A Model for Broadcast, Unicast and Multicast Communications of Mobile Ad Hoc Networks

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***Abstract*—We present a process calculus for the analysis of Mobile Ad Hoc Networks (MANETs) and their protocols. Our calculus captures the ability of a MANET node to broadcast a message to any other node within its physical transmission range, and to move in and out of the transmission range of other nodes in the network. In order to reason about cost-effective ad hoc routing protocols, we also model unicast and multicast communications. We show how to use our calculus to prove some useful connectivity properties of MANETs.**

***Keywords*-process algebra; behavioural equivalences; manets.**

I. Introduction

A Mobile Ad Hoc Network (MANET) is a collection of wireless mobile hosts which cooperate to establish communications without using any preset infrastructure of centralized administration. Each device in a MANET is free to move independently in any direction, and will therefore change its links to other devices frequently. Each node must forward traffic unrelated to its own usage, and then be a router. The devices communicate with each other via radio transceivers through the protocol IEEE 802.11 (WiFi) [11].

Energy efficiency is an important design criteria, since mobile nodes may be powered by batteries with limited capacity. Power failure of a node not only affects the node itself but also its ability to forward packets on behalf of others and thus the overall network lifetime. For this reason, many research efforts have been devoted to develop energy-aware routing protocols.

Energy efficient routing protocols use broadcast to transmit unicast and multicast data packets between nodes. The use of unicast and multicast has many benefits including power and bandwidth saving, and lower error rates. Indeed, since radio signals are likely to overlap with others in a geographical area, a straightforward broadcasting by flooding is usually very costly and results in serious redundancy, contention, and collisions. For this reason, modern ad hoc routing protocols indicates the real addresses of transmitted packets to reduce the number of control packets (see, for instance, [1], [9]).

In this paper we present the BUM calculus for the analysis of Broadcast, Unicast and Multicast communications of mobile ad hoc networks. This is an extension of CMN (Calculus of Mobile Ad Hoc Networks) [5] where the connectivity of a node is represented by a location and a transmission radius. In our calculus broadcast communications are limited to the transmission cell of the sender, while unicast and multicast communications are modelled by specifying, for each output action, the addresses of the intended recipients of the message. We show how to use this calculus to prove some useful connectivity properties of MANETs which can be exploited to control power/energy consumption and reduce interference.

The proofs of the results presented in the paper as well as a concrete example using the AODV protocol are reported in [3].

Plan of the paper: Section II presents the BUM calculus together with its semantics. Section III defines an equivalent lts-semantics, based on bisimilarity. Connectivity properties for MANETs are studied in Section IV.

II. The calculus

We introduce the BUM calculus that models mobile ad hoc networks as a collection of nodes, running in parallel, and using channels to broadcast messages. Our calculus extends CMN [5] to support multicast and unicast communications, and allows one to model the arbitrary and unexpected connections and disconnections of nodes.

We use letters c and d for *channels*; m and n for *nodes*; l, k and h for *locations*; r for *transmission radii*; x, y and z for *variables*. *Closed values* contain nodes, locations, transmission radii and any basic value (booleans, integers, ...). *Values* include also variables. We use u and v for closed values and w for (open) values. We denote by $\tilde{v}$, $\tilde{w}$ tuples of values.

The syntax of BUM is shown in Table I. Networks are collections of nodes (which represent devices), running in parallel, using channels to communicate messages. As usual, $\mathbf{0}$ denotes the empty network and $M_1|M_2$ represents the parallel composition of two networks. Processes are sequential and live within the nodes. Process $\mathbf{0}$ denotes the inactive process. Process $c(\tilde{x}).P$ can receive a tuple $\tilde{w}$ of (closed) values via channel c and continue as $P\{\tilde{w}/\tilde{x}\}$, i.e., as P with $\tilde{w}$ substituted for $\tilde{x}$ (where $|\tilde{x}| = |\tilde{w}|$). Process $\bar{c}_L\langle\tilde{w}\rangle.P$ can send a tuple of (closed) values $\tilde{w}$ via channel c and continue as P. The tag L is used to maintain the set of locations of the intended recipients: $L = \infty$ represents a broadcast transmission, while a finite set of locations L denotes a multicast communication (unicast if L is a singleton). Syntactically, L may be a variable, but it must be a set of locations when the output prefix is ready to fire. Process $[w_1 = w_2]P, Q$ behaves as P if $w_1 = w_2$, and as Q otherwise. We write $A\langle\tilde{w}\rangle$ to denote a possibly recursive process defined as $A(\tilde{x}) \stackrel{\text{def}}{=} P$, with $|\tilde{x}| = |\tilde{w}|$, where $\tilde{x}$ contains all channels and variables that appear free in P.

Each node has a location and a transmission radius. Nodes cannot be created or destroyed. We write $n[P]^{\mu}_{\lambda,r}$ for a node named n (this is the logic location of the device in the

Networks		Processes	
M,N ::= $\mathbf{0}$	Empty network	P,Q,R ::= $\mathbf{0}$	Inactive process
$\mid M_1 \mid M_2$	Parallel composition	$\mid c(\tilde{x}).P$	Input
$\mid n[P]^{\mu}_{\lambda,r}$	Node (or device)	$\mid \bar{c}_L\langle\tilde{w}\rangle.P$	Output
		$\mid [w_1 = w_2]P, Q$	Matching
		$\mid A\langle\tilde{w}\rangle$	Recursion

TABLE I: Syntax

network), located at λ, with transmission radius r, mobility tag μ, and executing a process P. The tag μ is $\mathtt{m}$ for mobile nodes, and $\mathtt{s}$ for stationary nodes; λ denotes the physical location of the node, and it is: a generic location l if the device is a mobile node connected to the network; a fixed location l_n if the device is a stationary, connected, node n; the tag nil if the device is a mobile node disconnected; the tag $nil(l_n)$ if the device is a stationary, disconnected, node located at l_n.

In the process $c(\tilde{x}).P$, the tuple $\tilde{x}$ is bound in P. We denote by $fv(\cdot)$ and $fc(\cdot)$ free variables and channels, respectively, and identify processes and networks up to α-conversion. We denote by $\prod_{i\in I} M_i$ the parallel composition of networks M_i, for $i \in I$. We write c_l for $c_{\{l\}}$, $\bar{c}_L\langle w\rangle$ for $\bar{c}_L\langle w\rangle.\mathbf{0}$, $\mathbf{0}$ for $n[\mathbf{0}]^{\mu}_{\lambda,r}$ and $[w_1 = w_2]P$ for $[w_1 = w_2]P, \mathbf{0}$. We assume that there are no free variables in a network (while there can be free channels). Moreover, we assume that in any network each node identifier is unique.

Reduction Semantics. The dynamics of the calculus is specified by the *reduction relation* ($\rightarrow$) over networks, described in Table II. As usual, it relies on an auxiliary relation, called structural congruence ($\equiv$), such that for instance $M|N \equiv N|M$, $(M|N)|M' \equiv M|(N|M')$ and $M|\mathbf{0} \equiv M$ (see [3] for full details). We assume the possibility of comparing locations in order to determine whether a node lies or not within the transmission cell of another node. This is done through function $d(\cdot,\cdot)$ which takes two locations and returns their distance. If one of the arguments is nil or $nil(l_n)$ it returns the value ∞, modelling the fact that nodes which are not connected to the networks cannot receive any message.

Rule (R-Bcast) models the transmission of a tuple $\tilde{v}$ through a channel c_L. The set L associated to channel c indicates the locations of the intended recipients. Indeed, nodes communicate using radio frequencies that enable only broadcast messages. However a node may decide to communicate with a specific group of nodes L. The cardinality of this set indicates the kind of communication that is used: if $L = \infty$ then the recipients set is the whole network and a broadcast transmission is performed, while if L is a finite set (resp., a singleton) then a multicast (resp., a unicast) communication is realized. In our calculus transmission is a *non-blocking action*: transmission proceeds even if there are no nodes listening for messages. The messages transmitted will be received only by those nodes which lie in the transmission area of the sender. Rule (R-Move) models arbitrary and unpredictable movements of mobile nodes. δ denotes the maximum distance that a node can cover in a computational step. Specific rules modelling arbitrary connections and disconnections of nodes are also defined. Notice that stationary nodes can only disconnect or connect, they cannot move. We denote by $\longrightarrow^*$ the reflexive and transitive closure of $\longrightarrow$.

Behavioral Semantics. The central actions of our calculus are transmission and reception of messages. However, only the transmission of messages can be observed. An observer cannot be sure whether a recipient actually receives a given value. Instead, if a node receives a message, then surely someone must have sent it. Following [6], we use the term *barb* as a synonymous of observable. In our definition of barb a transmission is observable only if at least one location in the set of the intended recipients is able to receive the message.

Definition 2.1: [Barb] We write $M \downarrow_c$ if $M \equiv (n[\bar{c}_L\langle\tilde{v}\rangle.P]^{\mu}_{l,r}|M')$, when $\exists k \in L \wedge d(l,k) \leqslant r$. We write $M \Downarrow_c$ if $M \longrightarrow^* M' \downarrow_c$.
Notice that, if $M \equiv (n[\bar{c}_L\langle\tilde{v}\rangle.P]^{\mu}_{l,r}|M')$ and $M \downarrow_c$ then at least one of the recipients in L is able to receive the message.

To define our observation equivalence we will ask for the largest relation which satisfies the following properties. Let $\mathcal{R}$ be a relation over networks:

Barb preservation. $\mathcal{R}$ is *barb preserving* if $M\,\mathcal{R}\,N$ and $M \downarrow_c$ implies $N \Downarrow_c$.

Reduction closure. $\mathcal{R}$ is *reduction closed* if $M\,\mathcal{R}\,N$ and $M \longrightarrow M'$ implies that there exists N' such that $N \longrightarrow^* N'$ and $M'\,\mathcal{R}\,N'$.

Contextuality. $\mathcal{R}$ is *contextual* if $M\,\mathcal{R}\,N$ implies $\mathcal{C}[M]\,\mathcal{R}\,\mathcal{C}[N]$ for any context $\mathcal{C}[\cdot]$, where $\mathcal{C}[\cdot] ::= [\cdot] \mid [\cdot]|M \mid M|[\cdot]$.

Definition 2.2: [Reduction barbed congruence] Reduction barbed congruence, written $\cong$, is the largest symmetric relation over networks, which is reduction closed, barb preserving, and contextual.

III. Bisimulation-based Proof Method

We develop a proof technique for the relation $\cong$. More precisely, we define a LTS semantics for BUM terms, which is built upon the rules in Table III. Transitions for processes are of the form $P \xrightarrow{\eta} P'$, where η ranges over input and output actions of the form $c\tilde{v}$ and $\bar{c}_L\tilde{v}$, respectively. Transitions for networks are of the form $M \xrightarrow{\gamma} M'$, where γ is as follows: $\gamma ::= c?\tilde{v}@l \mid c_L!\tilde{v}[l,r] \mid c!\tilde{v}@K \mid \tau$.

Rules for processes are simple and they do not need deeper explanations. Let us illustrate the rules for networks. Rule (Snd) models the sending, with transmission radius r, of the

$$\text{(R-Bcast)}\ \frac{\forall i \in I. d(l,l_i) \le r \wedge |\tilde{x}| = |\tilde{v}|}{n[\bar{c}_L\langle\tilde{v}\rangle.P]^{\mu}_{l,r} | \prod_{i\in I} n_i[c(\tilde{x_i}).P_i]^{\mu_i}_{l_i,r_i} \to n[P]^{\mu}_{l,r} | \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x_i}\}]^{\mu_i}_{l_i,r_i}}$$

$$\text{(R-sDisc)}\ \frac{-}{n[P]^{\mathtt{s}}_{l_n,r} \to n[P]^{\mathtt{s}}_{nil(l_n),r}} \quad \text{(R-sConn)}\ \frac{-}{n[P]^{\mathtt{s}}_{nil(l_n),r} \to n[P]^{\mathtt{s}}_{l_n,r}}$$

$$\text{(R-mDisc)}\ \frac{-}{n[P]^{\mathtt{m}}_{l,r} \to n[P]^{\mathtt{m}}_{nil,r}} \quad \text{(R-mConn)}\ \frac{-}{n[P]^{\mathtt{m}}_{nil,r} \to n[P]^{\mathtt{m}}_{l,r}}$$

$$\text{(R-Move)}\ \frac{d(l,k) \le \delta}{n[P]^{\mathtt{m}}_{l,r} \to n[P]^{\mathtt{m}}_{k,r}} \quad \text{(R-Par)}\ \frac{M \to M'}{M|N \to M'|N} \quad \text{(R-Struct)}\ \frac{M \equiv N\ N \to N'\ N' \equiv M'}{M \to M'}$$

TABLE II: Reduction Semantics

tuple $\tilde{v}$ through channel c to a specific set L of recipients, while rule (Rcv) models the reception of $\tilde{v}$ at l via channel c. Rule (Bcast) models the broadcast message propagation: all the nodes lying within the transmission cell of the transmitter may receive the message, regardless of the fact that they are in L. Rule (Obs) models the observability of a transmission: every output action may be detected (and hence *observed*) by any node located within the transmission cell of the sender. We are interested in observing the output actions reaching at least one of the intended recipients. The action $c!\tilde{v}@K$ represents the transmission of the tuple $\tilde{v}$ of messages via c to a set K of recipients in L, located within the transmission cell of the transmitter. When $K \neq \emptyset$ this is an observable action corresponding to the barb $\downarrow_c$. Rule (Lose) models message loss. Rule (Move) models migration of a mobile node from a location l to a new location k, where δ represents the maximum distance that a node can cover in a single computational step. Arbitrary and unpredictable connections and disconnections of both stationary and mobile nodes are modeled by rules (sDisc), (sConn), (mDisc), (mConn). Note that while a mobile node may reconnect in any arbitrary location, a stationary one is bound to its specific location.

Lemma 3.1: Let M be a network.

- If $M \xrightarrow{c?\tilde{v}@l} M'$, then there are n, P, μ, l, r and M_1, such that $M \equiv (n[c(\tilde{x}).P]^{\mu}_{l,r}|M_1)$ and $M' \equiv (n[P\{\tilde{v}/\tilde{x}\}]^{\mu}_{l,r}|M_1)$.
- If $M \xrightarrow{c_L!\tilde{v}[l,r]} M'$, then there are n, P, μ, l, r, M_1 and I (possibly empty), with n_i, P_i, μ_i, l_i, r_i, with $d(l,l_i) \le r$ for all $i \in I$, such that: $M \equiv (n[\bar{c}_L\langle\tilde{v}\rangle.P]^{\mu}_{l,r}| \prod_{i\in I} n_i[c(\tilde{x_i}).P_i]^{\mu_i}_{l_i,r_i}|M_1)$ and $M' \equiv (n[P]^{\mu}_{l,r}| \prod_{i\in I} n_i[P_i\{\tilde{v}/\tilde{x_i}\}]^{\mu_i}_{l_i,r_i}|M_1)$.

Lemma 3.2: Let M be a network. It holds that (i) $M \downarrow_c$ if and only if $M \xrightarrow{c!\tilde{v}@K}$ for some tuple of values $\tilde{v}$ and set of locations K; (ii) if $M \xrightarrow{\tau} M'$ then $M \longrightarrow M'$; (iii) if $M \longrightarrow M'$ then $M \xrightarrow{\tau}\equiv M'$.

We define a labelled bisimilarity that is a complete characterization of our reduction barbed congruence. It is built upon the following actions: $\alpha ::= c?\tilde{v}@l \mid c!\tilde{v}@K \mid \tau$.

Since we are interested in weak behavioral equivalences, that abstract over τ-actions, we introduce the notion of *weak action*. We denote by $\Rightarrow$ the reflexive and transitive closure of $\xrightarrow{\tau}$; we use $\xRightarrow{c?\tilde{v}@l}$ to denote $\Rightarrow\xrightarrow{c?\tilde{v}@l}\Rightarrow$; we use $\xRightarrow{c?\tilde{v}@F}$ to denote $\xRightarrow{c?\tilde{v}@l_1} \dots \xRightarrow{c?\tilde{v}@l_n}$ for $F = \{l_1, \dots, l_n\}$; we use $\xRightarrow{c!\tilde{v}@K}$ to denote $\xRightarrow{c?\tilde{v}@F_1}\xrightarrow{c!\tilde{v}@K_1}\xRightarrow{c?\tilde{v}@F'_1} \dots \xRightarrow{c?\tilde{v}@F_n}\xrightarrow{c!\tilde{v}@K_n}\xRightarrow{c?\tilde{v}@F'_n}$ for $K = \bigcup_{i=1}^n K_i$, $F = \bigcup_{i=1}^n (F_i \cup F'_i)$ and $F \cap K = \emptyset$; finally, $\xRightarrow{\hat{\alpha}}$ denotes $\Rightarrow$ if $\alpha = \tau$ and $\xRightarrow{\alpha}$ otherwise.

Notice that $\xRightarrow{c!\tilde{v}@K}$ means that a distributed observer receiving an instance of message $\tilde{v}$, at each location in K, in several computational steps, cannot assume that those messages belong to the same broadcast transmission, but they may be different transmissions of the same message. The presence of the weak input actions $\xRightarrow{c?\tilde{v}@F_i}$ are due to the fact that we want to ignore all the inputs executed by each location which is not included in the set of the intended receivers.

Definition 3.3: [Labelled bisimilarity] A binary relation $\mathcal{R}$ over networks is a *simulation* if $M\mathcal{R}N$ implies:

- If $M \xrightarrow{\alpha} M'$, $\alpha \neq c?\tilde{v}@l$, then $\exists\ N'$ such that $N \xRightarrow{\hat{\alpha}} N'$ with $M'\mathcal{R}N'$;
- If $M \xrightarrow{c?\tilde{v}@l} M'$ then $\exists\ N'$ such that either $N \xRightarrow{c?\tilde{v}@l} N'$ with $M'\mathcal{R}N'$ or $N \Rightarrow N'$ with $M'\mathcal{R}N'$.

We say that N *simulates* M if there is some simulation $\mathcal{R}$ such that $M\mathcal{R}N$. A relation $\mathcal{R}$ is a *bisimulation* if both $\mathcal{R}$ and its converse are simulations. *Labelled bisimilaty*, written $\approx$, is the largest bisimulation over networks.

Theorem 3.4: Let M and N be two networks. $M \cong N$ if and only if $M \approx N$.

IV. Properties of Mobile Ad Hoc Networks

In this section we use the BUM calculus to define and prove some useful properties of MANETs.

First observe that all the properties described in [5] can be proved also using our model; the properties we introduce here cannot be expressed in CMN. We assume that for each process P executed by a network node, it is possible to identify the set of all the intended recipients that may appear in an output action performed by P. We denote by $\mathtt{rcv}(P)$ the minimum set of locations ensuring that for each output action $\bar{c}_L\langle\tilde{w}\rangle$ performed by P it holds that $L \subseteq \mathtt{rcv}(P)$. Indeed, the tag L

Rules for Processes

$$\text{(Output)}\ \frac{-}{\bar{c}_L\langle\tilde{v}\rangle.P \xrightarrow{\bar{c}_L\tilde{v}} P} \qquad \text{(Input)}\ \frac{-}{c(\tilde{x}).P \xrightarrow{c\tilde{v}} P\{\tilde{v}/\tilde{x}\}} \qquad \text{(Rec)}\ \frac{P\{\tilde{v}/\tilde{x}\} \xrightarrow{\eta} P' \quad A(\tilde{x}) \stackrel{\text{def}}{=} P}{A\langle\tilde{v}\rangle \xrightarrow{\eta} P'}$$

$$\text{(Then)}\ \frac{P \xrightarrow{\eta} P'}{[\tilde{v} = \tilde{v}]P,Q \xrightarrow{\eta} P'} \qquad \text{(Else)}\ \frac{Q \xrightarrow{\eta} Q' \quad \tilde{v_1} \neq \tilde{v_2}}{[\tilde{v_1} = \tilde{v_2}]P,Q \xrightarrow{\eta} Q'}$$

Rules for Networks

$$\text{(Snd)}\ \frac{P \xrightarrow{\bar{c}_L\tilde{v}} P'}{n[P]^{\mu}_{l,r} \xrightarrow{c_L!\tilde{v}[l,r]} n[P']^{\mu}_{l,r}} \qquad \text{(Rcv)}\ \frac{P \xrightarrow{c\tilde{v}} P'}{n[P]^{\mu}_{l,r} \xrightarrow{c?\tilde{v}@l} n[P']^{\mu}_{l,r}} \qquad \text{(Par)}\ \frac{M \xrightarrow{\gamma} M'}{M|N \xrightarrow{\gamma} M'|N}$$

$$\text{(Bcast)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M' \quad N \xrightarrow{c?\tilde{v}@l'} N' \quad d(l,l') \leq r}{M|N \xrightarrow{c_L!\tilde{v}[l,r]} M'|N'} \qquad \text{(Obs)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M' \quad K \subseteq \{k : d(l,k) \leq r \wedge k \in L\} \quad K \neq \emptyset}{M \xrightarrow{c!\tilde{v}@K} M'}$$

$$\text{(Lose)}\ \frac{M \xrightarrow{c_L!\tilde{v}[l,r]} M'}{M \xrightarrow{\tau} M'} \qquad \text{(sDisc)}\ \frac{-}{n[P]^{\mathtt{s}}_{l_n,r} \xrightarrow{\tau} n[P]^{\mathtt{s}}_{nil(l_n),r}} \qquad \text{(sConn)}\ \frac{-}{n[P]^{\mathtt{s}}_{nil(l_n),r} \xrightarrow{\tau} n[P]^{\mathtt{s}}_{l_n,r}}$$

$$\text{(Move)}\ \frac{d(l,k) \leq \delta}{n[P]^{\mathtt{m}}_{l,r} \xrightarrow{\tau} n[P]^{\mathtt{m}}_{k,r}} \qquad \text{(mDisc)}\ \frac{-}{n[P]^{\mathtt{m}}_{l,r} \xrightarrow{\tau} n[P]^{\mathtt{m}}_{nil,r}} \qquad \text{(mConn)}\ \frac{-}{n[P]^{\mathtt{m}}_{nil,r} \xrightarrow{\tau} n[P]^{\mathtt{m}}_{l,r}}$$

TABLE III: LTS rules

associated to an output action occurring in P can be either a variable or a set of locations, then we are not able to statically calculate $\mathtt{rcv}(P)$. However, since an ad hoc network is usually designed to guarantee the communications within a specific area, we can reasonably assume that the underling protocol will always multicast messages to recipients located within the interested area and we can abstractly represent them by a finite set of locations.

Radius of maximum observability. We can define a "*radius of maximum observability*", that is a radius ensuring the correct reception of a message from all the locations in the recipients set. In particular, we define the "*minimum radius of maximum observability*", which corresponds to the distance between the sender of the message and the most distant recipient. Clearly, this property is relevant only for stationary nodes, since mobile nodes can always move within the transmission cell of the transmitter to receive the communication.

Theorem 4.1: [Radius of maximum observability] Let $n[P]^{\mathtt{s}}_{\lambda,r}$ be a stationary node located at l_n such that $\mathtt{rcv}(P) = L$ and $d(l_n,k) \leqslant r$ for all $k \in L$. Then $n[P]^{\mathtt{s}}_{\lambda,r} \approx n[P]^{\mathtt{s}}_{\lambda',r'}$ $\forall r' \geq r$. In this case, we say that r is a radius of maximum observability for $n[P]^{\mathtt{s}}_{\lambda,r}$.

Definition 4.2: [Minimum radius of maximum observability] Let $n[P]^{\mathtt{s}}_{\lambda,r}$ be a stationary node located at l_n such that $\mathtt{rcv}(P) = L$. We say that r is the *minimum radius of maximum observability* for $n[P]^{\mathtt{s}}_{\lambda,r}$, if r is a radius of maximum observability, and for all $r' < r$ it holds that r' is not a radius of maximum observability for $n[P]^{\mathtt{s}}_{\lambda,r}$.

The notion of minimum radius of maximum observability is relevant when dealing with the problem of power saving, since it provides us a way of reducing the transmission power of a node without loosing connectivity.

Simulation of stationary nodes in different locations. The tag L associated to each output action allows us to express a property of simulation for stationary devices in different locations. Two stationary nodes, placed at different locations (with therefore different neighbors), but communicating with the same set of intended recipients, result to be observational equivalent.

Theorem 4.3: [Simulation of stationary nodes at different locations] Let $n[P]^{\mathtt{s}}_{\lambda,r}$ and $m[P]^{\mathtt{s}}_{\lambda',r'}$ be two stationary nodes located at l_n and l_m, respectively. Assume $\mathtt{rcv}(P) = L$, $K = \{k \mid d(l_n,k) \leq r \wedge k \in L\}$ and $K' = \{k \mid d(l_m,k) \leq r' \wedge k \in L\}$. It holds that

1) If $K' \subseteq K$, then $n[P]^{\mathtt{s}}_{\lambda,r}$ simulates $m[P]^{\mathtt{s}}_{\lambda',r'}$;
2) If $K = K'$, then $n[P]^{\mathtt{s}}_{\lambda,r} \approx m[P]^{\mathtt{s}}_{\lambda',r'}$.

This property is useful, e.g., to minimize the number of routers within a network area while ensuring the correct communication between a given set of locations. If two different routers exhibit the same behaviour, then one of them can be turn off, thus allowing one to save both power and physical resources.

Range repeaters. Range repeaters are devices which regenerate a network signal in order to extend the range of the existing network infrastructure. Here we generalize the definition of repeater given in [5] and introduce a notion of *complete* range repeater. For simplicity we consider here range repeaters with one channel; range repeaters with two channels, receiving values through an input channel and retransmitting them through an output channel, are studied in [3].

Definition 4.4: [Range repeater] Let c be a channel, l_{rr} be a fixed location, r be a transmission radius and L be a set of locations. A *repeater with one channels* c *relative to* L is a stationary device, denoted by $rr[c \hookrightarrow_L c]^{\mathtt{s}}_{l_{rr},r}$, where

$$c \hookrightarrow_L c \stackrel{\text{def}}{=} c(x).\bar{c}_L\langle x\rangle.c \hookrightarrow_L c.$$

Range repeaters are usually exploited to enlarge the transmission cell of a stationary node and, if such a node always communicates with the same set of devices, each time through the same channel, by using a range repeater we can simulate the presence of the sender in the location of the repeater.

Theorem 4.5: [Range repeaters] Let $n[P]^{\mathtt{s}}_{l,r}$ be stationary with $fc(P) \subseteq \{c\}$ and $\mathtt{rcv}(P) = L$. Let $rr[c \hookrightarrow_L c]^{\mathtt{s}}_{l_{rr},r'}$ be a range repeater with $d(l, l_{rr}) \leq r$ and $d(l, l_{rr}) \leq r'$. Then $n[P]^{\mathtt{s}}_{l,r} \mid rr[c \hookrightarrow_L c]^{\mathtt{s}}_{l_{rr},r'}$ simulates $n[P]^{\mathtt{s}}_{l_{rr},r'}$.

We introduce the notion of *complete range repeater*, that is a repeater which has a radius large enough to reach all its intended recipients.

Definition 4.6: [Complete range repeater] A range repeater $rc[c \hookrightarrow_L c]^{\mathtt{s}}_{l_{rc},r}$ is said *complete* with respect to L if $L \subseteq K$ where $K = \{k : d(l_{rc}, k) \leqslant r\}$.

Theorem 4.7: [Complete range repeaters] Let $n[P]^{\mathtt{s}}_{l,r}$ be a stationary node such that $fc(P) \subseteq \{c\}$ for some channel c and $\mathtt{rcv}(P) = L$. Let $rc[c \hookrightarrow_L c]^{\mathtt{s}}_{l_{rc},r'}$ be a complete range repeater with respect to L and $d(l, l_{rc}) \leq r$. Then all the recipients in L are reachable by n, i.e., $\forall k \in L$, it holds that $d(k, l) \leqslant (r + r')$.

Interference. As stated in the introduction, one of the most critical problems in managing an ad hoc network is the power consumption. A technique based on topology control can be used to reduce the initial topology of the network in order to save energy and extend the lifetime of the network. Choosing a low transmission power for a node will reduce its connectivity within the network, but it will also reduce its power consumption. The foremost approach to achieve substantial energy consumption while preserving network connectivity consists in minimizing interference between network nodes.

Following the definition of interference introduced in [2], we can define the level of interference relative to a transmission as the number of network nodes listening the transmitted message, but not interested in receiving it.

Definition 4.8: [Interference] Let $c_L!\tilde{v}[l, r]$ be an output action, and $K = \{k : d(l, k) \leq r\}$. The *level of interference* relative to $c_L!\tilde{v}[l, r]$ is defined as: $I(c_L!\tilde{v}[l, r]) = |K - L|$.

If the set of nodes not interested in receiving the message is empty, i.e., if $I(c_L!\tilde{v}[l, r]) = 0$, then we say that the communication is interference-free.

Using our model, we can formalize the absence of interference for a network node by comparing the behaviour of that node with the behaviour of the same node but broadcasting its messages to the whole network. If the two behaviours are bisimilar, then we can state that only the specified recipients of the node are able to listen the transmitted messages, i.e., the node transmissions are not received by any other node which is not in the recipients set.

We define the broadcasting version of a process P, noted $brd(P)$, as:

- if $P = \mathbf{0}$ then $brd(P) = \mathbf{0}$;
- if $P = c(\tilde{x}).P'$ then $brd(P) = c(\tilde{x}).brd(P')$;
- if $P = \bar{c}_L\langle\tilde{v}\rangle.P'$ then $brd(P) = \bar{c}_\infty\langle\tilde{v}\rangle.brd(P')$.
- if $P = [w_1 = w_2]Q, R$ then $brd(P) = [w_1 = w_2]brd(Q), brd(R)$.

Definition 4.9: [Absence of Interference] A node $n[P]^{\mu}_{l,r}$ is *interference-free* if $n[P]^{\mu}_{l,r} \cong n[brd(P)]^{\mu}_{l,r}$.

Theorem 4.10: If $n[P]^{\mu}_{l,r}$ is *interference-free* then for all output actions $c_L!\tilde{v}[l, r]$ performed by $n[P]^{\mu}_{l,r}$ it holds that $I(c_L!\tilde{v}[l, r]) = 0$.

V. Conclusion

Ad hoc networks is a new area of mobile communication networks that has attracted significant attention due to its challenging research problems. Many researchers have proposed formal models [10], [4], [7], [8], [5] in order to reason on MANETs properties and problems. The main limitations of CMN are the absence of rules for arbitrary connections and disconnections of nodes, and the impossibility of representing multicast and unicast communications.

In this paper we extended CMN by associating a tag to each transmission; the tag represents a set of recipients and enables us to prove some important connectivity properties.

Acknowledgments

Work partially supported by the MIUR Project IPODS "Interacting Processes in Open-ended Distributed Systems".

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