Storia dell’informatica

Le radici logiche dell’informatica.

Parte III: L’algebrizzazione della logica

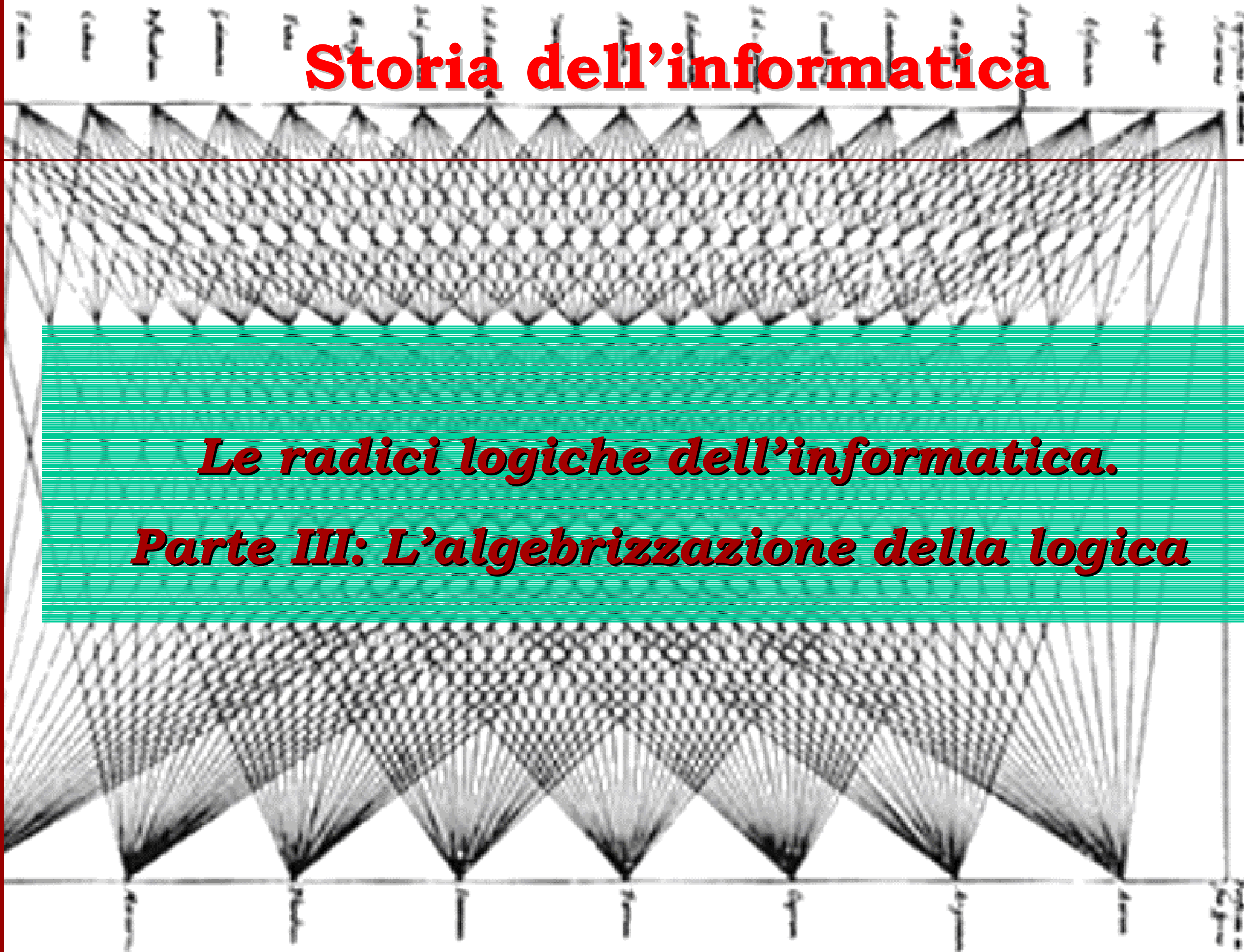


Illustration from Leibniz's Ars Combinatoria, or De Arte Combinatoria.

Gottfried Wilhelm Leibniz (1646-1716)

Educated in law and philosophy, and serving as factotum to two major German noble houses (one becoming the British royal family while he served it), Leibniz played a major role in the European politics and diplomacy of his day.

He occupies an equally large place in both the history of philosophy and the history of mathematics. He invented calculus independently of Newton, and his notation is the one in general use since.

In philosophy, he is most remembered for optimism, i.e., his conclusion that our universe is, in a restricted sense, the best possible one God could have made.

He was, along with Descartes and Spinoza, one of the three great 17th century rationalists, but his philosophy also both looks back to the Scholastic tradition and anticipates logic and analysis.

Leibniz’s Writings

When Leibniz died at the age of 70, he left behind an extraordinarily extensive and widespread collection of papers, only a small part of which had been published during his lifetime. Leibniz wrote in three languages: scholastic Latin, French, and (least often) German.

The bibliography of Leibniz’s printed works contains 882 items, but only 325 papers had been published by Leibniz himself.

Much more impressive than this group of printed works is Leibniz’s correspondence. The catalogue Bodemann [1889] contains more than 15,000 letters which Leibniz exchanged with more than 1,000 correspondents all over Europe, and the whole correspondence can be estimated to comprise some 50,000 pages.

Furthermore, there is the collection of Leibniz’s scientific, historical, and political manuscripts in the Leibniz-Archive in Hannover which was described in another catalogue (Bodemann [1895]).

Leibniz’s Logical Works

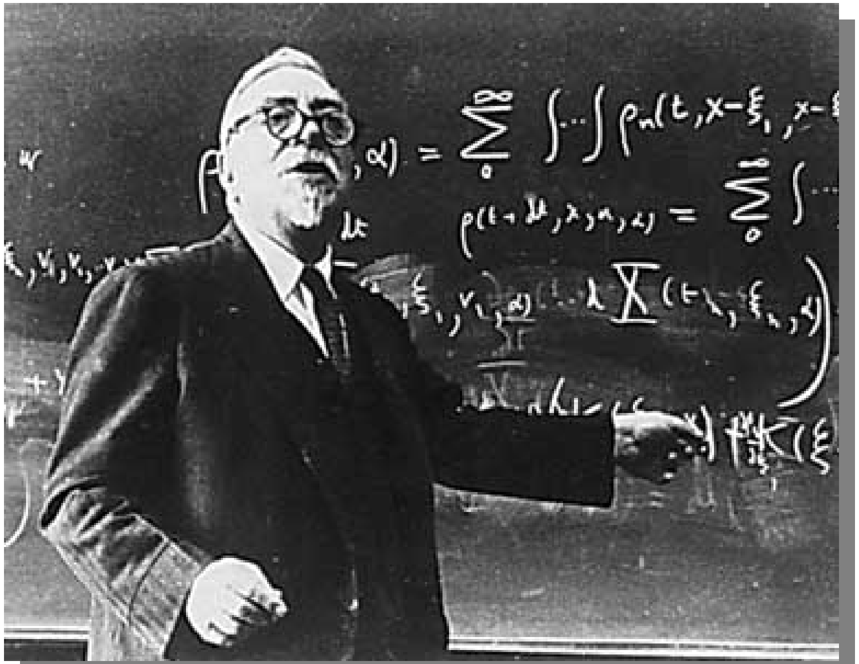
Throughout his life, Leibniz published not a single line on *logic*, except perhaps for the mathematical Dissertation “De Arte Combinatoria” or the Juridical Disputation “De Conditionibus”.

Leibniz’s main aim in logic was to extend Aristotelian syllogistics to a “Universal Calculus”. And although we know of several drafts for such a logic which had been elaborated with some care and which seem to have been composed for publication, Leibniz appears to have remained unsatisfied with these attempts.

So Leibniz’s genuinely logical essays appeared only posthumously.

It was not until 1903 that the majority of the logical works were published in Couturat’s most valuable edition of the *Opuscules et fragments inédits de Leibniz*.

Leibniz’s Project



«The history of the modern computing machine goes back to Leibniz and Pascal. Indeed, the general idea of a computing machine is nothing but a mechanization of Leibniz's ***calculus ratiocinator****.»*

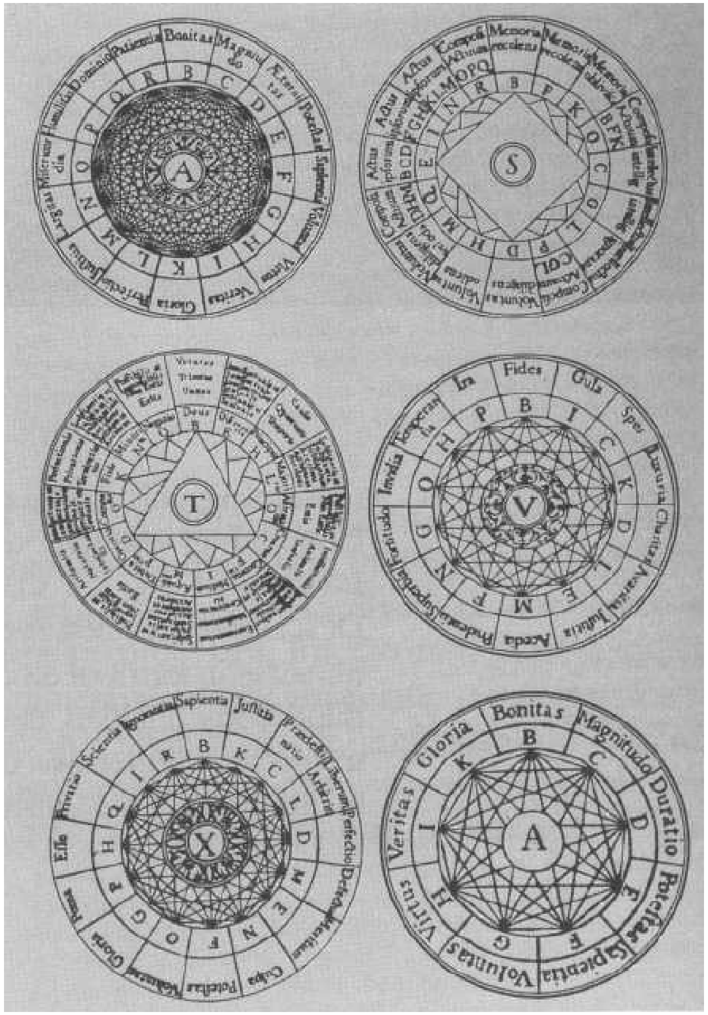
(Norbert Wiener, 1948)

Precursors of Leibniz: Raymond Lull (1235-1315)

In his *Ars Magna* the Spanish theologian **Raymond Lull** (1235-1315) used geometrical diagrams and primitive logical devices to try to demonstrate the truths of Christianity.

He believed that each domain of knowledge involves a finite number of basic principles, so that by enumerating the permutations of these basic principles in pairs, triples, and larger combinations a list of the basic building blocks for theological discourse could be assembled.

Lullian Circles

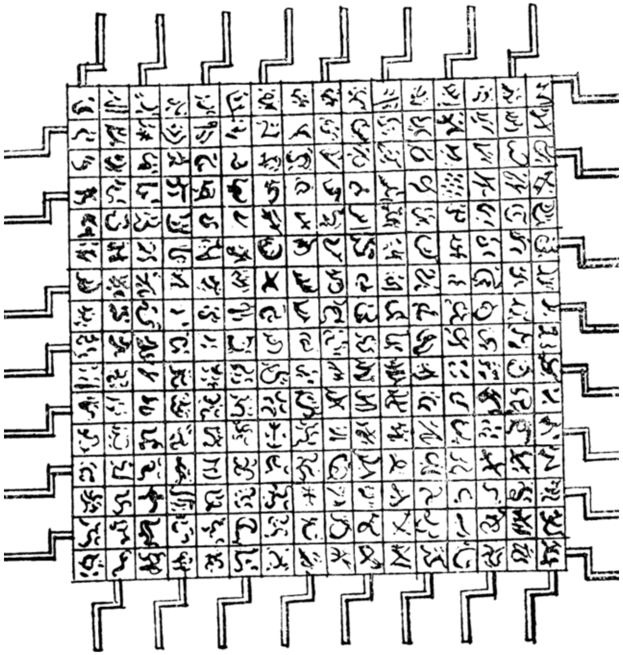


Lull mechanized the process of forming these permutations by constructing devices with two or more concentric circles, each listing the basic principles around the circumference. The permutations could then be formed by spinning the dials so as to line up different permutations.

The method was an early attempt to use logical means to produce knowledge. Lull hoped to show that Christian doctrines could be obtained artificially from a fixed set of preliminary ideas.

The Machine of the Grand Academy of Lagado...

«Era un grande quadrato di venti piedi per venti, collocato al centro della stanza. La sua superficie era fatta di piccoli cubi di legno, di dimensioni variabili, ma grossi in media come un ditale, e legati per mezzo di un filo di ferro. Su ciascuna faccia di questi cubi era attaccato un pezzo di carta con su scritta una parola in laputiano. C'erano tutte le parole della lingua, nei loro differenti tempi, modi o casi, ma senza alcun ordine. Il professore mi pregò di fare attenzione, perché stava per far funzionare la macchina. A un ordine, ciascun allievo prese una delle quaranta manovelle di ferro disposte ai lati del telaio e le fece fare un giro brusco, in modo che la disposizione delle parole si trovò completamente cambiata; poi trentasei di loro furono incaricati di leggere a bassa voce le differenti righe che apparivano sul quadro, e quando trovavano tre o quattro parole che, messe l'una di seguito all'altra, costituivano un elemento di frase, le dettavano ai quattro altri giovani che servivano come segretari. Questa operazione fu ripetuta tre o quattro volte, e l'apparecchio era concepito in modo che, a ogni giro di manovella, le parole formassero combinazioni diverse, col girare dei cubi su se stessi.»



(J. Swift, *I Viaggi di Gulliver*, 1726. Parte III, Cap. V)

... and Its "Markov Chain" Realization
(C. E. Shannon, *A Mathematical Theory of Communication*, 1948)

1. Zero-order approximation (symbols independent and equiprobable).

   XFOML RXKHRJFFJUJ ZLPWCFWKCYJ FFJEYVKCQSGHYD QPAAMKBZAACIBZL-HJQD.

2. First-order approximation (symbols independent but with frequencies of English text).

   OCRO HLI RGWR NMIELWIS EU LL NBNESEBYA TH EEI ALHENHTTPA OOBTTVA NAH BRL.

3. Second-order approximation (digram structure as in English).

   ON IE ANTSOUTINYS ARE T INCTORE ST BE S DEAMY ACHIN D ILONASIVE TU-COOWE AT TEASONARE FUSO TIZIN ANDY TOBE SEACE CTISBE.

4. Third-order approximation (trigram structure as in English).

   IN NO IST LAT WHEY CRATICT FROURE BIRS GROCID PONDENOME OF DEMONS-TURES OF THE REPTAGIN IS REGOACTIONA OF CRE.

5. First-order word approximation. Rather than continue with tetragram, ... , n-gram structure it is easier and better to jump at this point to word units. Here words are chosen independently but with their appropriate frequencies.

   REPRESENTING AND SPEEDILY IS AN GOOD APT OR COME CAN DIFFERENT NAT-URAL HERE HE THE A IN CAME THE TO OF TO EXPERT GRAY COME TO FURNISHES THE LINE MESSAGE HAD BE THESE.

6. Second-order word approximation. The word transition probabilities are correct but no further structure is included.

   THE HEAD AND IN FRONTAL ATTACK ON AN ENGLISH WRITER THAT THE CHAR-ACTER OF THIS POINT IS THEREFORE ANOTHER METHOD FOR THE LETTERS THAT THE TIME OF WHO EVER TOLD THE PROBLEM FOR AN UNEXPECTED.

The first two samples were constructed by the use of a book of random numbers in conjunction with (for example 2) a table of letter frequencies. This method might have been continued for (3), (4) and (5), since digram, trigram and word frequency tables are available, but a simpler equivalent method was used.

To construct (3) for example, one opens a book at random and selects a letter at random on the page. This letter is recorded. The book is then opened to another page and one reads until this letter is encountered. The succeeding letter is then recorded. Turning to another page this second letter is searched for and the succeeding letter recorded, etc. A similar process was used for (4), (5) and (6). It would be interesting if further approximations could be constructed, but the labor involved becomes enormous at the next stage.

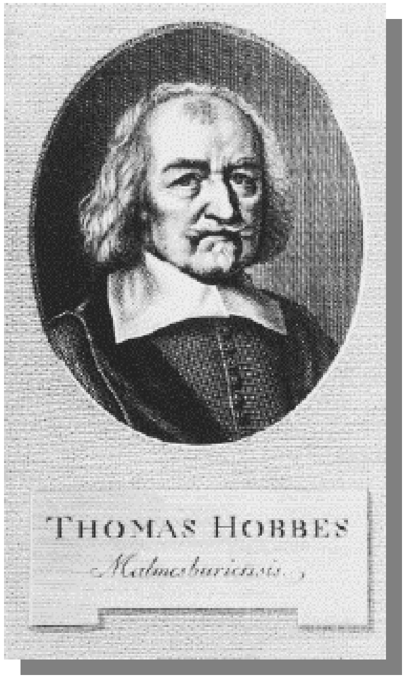
Hobbes' Legacy

«Quel profondissimo scrutatore dei principi in tutte le cose che fu Thomas Hobbes, sostenne giustamente che ogni operazione della nostra mente è un calcolo e che da essa si ottiene o la somma addizionando o la differenza, sottraendo[…]

Come sono dunque due i segni primari degli algebristi e degli analisti, il + e il -, così due sono le copule, è e non-è: nel primo caso la mente compone, nel secondo divide.»

(G. W. Leibniz, *De arte combinatoria,* 1666)

Thomas Hobbes (1588-1679)



"Per ragionamento, poi, io intendo il calcolo [*computatio*]. Calcolare significa raccogliere la somma di più cose aggiunte l'una all'altra, oppure, se si detrae una cosa dall'altra, conoscere quel che rimane. Quindi ragionare è il medesimo che addizionare e sottrarre, e se poi uno vi aggiungesse moltiplicare e dividere, non mi opporrei, dal momento che la moltiplicazione equivale all'addizione di termini uguali, e la divisione alla sottrazione di termini uguali tante volte quanto è possibile. Ogni ragionamento quindi si riduce a due operazioni dell'animo, l'addizione e la sottrazione." [*Computatio sive logica*, p. 3]

"Non si deve dunque pensare che il calcolare, cioè il ragionare, abbia luogo soltanto con i numeri, come se l'uomo si distinguesse dal resto degli esseri animati per la sola facoltà del numerare […] infatti si può aggiungere o togliere grandezza a grandezza, corpo a corpo, moto a moto, tempo a tempo, gradi di qualità a gradi di qualità, azione ad azione, concetto a concetto, proporzione a proporzione, discorso a discorso, nome a nome (nelle quali attività è contenuto ogni genere di filosofia)."

In the late 17th century, logic both as an academic discipline and as a formal science basically coincided with Aristotelian syllogistics.

Thus also Leibniz's logical work was to a large extent related to the theory of the syllogism, but at the same time it aimed at the construction of a much more powerful "universal calculus".

This calculus should primarily serve as a general tool for determining which formal inferences (not only of syllogistic form) are *logically valid*.

Moreover, Leibniz was looking for a "universal characteristic" by means of which he hoped to become able to apply the logical calculus to arbitrary (scientific) propositions so that their *factual truth* could be "calculated" in a purely mechanical way.

Calculemus!

“Io penso che mai le controversie possono essere condotte a termine e che mai si puo’ imporre silenzio alle sette se non siamo ricondotti dai ragionamenti complicati ai calcoli semplici, dai vocaboli di significato incerto e vago a caratteri determinati ...
Si deve fare in modo che ogni paralogismo non sia null’altro che un errore di calcolo ...
Fatto cio’, quando sorgano controversie non ci sara’ piu’ bisogno di dispute fra due filosofi di quanto non ce ne sia fra due computisti. Bastera’ infatti prendere la penna, sedersi all’abaco e dirsi vicendevolmente: calcoliamo!”

G. W. Leibniz

Leibniz’s Project

Though modern logic is really due to Boole and De Morgan, Leibniz was the first to have a really distinct plan of a system of mathematical logic.

The principles of the logic of Leibniz, and consequently of his whole philosophy, reduce to two:

1) All our ideas are compounded of a very small number of simple ideas which form the alphabet of human thoughts;

2) Complex ideas proceed from these simple ideas by a uniform and symmetrical combination which is analogous to arithmetical multiplication.

Alphabet of Human Thought

The idea of an **alphabet of human thought** originates in the 17th century, when proposals were first made for a universal language.

René Descartes suggested that the lexicon of a universal language should consist of primitive elements. The systematic combination of these elements, according to syntactical rules, would generate “an infinity of different words.”

Leibniz outlined his characteristica universalis, an artificial language in which grammatical and logical structure would coincide, which would allow much reasoning to be reduced to calculation.

The basic elements of his *characteristica* would be pictographic characters representing unambiguously a limited number of elementary concepts. Leibniz called the inventory of these concepts "the alphabet of human thought."

The Importance of Symbols

Leibniz thought symbols very important for human understanding.

He attached so much importance to the invention of good notations that he attributed to this alone the whole of his discoveries in mathematics.

His notation for the infinitesimal calculus affords a splendid example of his skill in this regard.

The dot was introduced as a symbol for multiplication by Leibniz. On July 29, 1698, he wrote in a letter to Johann Bernoulli: *"I do not like X as a symbol for multiplication, as it is easily confounded with x…"*
[Quoted in F Cajori, *A History of Mathematical Notations* (1928)]

Charles Peirce, a 19th century pioneer of semiotics, shared Leibniz's passion for symbols and notation, and his belief that these are essential to a well-running logic and mathematics.

“It is obvious that if we could find characters or signs suited for expressing all our thoughts as clearly and as exactly as arithmetic expresses numbers or geometry expresses lines, we could do in all matters *insofar as they are subject to reasoning* all that we can do in arithmetic and geometry. For all investigations which depend on reasoning would be carried out by transposing these characters and by a species of calculus.”

(*Preface to the General Science*, 1677.
Revision of Rutherford's translation in Jolley 1995: 234)

Thoughts and Numbers

More complex thoughts would be represented by combining in some way the characters for simpler thoughts.

Leibniz saw that the uniqueness of prime factorization suggests a central role for prime numbers in the universal characteristic, a striking anticipation of Gödel numbering.

Granted, there is no intuitive or mnemonic way to number any set of elementary concepts using the prime numbers.

Calculus Ratiocinator

There are two contrasting perspectives on what Leibniz meant by *calculus ratiocinator*.

The analytic view

The received view in analytic philosophy and formal logic, is that the *calculus ratiocinator* anticipates mathematical logic — an “algebra of logic”. That logic, a formal inference engine that can be designed so as to grant primacy to calculations, began with Frege's 1879 *Begriffsschrift* and Charles Peirce's writings on logic in the 1880s.

The synthetic view

A contrasting view, stemming from synthetic philosophy and fields such as cybernetics, electronic engineering and general systems theory is little appreciated in analytic philosophy. The synthetic view understands the *calculus ratiocinator* as referring to a "calculating machine." The cybernetician Norbert Wiener considered Leibniz's *calculus ratiocinator* a forerunner to the modern day digital computer:

The “General Algebra"

Near the end of his life, Leibniz wrote that combining metaphysics with mathematics and science through a universal character would require creating what he called:

“... a kind of general algebra in which all truths of reason would be reduced to a kind of calculus. At the same time, this would be a kind of universal language or writing, though infinitely different from all such languages which have thus far been proposed; for the characters and the words themselves would direct the mind, and the errors -- excepting those of fact -- would only be calculation mistakes. It would be very difficult to form or invent this language or characteristic, but very easy to learn it without any dictionaries”

(letter to Nicolas Remond, 10 January 1714)

Sample from One of Leibniz's Logical Calculi

Def. 3. A is in L, or L contains A, is the same as to say that L can be made to coincide with a plurality of terms taken together of which A is one. $B \oplus N = L$ signifies that B is in L and that B and N together compose or constitute L. The same thing holds for a larger number of terms.

Axiom 1. $B \oplus N = N \oplus B$.

Postulate. Any plurality of terms, as A and B, can be added to compose a single term $A \oplus B$.

Axiom 2. $A \oplus A = A$.

Prop. 5. *If A is in B and $A = C$; then C is in B.*

For in the proposition *A is in B* the substitution of A for B gives *C is in B*.

Prop. 6. *If C is in B and $A = B$ then C is in A.*

For in the proposition *C is in B* the substitution of A for B gives *C is in A*.

Prop. 7. *A is in A.*

For A is in $A \oplus A$ (by Def. 3). Therefore (by Prop. 6) A is in A.

. . . .

Prop. 20 *If A is in M and B is in N, then $A \oplus B$ is in $M \oplus N$.*

Leibniz Loses Heart

Leibniz rightly saw that creating the *characteristica* would be difficult, fixing the time required for devising it as follows: "*I think that some selected men could finish the matter in five years*".

But later in life, a more sober note emerged. In a March 1706 letter to the Electress Sophia of Hanover, the spouse of his patron, he wrote:

"It is true that I once planned a new method of calculation proper to subjects having nothing in common with mathematics, and if this manner of Logic were put into practice, all reasoning, even analogical ones, would be carried out in a mathematical way. Then modest intellects could, with diligence and good will, not accompany but at least follow greater ones. For one could always say "let us calculate" and judge properly, insofar as reason and the data can furnish us the means to do so. But I do not know whether I will ever be able to execute such a project, one requiring more than one hand, and it would even seem that humanity is not yet sufficiently mature to pretend to the advantages to which this method could lead."

In an another 1714 letter to Nicholas Remond, Leibniz wrote:

“I have spoken to the Marquis de l'Hôpital and others about my general algebra, but they have paid no more attention to it than if I had told them about a dream of mine. I should have to support it too by some obvious application, but to achieve this it would be necessary to work out at least a part of my characteristic, a task which is not easy, especially in my present condition and without the advantage of discussions with men who could stimulate and help me in work of this nature.”

(Loemker 1969, p. 656)

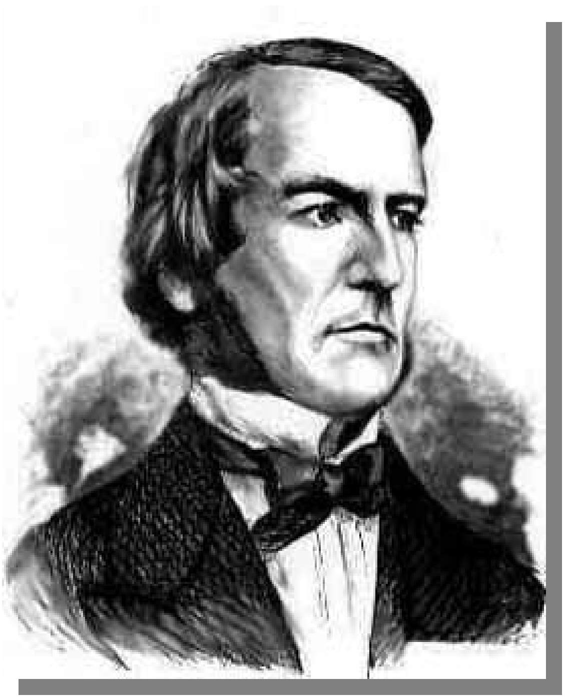
Leibniz As a Precursor of Boole

The rediscovery of Leibniz's logical work would not have been possible without the pioneering work Louis Couturat. On the other hand, Couturat is also (at least partially) responsible for the underestimation of the value of traditional logic in general and of Leibniz's logic in particular as it may be observed throughout the 20th century.

In the "Résumé et conclusion" of chapter 8, Couturat compares Leibniz's logical achievements with those of modern logicians, especially with the work of George Boole:

"Summing up, Leibniz had the idea […] of all logical operations, not only of multiplication, addition and negation, but even of subtraction and division. […] He found the correct algebraic translation of the four classical propositions […] He discovered the main laws of the logic calculus, in particular the rules of composition and decomposition […] In one word, he possessed almost all principles of the Boole-Schröder-logic, and in some points he was even more advanced then Boole himself." (Cf. Couturat [1901: 385-6])

George Boole (1815-1864)



George Boole was born in Lincoln, England on Nov. 2nd 1815. He inherited his father's passion for science and by the age of 14 could read Latin, Greek, French and German. But Boole's family fell on hard times, and he was forced find work to support them.

Boole discovered and taught himself mathematics while teaching in local schools. The papers that he published in the Cambridge Mathematical Journal earned him respect as a capable mathematician. In 1849, despite lacking a university degree, he was offered the first professorship of mathematics at Queen's College, Cork, in Ireland, where he taught until his death on Dec. 8th, 1864.

In 1854, Boole published his greatest and most influential work: "An Investigation Into the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probabilities" in which he brilliantly combined algebra with logic.

The Laws of Thought

«The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind.»

(G. Boole, *The Laws of Thought,* 1854)

Signs and Their Laws

A sign is an arbitrary mark, having a fixed interpretation, and susceptible of combination with other signs in subjection to fixed laws dependent upon their mutual interpretation.

All the operations of Language, as an instrument of reasoning, may be conducted by a system of signs composed of the following elements:

1. Literal symbols, as x, y, &c., representing things as subjects of our conceptions.

2. Signs of operation, as +, -, ×, standing for those operations of the mind by which the conceptions of things are combined or resolved so as to for new conceptions involving the same elements.

3. The sign of identity, =.

The “Intersection”

Let us then agree to represent the class of individuals to which a particular name or description is applicable, by a single letter, as x. If the name is “men,” for instance, let x represent “all men,” or the class “men.”

Again, if an adjective, as “good,” is employed as a term of description, let us represent by a letter, as y, all things to which the description “good” is applicable, i.e. “all good things,” or the class “good things.”

Let it further be agreed, that by the combination xy shall be represented that class of things to which the names or descriptions represented by x and y are simultaneously applicable.

Thus, if x alone stands for “white things,” and y for “sheep,” let xy stand for “white sheep;” and in like manner, if z stand for “horned things,” and x and y retain their previous interpretations, let zxy represent “horned white sheep,” i.e. that collection of things to which the name “sheep,” and the descriptions “white” and “horned” are together applicable.

Commutativity

First, it is evident, that according to the above combinations, the order in which two symbols are written is indifferent.

The expressions xy and yx equally represent that class of things to the several members of which the names or descriptions x and y are together applicable.

Hence we have,

$$xy = yx$$

Idempotence

As the combination of two literal symbols in the form xy expresses the whole of that class of objects to which the names or qualities represented by x and y are together applicable, it follows that if the two symbols have exactly the same signification, their combination expresses no more than either of the symbols taken alone would do.

In such case we should therefore have

$$xy = x.$$

As y is, however, supposed to have the same meaning as x, we may replace it in the above equation by x, and we thus get

$$xx = x.$$

Idempotence

Now in common Algebra the combination xx is more briefly represented by x^2. [...]

In accordance with this notation, then, the above equation assumes the form

$$x^2 = x$$

and is, in fact, the expression of a second general law of those symbols by which names, qualities, or descriptions, are symbolically represented.

Addition

We are not only capable of entertaining the conceptions of objects, as characterized by names, qualities, or circumstances, applicable to each individual of the group under consideration, but also of forming the aggregate conception of a group of objects consisting of partial groups, each of which is separately named or described. For this purpose we use the conjunctions "and," "or," &c. "Trees and minerals," "barren mountains, or fertile vales," are examples of this kind. [...]

In this and in all other respects the words "and" "or" are analogous with the sign + in algebra, and their laws are identical.

Thus the expression "men and women" is, conventional meanings set aside, equivalent with the expression "women and men." Let x represent "men," y, "women;" and let + stand for "and" and "or," then we have

$$x + y = y + x$$

Distributivity

Let the symbol z stand for the adjective “European,” then since it is, in effect, the same thing to say “European men and women,” as to say “European men and European women,” we have

$$z (x + y) = zx + zy.$$

And this equation also would be equally true were x, y, and z symbols of number, and were the juxtaposition of two literal symbols to represent their algebraic product, just as in the logical signification previously given, it represents the class of objects to which both the epithets conjoined belong.

Substraction...

But the very idea of an operation effecting some positive change seems to suggest to us the idea of an opposite or negative operation, having the effect of undoing what the former one has done. Thus we cannot conceive it possible to collect parts into a whole, and not conceive it also possible to separate a part from a whole.

This operation we express in common language by the sign except, as, “All men except Asiatics,” “All states except those which are monarchical.” Here it is implied that the things excepted form a part of the things from which they are excepted.

As we have expressed the operation of aggregation by the sign +, so we may express the negative operation above described by - minus.

Thus if x be taken to represent men, and y, Asiatics, i. e. Asiatic men, then the conception of “All men except Asiatics” will be expressed by x-y.

... and its Properties

As it is indifferent for all the essential purposes of reasoning whether we express excepted cases first or last in the order of speech, it is also indierent in what order we write any series of terms, some of which are affected by the sign -.

Thus we have, as in the common algebra,

$$x - y = -y + x.$$

Still representing by x the class “men,” and by y “Asiatics,” let z represent the adjective “white.” Now to apply the adjective “white” to the collection of men expressed by the phrase “Men except Asiatics,” is the same as to say, “White men, except white Asiatics.” Hence we have

$$z(x - y) = zx - zy.$$

This is also in accordance with the laws of ordinary algebra.

The symbol "="

The above sign, *is* or *are* may be expressed by the symbol =. The laws, or as would usually be said, the axioms which the symbol introduces, are next to be considered.

Instead of dwelling upon particular cases, we may at once affirm the general axioms:

1st. If equal things are added to equal things, the wholes are equal.

2nd. If equal things are taken from equal things, the remainders are equal.

And it hence appears that we may add or subtract equations, and employ the rule of transposition above given just as in common algebra.

Binary Variables

We have seen that $x^2 = x$. We know that $0^2 = 0$, and that $1^2 = 1$; and the equation $x^2 = x$, considered as algebraic, has no other roots than 0 and 1. [...]

Let us conceive, then, of an Algebra in which the symbols x, y, z, etc. admit indifferently of the values 0 and 1, and of these values alone.

The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. [...]

Upon this principle the method of the following work is established.

Nothing...

The symbol 0, as used in Algebra, satisfies the following formal law,

$$0 \times y = 0, \text{ or } 0y = 0$$

whatever number y may represent.

That this formal law may be obeyed in the system of Logic, we must assign to the symbol 0 such an interpretation that the class represented by 0y may be identical with the class represented by 0, whatever the class y may be.

A little consideration will show that this condition is satisfied if the symbol 0 represent **Nothing**.

... and Everything

The symbol 1 satisfies in the system of Number the following law,

$$1 \times y = y, \text{ or } 1y = y,$$

whatever number y may represent. And this formal equation being assumed as equally valid in the system of this work, in which 1 and y represent classes, it appears that the symbol 1 must represent such a class that all the individuals which are found in any proposed class y are also all the individuals 1y that are common to that class y and the class represented by 1.

A little consideration will here show that the class represented by 1 must be "**the Universe**," since this is the only class in which are found all the individuals that exist in any class.

Hence the respective interpretations of the symbols 0 and 1 in the system of Logic are Nothing and Universe.

The Complement (1-x)

If x represent any class of objects, then will 1 - x represent the contrary or supplementary class of objects., i.e. the class including all objects which are not comprehended in the class x.

The “Principle” of Contradiction... Derived!

That axiom of metaphysicians which is termed the principle of contradiction, and which affirms that it is impossible for any being to possess a quality, and at the same time not to possess it, is a consequence of the fundamental law of thought, whose expression is

$$x^2 = x.$$

Let us write this equation in the form $x - x^2 = 0$ whence we have

$$x\,(1 - x) = 0.$$

Hence $x\,(1 - x)$ will represent the class whose members are at once “men,” and “not men,” and the equation (1) thus express the principle, that a class whose members are at the same time men and not men does not exist. In other words, that it is impossible for the same individual to be at the same time a man and not a man.

Derivation of Aristotle's Syllogisms

Using the properties derived above, Boole was able to prove (algebraically!) the validity of Aristotle's syllogisms.

Barbara:

Every x is y	$x(1-y) = 0$ [or, $x=xy$]
Every y is z	$y(1-z) = 0$ [or, $y=yz$]
----------------	--------------------------
Every x is z	$x(1-z) = 0$ [or, $x=xz$]

In fact: $x = xy = x(yz) = (xy)z = xz$

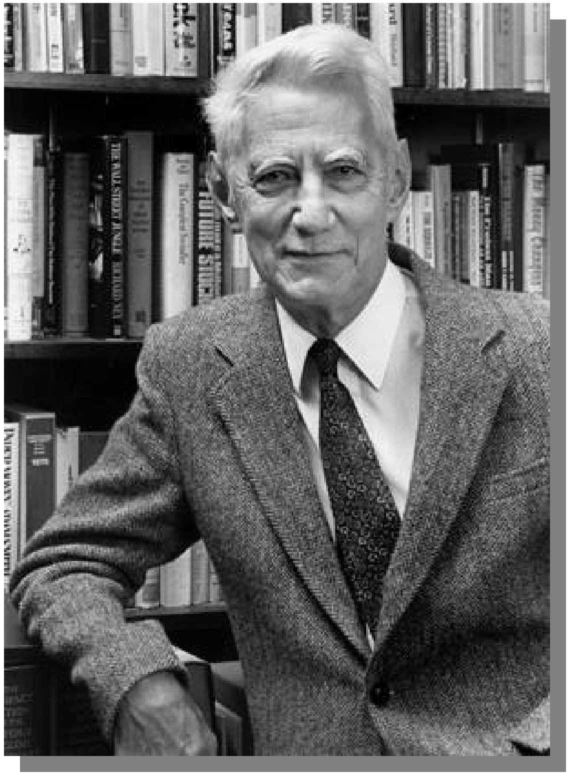
Celarent:

No X is Y	$xy=0$
Every Z is Y	$z=zy$ [or, $z(1-y) = 0$]
----------------	--------------------------
No X is Z	$xz=0$

In fact: $xz = x(zy) = x(yz) = (xy)z = 0z = 0$

Modern Developments of Boole’s Algebraic Logic

Claude Shannon:
The Father of the “Digital Age”



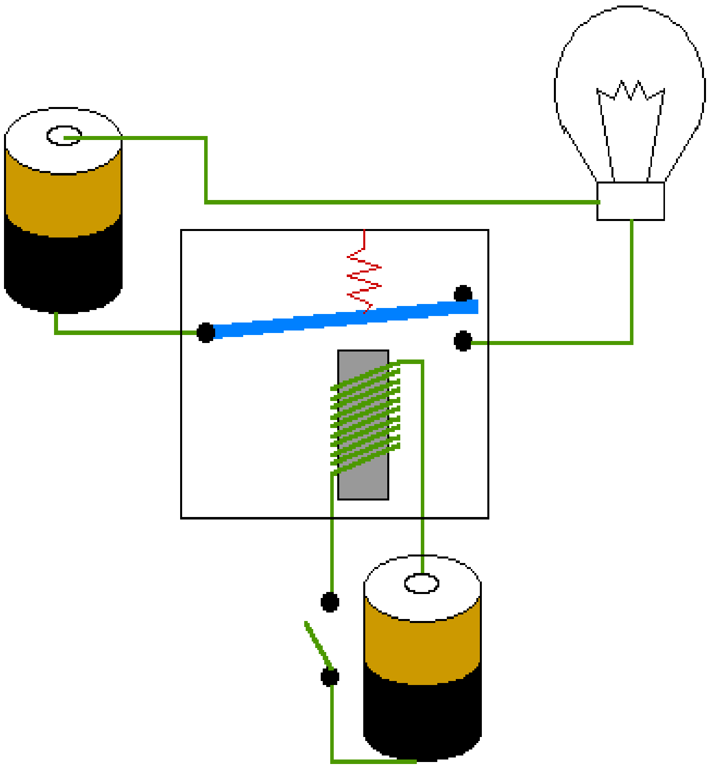
Claude Elwood Shannon was born in Petoskey, Michigan, on April 30th, 1916. He graduated from the University of Michigan in 1936 with bachelor's degrees in mathematics and electrical engineering. In 1940 he was awarded both a master's degree in electrical engineering and a Ph.D. in mathematics from the MIT.

Shannon joined the Mathematics Department at Bell Labs in 1941 with which he remained affiliated until 1972. He became a visiting professor at MIT in 1956, a permanent member of the faculty in 1958, and a professor emeritus in 1978.

He gave pioneering contributions in diverse fields such as Information Theory, Cryptography, Circuit Design, Game Theory, and developed one of the first computer programs to play chess.

Claude Shannon died on February 26th, 2001

The Electromechanical Relay



The relay technique is standard in the telephone exchange in the 1940s.

Shannon’s Master Thesis (1937)

In his 1937 MIT master's thesis, ***A Symbolic Analysis of Relay and Switching Circuits***, Shannon proved that Boolean algebra and binary arithmetic could be used to simplify the arrangement of the electromechanical relays then used in telephone routing switches, then turned the concept upside down and also proved that it should be possible to use arrangements of relays to solve Boolean algebra problems.

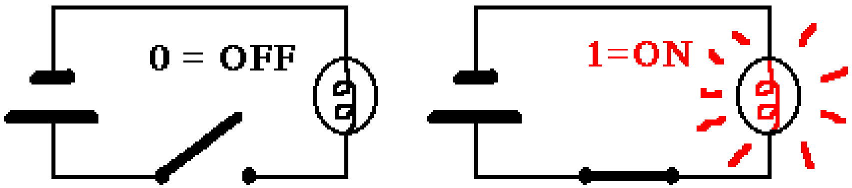
This concept, of utilizing the properties of electrical switches to do logic, is the basic concept that underlies all electronic digital computers, and the thesis became the foundation of practical digital circuit design when it became widely known among the electrical engineering community during and after World War II.

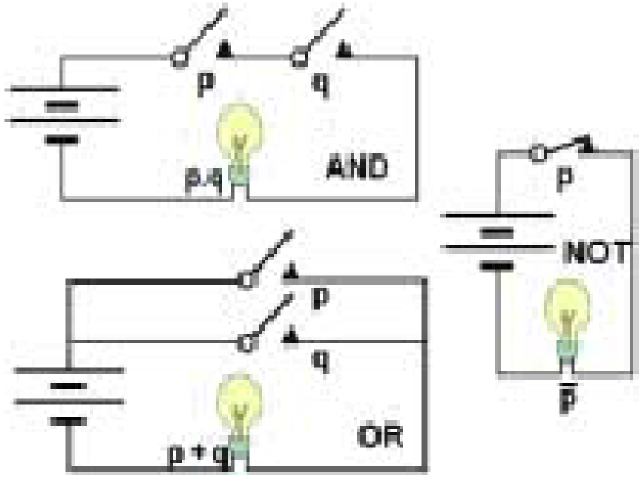
Professor Howard Gardner (Harvard) called Shannon's thesis "possibly the most important, and also the most famous, master's thesis of the century".

A version of the paper was published in the 1938 issue of the *Transactions of the American Institute of Electrical Engineers*, and in 1940, it earned Shannon the Alfred Noble American Institute of American Engineers Award.

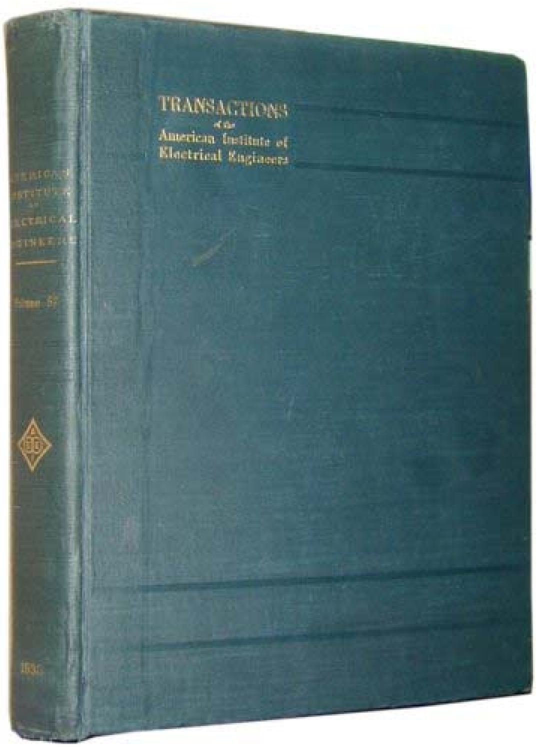
The Basic Idea

The fundamental unit of information is a yes-no situation. Either something *is* or *is not*. This can be easily expressed in Boolean two-value binary algebra by 1 and 0, so that 1 means "on" when the switch is closed and the power is on, and 0 means "off" when the switch is open and power is off.





Shannon's Master Thesis (1937)



"Claude E. Shannon, the founder of what is often called Information Theory, in his master's thesis showed in a masterful way how the analysis of complicated circuits for switching could be affected by the use of Boolean algebra. This surely must be one of the most important master's theses ever written... The paper was a landmark in that it helped to change digital circuit design from an art to a science."

Hermann Goldstine

SHANNON, Claude E. (1916-2001). A symbolic analysis of relay and switching circuits, In Transactions of the American Institute of Electrical Engineers, Vol. 57 (1938). pp. 713-23. Quarto, original publisher's blue cloth. $18,000.

Shannon’s Forerunners

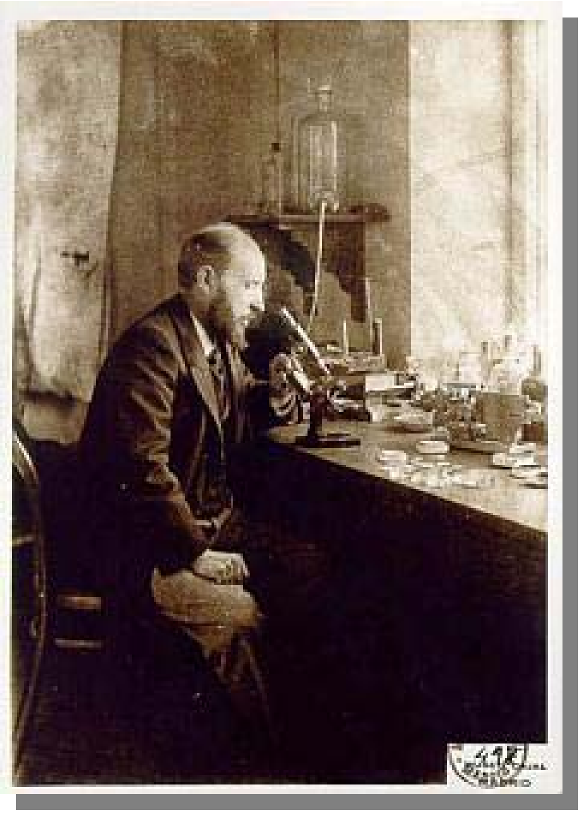
Indeed, Shannon was not the first to suggest the isomorphism between propositional calculus and relay and switching circuits .

The idea had been suggested in the Russian literature in **1910** by Paul Ehrenfest and followed up in **1934** by V**.** I. S ***.*** Sestakov.

It also appeared in a **1936** Japanese publication by Akira Nakasima and Masao Hanzawa.

However, none of these received the wide attention of Shannon's paper, mainly because his paper was published in English and presented a detailed account of the isomorphism in a way that highlighted its value to circuit design theory.

The “Neuron” as the Elementary Computational Unit in the Brain



Santiago Ramón y Cajal (1852–1934) was a famous Spanish histologist, physician, and Nobel laureate. He is considered to be one of the founders of modern neuroscience.

Ramón y Cajal's most famous studies were on the fine structure of the central nervous system. Cajal used a histological staining technique developed by his contemporary Camillo Golgi.

Using Golgi's method, he found that the nervous system is made up of billions of separate neurons and that these cells are polarized. Rather than forming a continuous web,

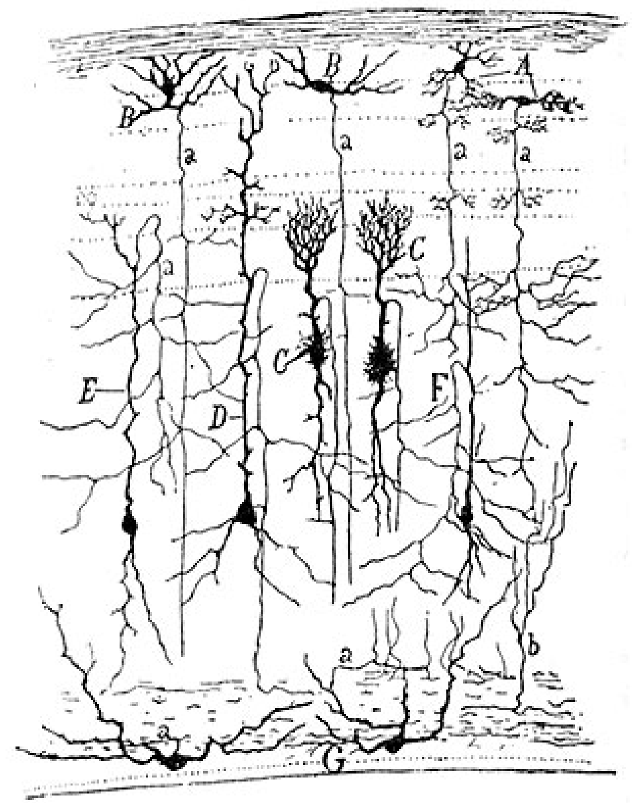
For this work Ramón y Cajal and Golgi shared the Nobel Prize in Physiology or Medicine in 1906.

The Structure of the Brain

The human cerebral cortex is composed of about 100 billion (10^{11}) neurons (nerve cells) of many different types.

Each neuron is connected to other 1000/10000 neurons, wich yields $10^{14}/10^{15}$ connections.

The cortex covers about 0.15 m^2 and is 2 – 5 mm thick



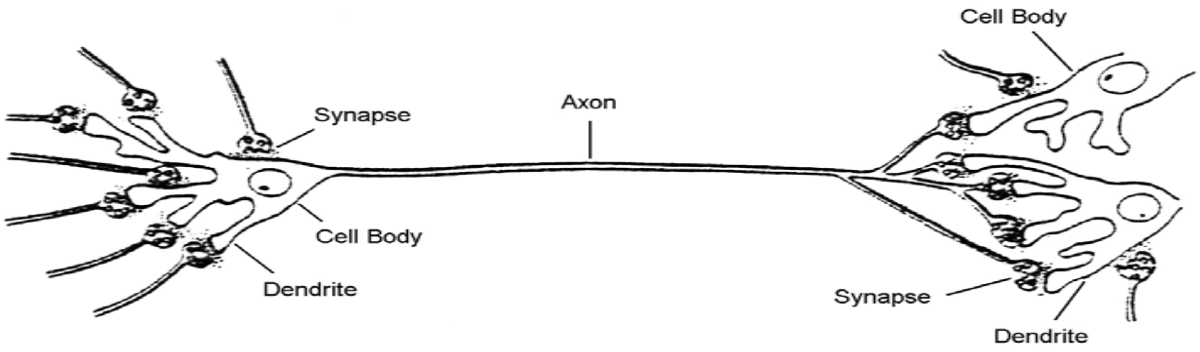
The Structure of Neurons

A “generic” neuron consists of:

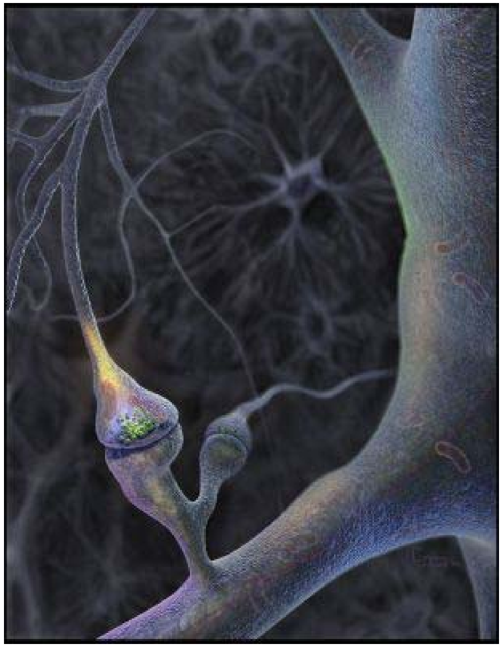
Cell Body (Soma): Computational unit. 5-10 microns in diameter

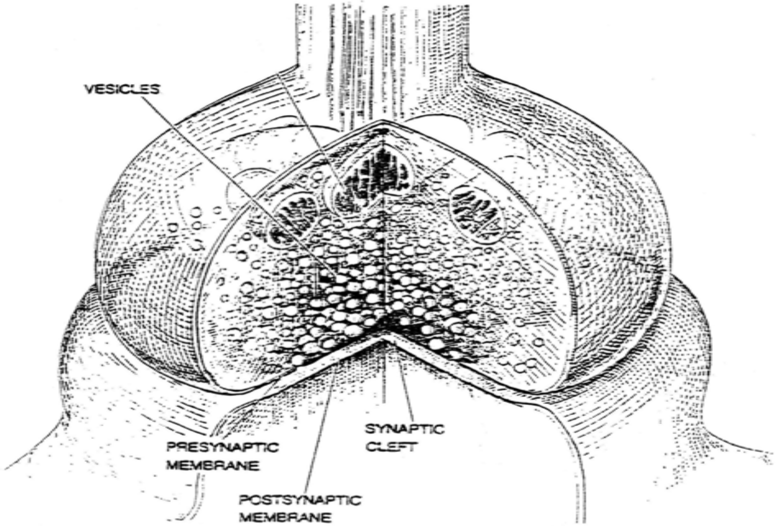
Axon: Output mechanism for a neuron; one axon/cell, but thousands of branches and cells possible for a single axon

Dendrites: Receive incoming signals from other nerve axons via synapse



Synapses





The **synapse** is the relay point where information is conveyed by chemical transmitters from neuron to neuron. A synapse consists of two parts: the knowblike tip of an axon terminal and the receptor region on the surface of another neuron. The membranes are separated by a synaptic cleft some 200 nanometers across. Molecules of chemical transmitter, stored in vesicles in the axon terminal, are released into the cleft by arriving nerve impulses. Transmitter changes electrical state of the receiving neuron, making it either more likely or less likely to fire an impulse.

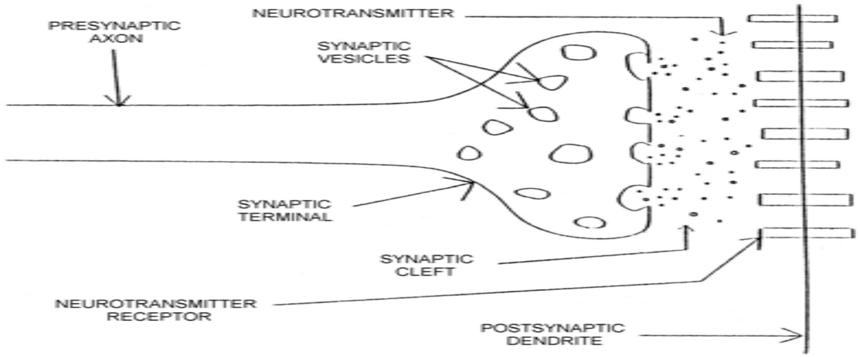
Neural Dynamics

The transmission of signal in the cerebral cortex is a complex process:

electrical — chemical — electrical

Simplifying :

1) The cellular body performs a “weighted sum” of the incoming signals
2) If the result exceeds a certain threshold value, then it produces an “action potential” which is sent down the axon (cell has “fired”), otherwise it remains in a rest state
3) When the electrical signal reaches the synapse, it allows the “neuro-transmitter” to be released and these combine with the “receptors” in the post-synaptic membrane
4) The post-synaptic receptors provoke the diffusion of an electrical signal in the post-synaptic neuron



Synaptic Efficacy

It is the amount of current that enters into the post-synaptic neuron, compared to the action potential of the pre-synaptic neuron.

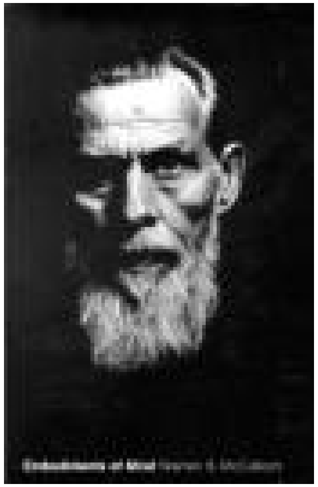
Learning takes place by modifying the synaptic efficacy.

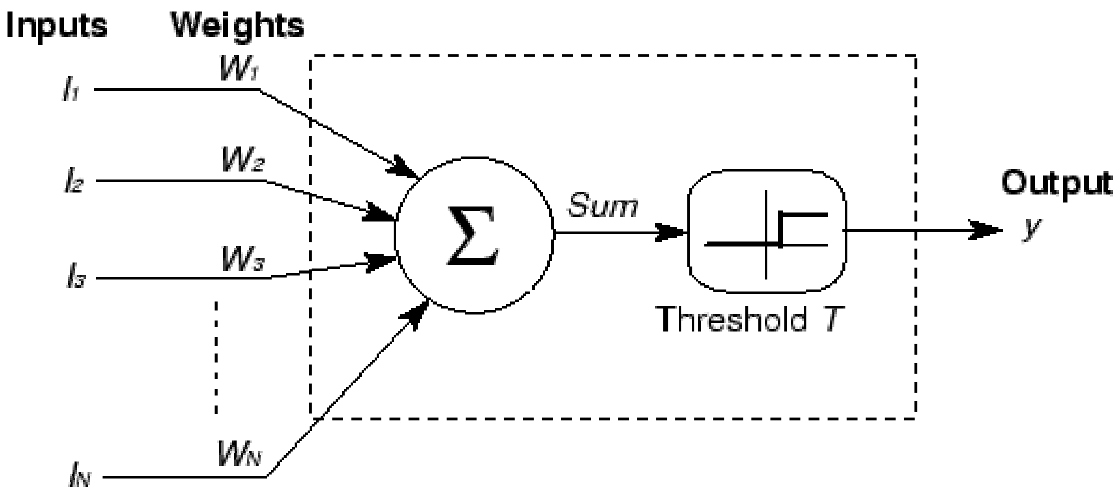
There are two types of synapses:

Excitatory: favor the generation of action potential in the post-synaptic neuron

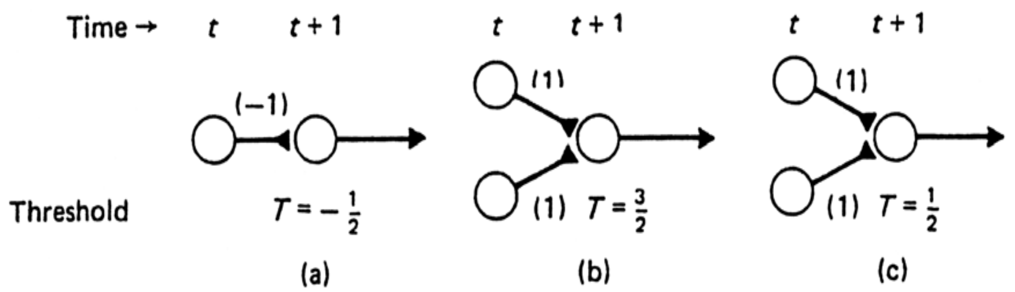
Inhibitory : hinder the generation of action potential

The McCulloch and Pitts "Neuron" (1943)

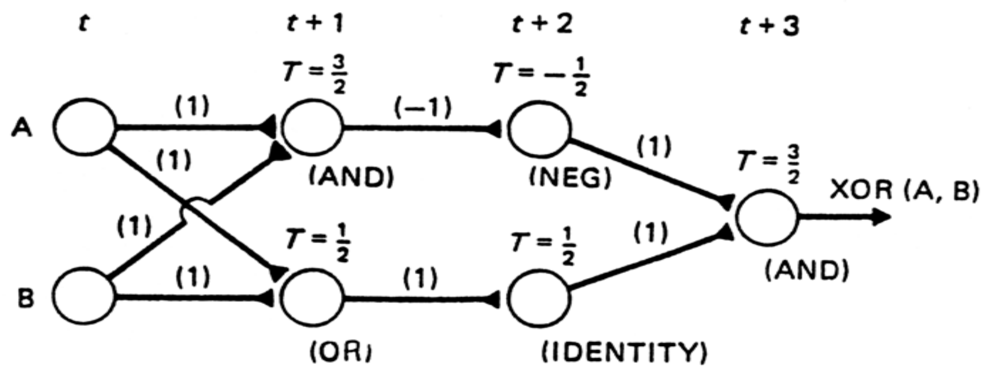




Neural Nets and Propositional Calculus



Three elementary logical operations (a) **negation**, (b) **and**, (c) **or**. In each diagram the states of the neurons on the left are at time *t* and those on the right at time *t* +1.



The construction for the **exclusive or**

Original Texts

G. Boole. *An Investigation of The Laws of Thought* (1854).
http://www.gutenberg.org/etext/15114

L. Couturat. *The Algebra of Logic* (1905).
http://www.gutenberg.org/etext/10836

C. E. Shannon. *A Symbolic Analysis of Relay and Switching Circuits* (1937).
http://hdl.handle.net/1721.1/11173

Further Readings