series of consecutive repeaters, which will connect the central server to the disaster area.

Theorem 4.5 (Range repeaters with one channel)
Let $n[P]_l$ be a stationary node such that $\mathtt{rcv}(P) = L$. Suppose that P uses exactly one channel c with a fixed transmission radius r (i.e., each output action will be of the form $c_{L',r}$ with $L' \subseteq L$) and $r \leq r_n$. Let $rr[c \hookrightarrow_{L,r} c]_k$ be a range repeater such that $d(l,k) \leq r$ and $r \leq r_{rr}$. Then:

$$n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k \text{ simulates } n[P]_k.$$

Proof
It is sufficient to prove that the relation

$$\mathcal{S} \stackrel{\text{def}}{=} \{(n[P]_k, n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k) : d(l,k) \leq r, \\ \mathtt{rcv}(P) \subseteq L \text{ and each output action of } P \text{ is of the form} \\ c_{L',r} \text{ with } L' \subseteq L\}\}$$

is a simulation.

Suppose that $n[P]_k \xrightarrow{c!\tilde{v}@K \triangleleft R} n[P']_k$ because $n[P]_k \xrightarrow{c_{L'}!\tilde{v}[k,r]} n[P']_k$ with $R \subseteq \{k : d(l,k) \leq r\}$ and $K = R \cap L$ and $P \xrightarrow{\bar{c}_{L',r}\tilde{v}} P'$ for some $L' \subseteq L$. Hence, from the fact that $n[P]_l \xrightarrow{c_{L'}!\tilde{v}[l,r]} n[P']_l$ and $rr[c \hookrightarrow_{L,r} c]_k \xrightarrow{c?\tilde{v}@k} rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k$ with $d(l,k) \leq r$, by applying rule (Bcast) we obtain

$$n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k \xrightarrow{c_{L'}!\tilde{v}[l,r]} \\ n[P']_l \mid rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k,$$

and, by applying rule (Lose),

$$n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k \xrightarrow{\tau} \\ n[P']_l \mid rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k$$

Since $rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xrightarrow{c_L!\tilde{v}[k,r]} rr[c \hookrightarrow_{L,r} c]_k$ we can deduce that $rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xrightarrow{c!\tilde{v}@K' \triangleleft R'}$

$rr[c \hookrightarrow_{L,r} c]_k$ for all $R' \subseteq \{k' : d(k,k') \le r\}$, $K' = R' \cap L'$. As $L' \subseteq L$ we can infer $rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xrightarrow{c!\tilde{v}@K \triangleleft R} rr[c \hookrightarrow_{L,r} c]_k$ and then

$$n[P]_l \mid rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xRightarrow{c!\tilde{v}@K \triangleleft R} n[P']_l \mid rr[c \hookrightarrow_{L,r} c]_k.$$

The fact that $(n[P']_k, n[P']_l \mid rr[c \hookrightarrow_{L,r} c]_k) \in \mathcal{S}$ follows immediately since $\texttt{rcv}(P') \subseteq L$ and each output action of P' is of the form $c_{L',r}$ with $L' \subseteq L$.

Suppose now that $n[P]_k \xrightarrow{c?\tilde{v}@k} n[P']_k$ because $P \xrightarrow{c\tilde{v}} P'$. Hence

$$n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k \xrightarrow{c?\tilde{v}@k} n[P]_l \mid rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k.$$

Moreover, since $n[P]_l \xrightarrow{c?\tilde{v}@l} n[P']_l$ and $rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xrightarrow{c_L!\tilde{v}[k,r]} rr[c \hookrightarrow_{L,r} c]_k$ with $d(l,k) \le r$, by applying rule (Bcast) and (Lose) we obtain

$$n[P]_l \mid rr[\bar{c}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} c]_k \xrightarrow{\tau} n[P']_l \mid rr[c \hookrightarrow_{L,r} c]_k$$

and then

$$n[P]_l \mid rr[c \hookrightarrow_{L,r} c]_k \xRightarrow{c?\tilde{v}@k} n[P']_l \mid rr[c \hookrightarrow_{L,r} c]_k.$$

The fact that $(n[P']_k, n[P']_l \mid rr[c \hookrightarrow_{L,r} c]_k) \in \mathcal{S}$ follows as above.

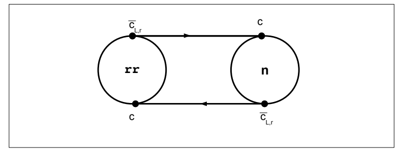
Finally, the case $n[P]_k \xrightarrow{\tau} n[P']_k$ is trivial. □

The simulation just described can be realised also with a range repeater with two channels. Using two channels, however, we need to adopt two range repeaters, respectively for input ($\texttt{in}[d \hookrightarrow_{L,r} c]_l$) and output ($\texttt{out}[c \hookrightarrow_{L,r} d]_l$) management. The diagrams in Figure 7 illustrate the use of the channels and the interaction between the nodes when range repeaters with one or two channels are adopted. This picture is inspired by the diagrams in [12], describing the behaviour of agents and the use of channels for data input and output. We emphasise that this kind of diagrams gives no information about the physical position of the nodes or about the network topology, but they only show the connections through which devices can exchange data.

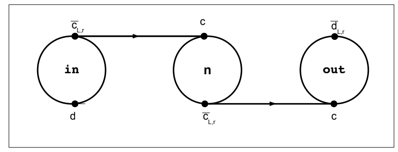
Theorem 4.6 (Range repeaters with two channels)
Let $n[P]_l$ be a stationary node such that $\texttt{rcv}(P) = L$. Suppose that P uses exactly one channel c with a fixed transmission radius r (i.e., each output action will be of the form $c_{L',r}$ with $L' \subseteq L$) and $r \le r_n$. Let $\texttt{out}[c \hookrightarrow_{L,r} d]_k$ and $\texttt{in}[d \hookrightarrow_{L,r} c]_k$ be two range repeaters such that $d(l,k) \le r$ and $r \le r_{rr}$. Then:

$$n[P]_l | \texttt{out}[c \hookrightarrow_{L,r} d]_k | \texttt{in}[d \hookrightarrow_{L,r} c]_k \text{ simulates } n[P\{d/c\}]_k$$

Proof
It is sufficient to prove that the following relation $\mathcal{S}$



(a) $rr[c \hookrightarrow_{L,r} c]_l$



(b) $\texttt{in}[d \hookrightarrow_{L,r} c]_l | n[P]_k | \texttt{out}[c \hookrightarrow_{L,r} d]_l$

Figure 7. Range repeaters: interactions between the nodes

$$\{(n[P\{d/c\}]_k, n[P]_l \,|\, \texttt{out}[c \hookrightarrow_{L,r} d]_k \,|\, \texttt{in}[d \hookrightarrow_{L,r} c]_k) : d(l,k) \le r,\ \texttt{rcv}(P) \subseteq L \text{ and each output action of } P \text{ is of the form } c_{L',r} \text{ with } L' \subseteq L\}\}$$

is a simulation.

Let $n[P\{d/c\}]_k \xrightarrow{d!\tilde{v}@K \triangleleft R} n[P'\{d/c\}]_k$ because $n[P\{d/c\}]_k \xrightarrow{d_{L'}!\tilde{v}[k,r]} n[P'\{d/c\}]_k$ with $R \subseteq \{k' : d(k',k) \le r\}$ and $K = R \cap L$ and $P \xrightarrow{\bar{c}_{L',r}\tilde{v}} P'$ for some $L' \subseteq L$. Hence, from $n[P]_l \xrightarrow{c_{L'}!\tilde{v}[l,r]} n[P']_l$ and $\texttt{out}[c \hookrightarrow_{L,r} d]_k \xrightarrow{c?\tilde{v}@k} \texttt{out}[\bar{d}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} d]_k$ with $d(l,k) \le r$ by applying rule (Bcast) we obtain

$$n[P]_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k \xrightarrow{c_{L'}!\tilde{v}[l,r]} n[P']_l \mid \texttt{out}[\bar{d}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k.$$

By rule (Lose) we have

$$n[P]_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k \xrightarrow{\tau} n[P']_l \mid \texttt{out}[\bar{d}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k.$$

By rule (Bcast) and rule (Obs) we obtain

$$n[P']_l \mid \texttt{out}[\bar{d}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k \xrightarrow{d!\tilde{v}@K' \triangleleft R'} n[P']_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k$$

for all $R' \subseteq \{k' : d(k,k') \le r\}$, $K' = R' \cap L$. From the fact that $L' \subseteq L$ we can infer

$$n[P']_l \mid \texttt{out}[\bar{d}_{L,r}\langle\tilde{v}\rangle.c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k \xrightarrow{d!\tilde{v}@K \triangleleft R} n[P']_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k$$

and then

$$n[P]_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k \xRightarrow{d!\tilde{v}@K \triangleleft R} n[P']_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k.$$

The fact that $(n[P'\{d/c\}]_k, n[P']_l \mid \texttt{out}[c \hookrightarrow_{L,r} d]_k \mid \texttt{in}[d \hookrightarrow_{L,r} c]_k) \in \mathcal{S}$ follows immediately since $\texttt{rcv}(P') \subseteq L$ and each output action of P' is of the form $c_{L',r}$ with $L' \subseteq L$.

The other cases are similar to corresponding cases of Theorem 4.5. □

We introduce now the notion of *complete range repeater*, that is a repeater which has a radius large enough to reach all its intended recipients.

Definition 4.7 (Complete range repeater)
A range repeater $rc[c \hookrightarrow_{L,r} c]_l$ is said *complete* with respect to L if $L \subseteq K$ where $K = \{k : d(l,k) \leq r\}$.

Consider the example depicted in Figure 6, where we suppose that a repeater is installed to allow the central server of the hospital to communicate with the ambulances. The repeater has been chosen in order to guarantee that its transmission radius (the red segment in the picture) covers the complete area of the disaster (the orange line). This is an example of a complete range repeater, whose radius is able to cover the entire earthquake area. The use of a complete range repeater reduces the problem of ensuring the communication between the central server and a set of locations, to the problem of ensuring the communication between only two devices `H` and `rr`: `H` will be then sure that, in whatever locations the ambulances will lie, they will be always reachable by `rr`.

5. INTERFERENCE

As mentioned in the introduction, reducing interference is one of the main goals of topology control besides direct energy conservation by restriction of transmission power.

Hereafter, we formalize the notion of *interference* for mobile ad-hoc networks. We consider two different approaches:

1. First we introduce a notion of *sender-centered interference* which measures the amount of noise caused by a certain transmission.
2. Then we formalize the concept of *receiver-centered interference* which measures the amount of noise caused on a given transmission.

These two definitions are based on the notion of observability that pertains to the semantics of our calculus: what we observe of a transmission is its ability to reach the set of its intended receivers.

Given a location l and a transmission radius r we denote as $D[l,r]$ the set of all locations in $\mathbf{Loc}$ lying within the disk with centre l and radius r.

Definition 5.1 (Interference Set)
Let $M \equiv \prod_{i \in I} n_i[P_i]_{l_i}$ be a network. Then for each $n \in \mathbf{N}$

$$I(n,M) = \begin{cases} D[l_i, r] - L, & \text{if } \exists i \in I \text{s.t. } n_i = n \text{ and} \\ & P_i = \bar{c}_{L,r}\langle\tilde{v}\rangle.P' \\ \emptyset, & \text{otherwise} \end{cases}$$

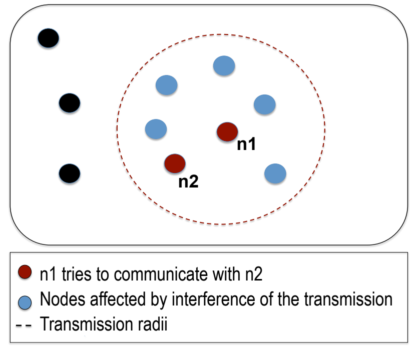


Figure 8. Example of interference caused by a transmission

5.1. Sender-centered Interference

Following the definition introduced in [3], the notion of *sender-centered interference* arises from a natural question: How many nodes are disturbed by a given communication over the network?

Consider the situation depicted in Figure 8 where a node $n1$ is intended to transmit a message to $n2$. We can define the notion of sender-centered interference as the number of nodes listening to the message, but not interested in receiving it.

Definition 5.2 (Level of Sender-centered Interference)
Let M be a network, its *level of sender-centered interference* is defined as:

$$I_{send}(M) = |\cup_{n \in \mathbf{N}} I(n,M)|.$$

If no nodes in the network are causing interference, i.e., $I_{send}(M) = 0$, then we can affirm that the network M does not provoke any interference.

The E-BUM calculus allows us to observe the case in which a transmission reaches only its intended receivers, without any interference. Indeed, we can compare the behaviour of a node communicating with a given set L of recipients, with the behaviour of the same node but broadcasting all its communications to the whole network. If the two behaviours are related by $\cong$, then we can affirm that the node transmissions do not provoke any interference, in other words they do not disturb any other node in the network.

Let us first define the broadcasting version of a process P, denoted by $brd(P)$, as follows:

- if $P = \mathbf{0}$ then $brd(P) = \mathbf{0}$
- if $P = c(\tilde{x}).P'$ then $brd(P) = c(\tilde{x}).brd(P')$
- if $P = \bar{c}_{L,r}\langle\tilde{v}\rangle.P'$ then $brd(P) = \bar{c}_{\infty,r}\langle\tilde{v}\rangle.brd(P')$
- if $P = [w_1 = w_2]Q, R$ then $brd(P) = [w_1 = w_2]brd(Q), brd(R)$.

Given a network $M \equiv \prod_{i \in I} n_i[P_i]_{l_i}$, then we write $brd(M)$ for $\prod_{i \in I} n_i[brd(P_i)]_{l_i}$.

We provide an efficient proof technique for verifying the absence of sender-centered interference for a specific node n.

Definition 5.3 (Absence of sender-centered Interference)
We say that a network M is free of *sender-centered interference* if $M \cong brd(M)$

Theorem 5.4
If M is *free of sender-centered interference* then the *level of sender-centered interference* will be always void. Formally, for each M' reachable by M then

$$I_{send}(M') = 0.$$

Proof
We proceed by contradiction, proving that, if $I_{send}(M') \neq 0$ for some M' s.t. $M \stackrel{\alpha}{\Longrightarrow} M'$ for some action α, then $M \not\approx brd(M)$. Suppose that $\exists M'$ s.t. $M \stackrel{\alpha}{\rightarrow} M'$ and $I_{send}(M') \neq 0$ (we consider a one-step execution of M, but the procedure can be easily extended to multi-step executions).

That means $M' \equiv n[\bar{c}_{L,r}\langle\tilde{v}\rangle.P]_l \mid \hat{M}$ and $I(n, M') \neq \emptyset$.

$I(n, M') \neq \emptyset$ implies $\exists k \in D[l, r] - L$.

Since $M \approx brd(M)$, $brd(M) \stackrel{\alpha}{\Longrightarrow} brd(M')$, with $M' \approx brd(M')$.

Since $\exists k \in D[l, r] - L$, that means $\exists K$ s.t. $k \in K$ and $brd(M') \xrightarrow{c!\tilde{v}@K \triangleleft K}$,

while

$M' \stackrel{c!\tilde{v}@K \triangleleft K}{\not\Longrightarrow}$, because $k \notin L$, that implies $M \not\approx brd(M)$ that is a contradiction.

□

We may be interested in verifying the absence of sender-centered interference relative to a specific set of nodes S. This can be done by defining the broadcasting version of a process P relative to S, noted $brd(S, P)$. The definition of $brd(S, P)$ is analogous to the one of $brd(P)$ except for the third item that is

- if $P = \bar{c}_{L,r}\langle\tilde{v}\rangle.P'$ then $brd(P, S) = \bar{c}_{L \cup S,r}\langle\tilde{v}\rangle.brd(P', S)$.

Given a network $M \equiv \prod_{i \in I} n_i[P_i]_{l_i}$, then we write $brd(M, S)$ for $\prod_{i \in I} n_i[brd(P_i, S)]_{l_i}$.

In this case, we obtain that a network M is free of *sender-centered interference relative to* S if $M \cong brd(M, S)$.

5.2. Receiver-centered interference

We now formalize the notion of interference at the intended receiver of a message (see [19, 7]). Consider the situation depicted in Figure 9: $n1$ is trying to transmit a message to $n2$, but $n2$ lies in the transmission cell of three other devices, which, due to their transmissions, may prevent $n2$ to receive the message sent by $n1$.

Following we introduce the level of receiver-centered interference as an upper bound on the quantity of noise possibly provoked by a network M to a location l.

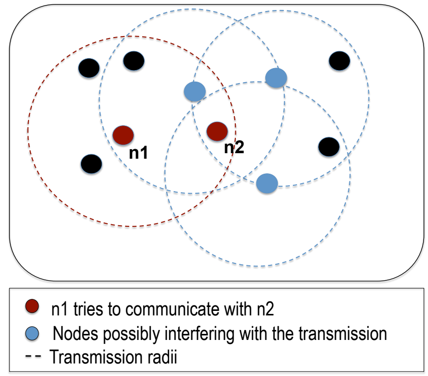


Figure 9. Example of interference suffered by a transmission

Definition 5.5 (Level of Receiver-centered Interference)
Let M be a network, the *level of receiver-centered interference* with respect to l (written $I_{rec}(l, M)$) will be:

$$I_{rec}(l, M) =\mid \{n \in \mathbf{N} \mid l \in I(n, M)\} \mid .$$

As we have done above, we use the E-BUM calculus to provide an efficient proof technique for the ideal situation where a location l is reached only by those nodes which are interested in communicating with it.

Definition 5.6 (Absence of receiver-centered Interference)
We say that a location l is *free of receiver-centered interference* with respect to a network M if,

$$M \approx brd(M, l).$$

Notice that, by contextuality, if l is *free of receiver-centered interference* with respect to M then for any node n placed at location l, and for any radius r and process P, we have

$$n[P]_l \mid M \approx n[P]_l \mid brd(M, l).$$

The following theorem proves the soundness of the above technique.

Theorem 5.7
Given a network M and a location l, if l is *free of receiver-centered interference* with respect to M then $I_{rec}(l, M') = 0$ for each M' such that $M \stackrel{\alpha}{\Longrightarrow} M'$.

Proof
Proof is analogous to that of Theorem 5.4. □

6. CONCLUSION

Ad-hoc networks is a new area of mobile communication networks that has attracted significant attention due to its challenging problems. Many researchers have proposed formal models, such as process algebras, in order to reason

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on properties and problems of this kind of networks (see, e.g., [18, 8, 13, 14]).

The main goal of our work was to provide a formal model in order to reason about the problem of limiting the power consumption of communications. One of the most critical challenges in managing mobile ad-hoc networks is actually to find a good equilibrium between network connectivity and power saving. The ability of a node to control (and hence limit) the power of its transmissions is represented by the introduction of a variable radius. Even though not all the devices have the ability of adjusting their transmission power, modern technologies are quickly evolving, and at the moment there already exist devices which enable one to choose among two or more different power levels. For this reason many researches have proposed algorithms and protocols with the aim of providing a way to decide the best transmission power for a node's communication in a given environment (see, e.g., [2, 4, 6, 9, 17, 20]).

In this paper we presented a calculus (E-BUM) which, by its characteristics of modelling broadcast, multicast and unicast communications, and the ability of a node to change its transmission power in accordance with the protocol it is executing, results to be a valid formal model for an accurate analysis, evaluation and comparison of the energy-aware protocols and algorithms specifically developed for mobile ad-hoc networks.

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