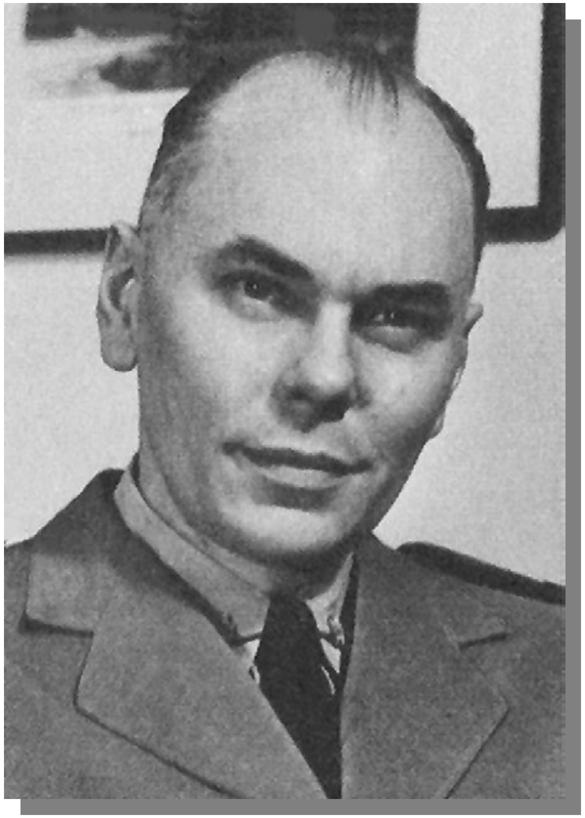
Storia dell’informatica

Le radici logiche dell’informatica

Parte I: Aristotele

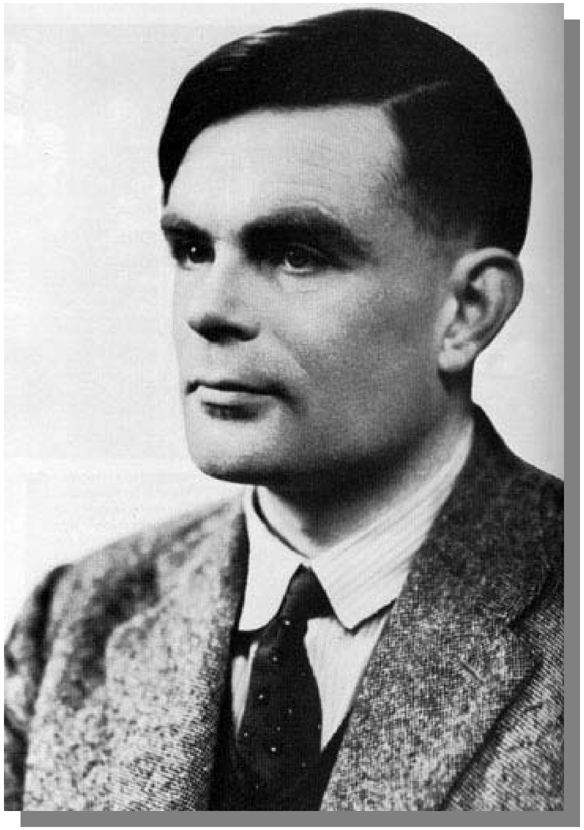
Logic and the Computer



«If it should turn out that the basic logics of a machine designed for the numerical solution of differential equations coincide with the logics of a machine intended to make bills for a department store, I would regard this as the most amazing coincidence I have ever encountered.»

Howard Aiken, 1956

The Computer as a "Universal Machine"



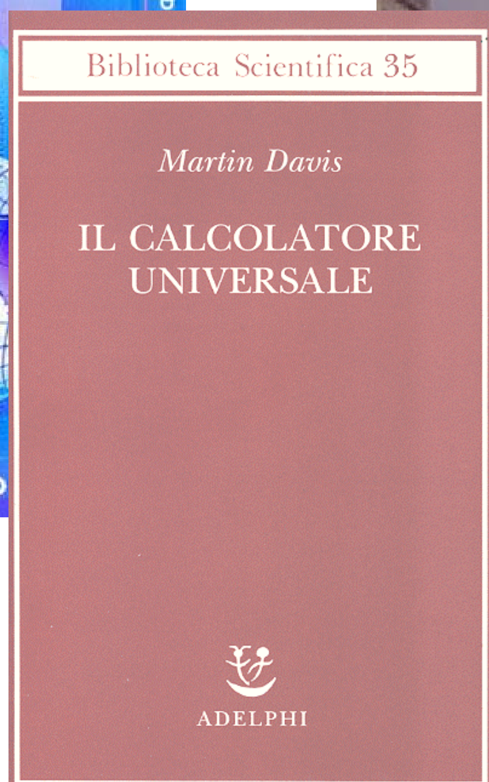
«Let us now return to the analogy of the theoretical computing machines […]

It can be shown that a single special machine of that type can be made to do the work of all. It could in fact be made to work as a model of any other machine.

The special machine may be called the universal machine.»

Alan Turing, 1947

The Universal Computer
The Road from Leibniz to Turing



Introduction / 1

In the fall of 1945, as the ENIAC, a gigantic calculating engine containing thousands of vacuum tubes, neared completion at the Moore School of Electrical Engineering in Philadelphia, a group of experts met regularly to discuss the design of its proposed successor, the EDVAC.

As the weeks went by, the meetings grew increasingly acrimonious, with the experts finding themselves divided into two groups they dubbed the "engineers" and the "logicians."

John Presper Eckert, leader of the "engineers," was justly proud of his accomplishment with the ENIAC. It had been thought impossible for 15,000 hot vacuum tubes to work together long enough without any of them failing, for anything useful to be accomplished. Nevertheless, by using careful conservative design principles, Eckert had succeeded brilliantly in accomplishing this feat.

Things came to a head when, much to Eckert's displeasure, the group's leading "logician," the eminent mathematician John von Neumann, circulated, under his own name, a draft report on the proposed EDVAC that, paying little attention to engineering details, set forth the fundamental *logical* computer design known to this day as the von Neumann architecture.

Introduction / 2

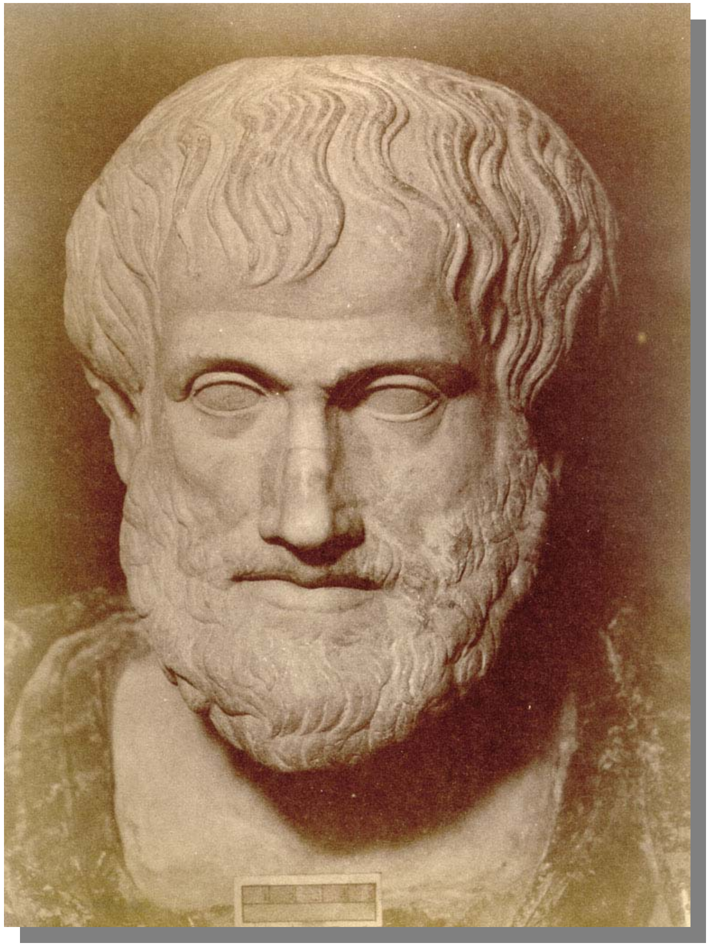
Although an engineering tour de force, the ENIAC was a logical mess. It was von Neumann's expertise as a logician and what he had learned from the English logician Alan Turing that enabled him to understand the fundamental fact that a computing machine is a logic machine.

In its circuits are embodied the distilled insights of a remarkable collection of logicians, developed over centuries.

Nowadays, when computer technology is advancing with such breathtaking rapidity, as we admire the truly remarkable accomplishments of the engineers, it is all too easy to overlook the logicians whose ideas made it all possible.

This book tells their story.

Aristotle, the Father of Logic



Aristotle's logical works contain the earliest formal study of logic that we have.

It is therefore all the more remarkable that together they comprise a highly developed logical theory, one that was able to command immense respect for many centuries.

Kant held that nothing significant had been added to Aristotle's views in the intervening two millennia.

Aristotle’s Logical Works: The *Organon*

The ancient commentators grouped together several of Aristotle's treatises under the title ***Organon*** ("Instrument") and regarded them as comprising his logical works:

1. *Categories*
2. *On Interpretation*
3. *Prior Analytics*
4. *Posterior Analytics*
5. *Topics*
6. *On Sophistical Refutations*

In fact, the title *Organon* reflects a much later controversy about whether logic is a part of philosophy (as the Stoics maintained) or merely a tool used by philosophy (as the later Peripatetics thought).

To these works should be added the *Rhetoric*, which explicitly declares its reliance on the *Topics*.

A New Science Out of Nothing

At the end of the Organon, Aristotle says:

«In the case of all discoveries the results of previous labours that have been handed down from others have been advanced gradually by those who have taken them over, whereas the original discoveries generally make an advance that is small at first though much more useful than the development which later springs out of them […]

Of this enquiry, on the other hand, it was not the case that part of the work had been thoroughly done before, while part had not. Nothing existed at all.»

(*On Sopistical Refutations,* 183b 17-22, 34-184b3)

Two Millennia Afterwards…

«That logic has already, from the earliest times, proceeded upon this sure path [of a science] is evidenced by the fact that since Aristotle it has not required to retrace a single step [...]

It is remarkable also that to the present day this logic has not been able to advance a single step, and is thus to all appearance a closed and completed body of doctrine.»

Immanuel Kant, 1787

The Subject of Logic: “Syllogisms”

All Aristotle's logic revolves around one notion: the **deduction** (*sullogismos*). In Aristotle’s words:

«A syllogism is a discourse (logos) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so.»

(*Prior Analytics* I.2, 24b18-20)

Each of the “things supposed” is a **premise** (*protasis*) of the argument, and what “results of necessity” is the **conclusion** (*sumperasma*).

The core of this definition is the notion of “resulting of necessity”.

This corresponds to a modern notion of logical consequence: X results of necessity from Y and Z if it would be impossible for X to be false when Y and Z are true.

Deduction and Induction

Deductions are one of two species of argument recognized by Aristotle.

The other species is **induction** (*epagôgê*).

He has far less to say about this than deduction, doing little more than characterize it as “*argument from the particular to the universal*”.

However, induction (or something very much like it) plays a crucial role in the theory of scientific knowledge in the *Posterior Analytics*.

It is induction, or at any rate a cognitive process that moves from particulars to their generalizations, that is the basis of knowledge of the indemonstrable first principles of sciences.

The Structure of Assertions

Syllogisms are structures of sentences each of which can meaningfully be called true or false: **assertions** (*apophanseis*), in Aristotle's terminology.

According to Aristotle, every such sentence must have the same structure: it must contain a **subject** (*hupokeimenon*) and a **predicate** and must either affirm or deny the predicate of the subject.

Thus, every assertion is either the **affirmation** (*kataphasis*) or the **denial** (*apophasis*) of a single predicate of a single subject.

The Sructure of Assertions / 2

In *On Interpretation*, Aristotle argues that a single assertion must always either affirm or deny a single predicate of a single subject.

Thus, he does not recognize sentential compounds, such as conjunctions and disjunctions, as single assertions. This appears to be a deliberate choice on his part: he argues, for instance, that a conjunction is simply a collection of assertions, with no more intrinsic unity than the sequence of sentences in a lengthy account (e.g. the entire *Iliad*, to take Aristotle's own example).

Since he also treats denials as one of the two basic species of assertion, he does not view negations as sentential compounds. His treatment of conditional sentences and disjunctions is more difficult to appraise, but it is at any rate clear that Aristotle made no efforts to develop a sentential logic. Some of the consequences of this for his theory of demonstration are important

Terms

Subjects and predicates of assertions are **terms**.

A term (*horos*) can be either **individual**, e.g. *Socrates*, *Plato,* or **universal**, e.g. *human*, *horse*, *animal*, *white*.

Subjects may be either individual or universal, but predicates can only be universals, e.g.:

- *Socrates is human*
- *Plato is not a horse*
- *horses are animals*
- *humans are not horses*

The word **universal** (*katholou*) appears to be an Aristotelian coinage. Literally, it means "of a whole"; its opposite is therefore "of a particular".

Universal terms are those which can properly serve as predicates, while particular terms are those which cannot.

In *On Interpretation*, Aristotle spells out the relationships of contradiction for sentences with universal subjects as follows:

	Affirmation	**Denial**
Universal	Every A is B	No A is B
Universal	Some A is B	Not every A is B

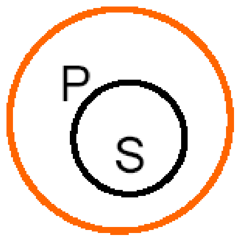
Abbreviations

For clarity and brevity, the following abbreviations for Aristotelian categorical sentences are typically used (note that the predicate term comes *first* and the subject term *second*):

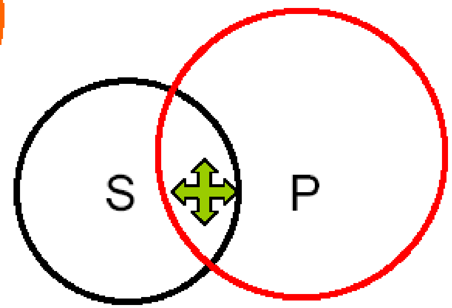
Univ. Aff.	**A**ab	a belongs to all b (Every b is a)
Univ. Neg.	**E**ab	a belongs to no b (No b is a)
Part. Aff.	**I**ab	a belongs to some b (Some b is a)
Part. Neg.	**O**ab	a does not belong to all b (Some b is not a)

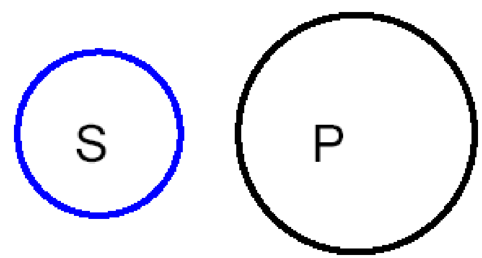
Using Venn Diagrams

A: Universal Affermative (“*Every S is P”*)

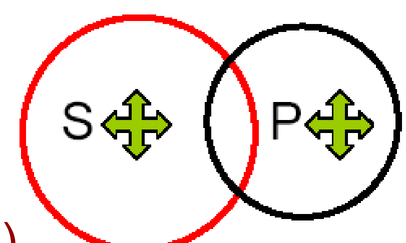


I: Particular Affermative (“*Some S is P*”)





E: Universal Negative (“*No S is P*”)

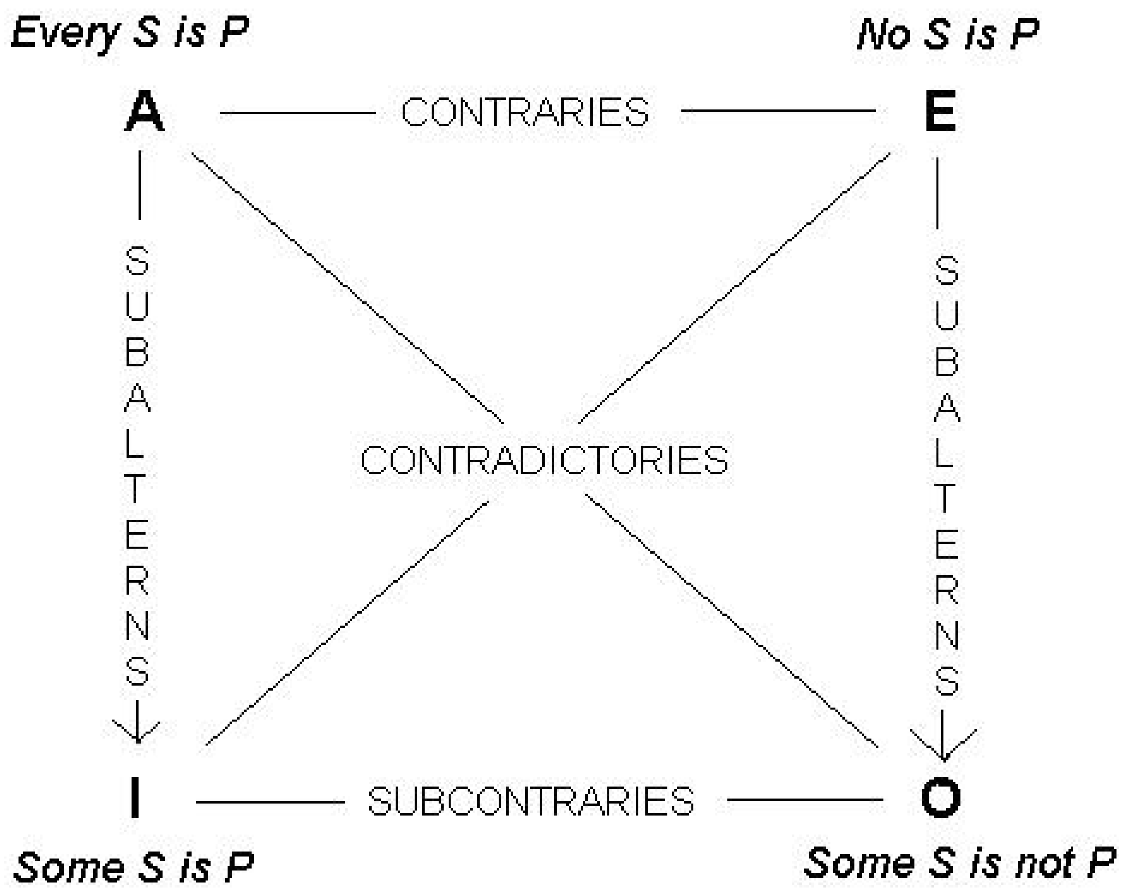


O: Particular Negative (“*Some S is not P*”)

Opposition Between Propositions

- Two propositions are **contradictory** iff they cannot both be true and they cannot both be false.
- Two propositions are **contraries** iff they cannot both be true but can both be false.
- Two propositions are **subcontraries** iff they cannot both be false but can both be true.
- A proposition is a **subaltern** of another iff it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

The Square of Oppositions



Aristotle’s Syllogistic

Aristotle's most famous achievement as logician is his theory of inference, traditionally called the **syllogistic** (though not by Aristotle).

That theory is in fact the theory of inferences of a very specific sort: inferences with two premises, each of which is a categorical sentence, having exactly one term in common, and having as conclusion a categorical sentence the terms of which are just those two terms not shared by the premises.

Aristotle calls the term shared by the premises the **middle term** (*meson*) and each of the other two terms in the premises an **extreme** (*akron*). The middle term must be either subject or predicate of each premise, and this can occur in three ways: the middle term can be the subject of one premise and the predicate of the other, the predicate of both premises, or the subject of both premises.

Aristotle refers to these term arrangements as **figures** (*schêmata*):

The Three Figures

	First Figure		Second Figure		Third Figure	
	Predicate	**Subject**	**Predicate**	**Subject**	**Predicate**	**Subject**
Premise	a	b	a	b	a	c
Premise	b	c	a	c	b	c
Conclusion	a	c	b	c	a	b

The List of Deductions

In *Prior Analytics* I.4-6, Aristotle shows that the premise combinations given in the following table yield deductions and that all other premise combinations fail to yield a deduction.

In this table, "|- " separates premises from conclusion; it may be read "therefore".

The second column lists the medieval mnemonic name associated with the inference (these are still widely used, and each is actually a mnemonic for Aristotle's proof of the mood in question).

Form	Mnemonic
FIRST FIGURE	
Aab, Abc ⊢ Aac	*Barbara*
Eab, Abc ⊢ Eac	*Celarent*
Aab, Ibc ⊢ Iac	*Darii*
Eab, Ibc ⊢ Oac	*Ferio*
SECOND FIGURE	
Eab, Aac ⊢ Ebc	*Cesare*
Aab, Eac ⊢ Ebc	*Camestres*
Eab, Iac ⊢ Obc	*Festino*
Aab, Oac ⊢ Obc	*Baroco*
THIRD FIGURE	
Aac, Abc ⊢ Iab	*Darapti*
Eac, Abc ⊢ Oab	*Felapton*
Iac, Abc ⊢ Iab	*Disamis*
Aac, Ibc ⊢ Iab	*Datisi*
Oac, Abc ⊢ Oab	*Bocardo*
Eac, Ibc ⊢ Oab	*Ferison*

Barbara and Celarent

«Whenever three terms are so related to one another that the last is contained in the middle as in a whole, and the middle is either contained in, or excluded from, the first as in or from a whole, the extremes must be related by a perfect syllogism.

I call that term middle which is itself contained in another and contains another in itself: in position also this comes in the middle. By extremes I mean both that term which is itself contained in another and that in which another is contained.

If A is predicated of all B, and B of all C, A must be predicated of all C: we have already explained what we mean by 'predicated of all'. Similarly also, if A is predicated of no B, and B of all C, it is necessary that no C will be A.»

(*Prior Analytics*, I.4)

Examples

Barbara:

Every S is M	Every Greek is man
Every M is P	Every man is mortal
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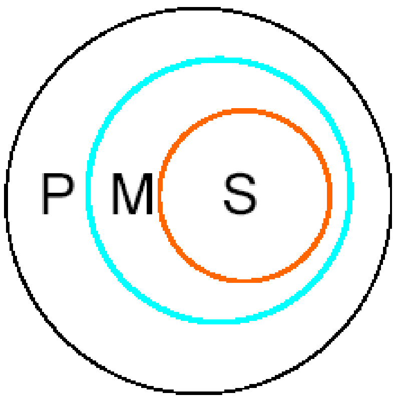
Every S is P	Every Greek is mortal



Celarent:

No S is P	No stone is animal
Every M is P	Every man is an animal
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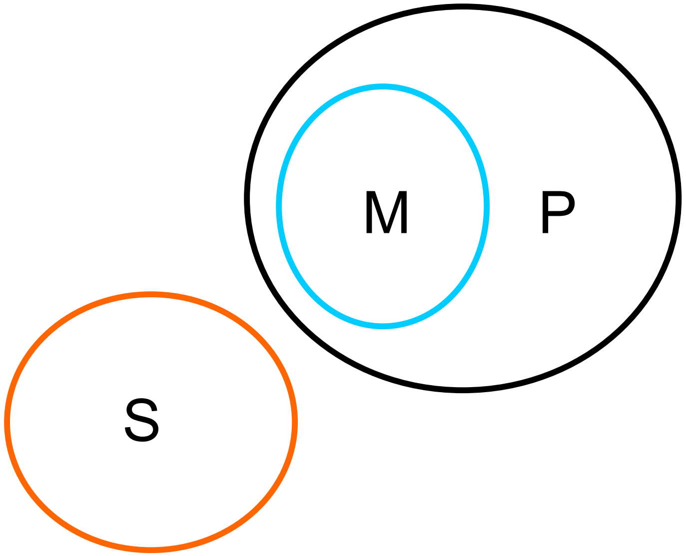
No S is M	No stone is a man



Invalid Conclusions

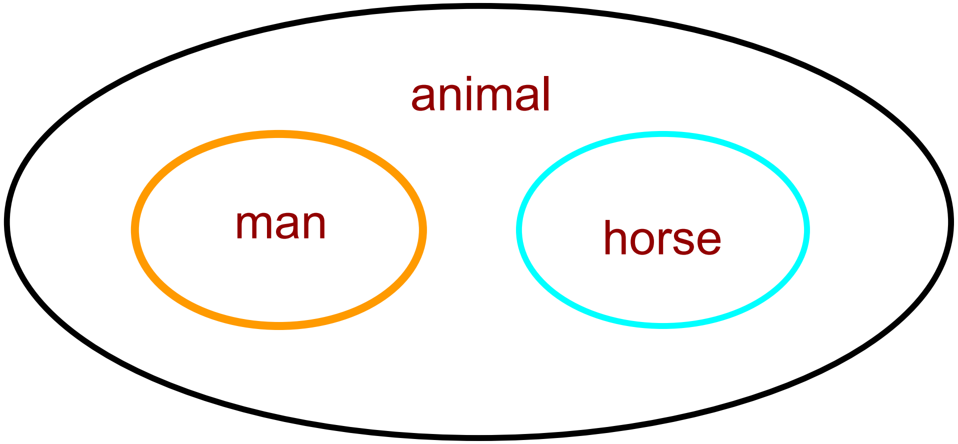
«But if the first term belongs to all the middle, but the middle to none of the last term, there will be no syllogism in respect of the extremes; for nothing necessary follows from the terms being so related; for it is possible that the first should belong either to all or to none of the last, so that neither a particular nor a universal conclusion is necessary.

But if there is no necessary consequence, there cannot be a syllogism by means of these premisses.

As an example of a universal affirmative relation between the extremes we may take the terms animal, man, horse; of a universal negative relation, the terms animal, man, stone.»

(*Prior Analytics*, I.4)

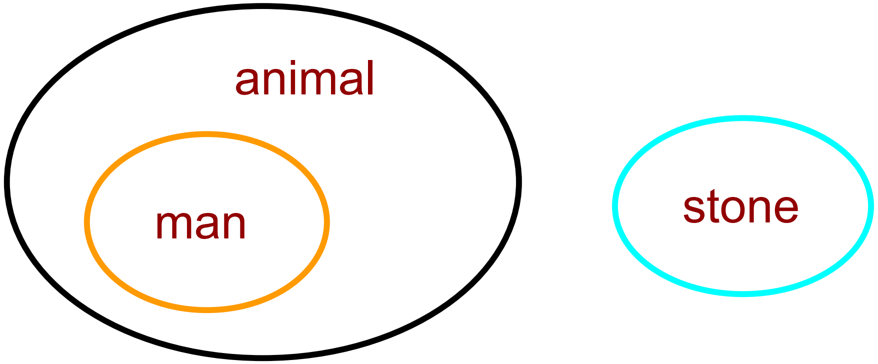
Examples



animal – man – horse

Every man is an animal
No horse is a man

Every horse is an animal



animal – man – stone

Every man is an animal
No stone is a man

No stone is an animal

Metatheoretical Results

Having established which deductions in the figures are possible, Aristotle draws a number of metatheoretical conclusions, including:

1. No deduction has two negative premises
2. No deduction has two particular premises
3. A deduction with an affirmative conclusion must have two affirmative premises
4. A deduction with a negative conclusion must have one negative premise.
5. A deduction with a universal conclusion must have two universal premises
6. All deductions can be reduced to the two universal deductions in the first figure.

Syllogisms with Modalities

Aristotle follows his treatment with a much longer, and much more problematic, discussion of what happens to the arguments when we add the qualifications "necessarily" and "possibly" to their premises in various ways.

In contrast to the “assertoric” syllogistic, this **modal** syllogistic appears to be much less satisfactory and is certainly far more difficult to interpret.

Modern modal logic treats necessity and possibility as interdefinable: "necessarily P" is equivalent to "not possibly not P", and "possibly P" to "not necessarily not P".

In *Prior Analytics*, he makes a distinction between two notions of possibility. On the first, which he takes as his preferred notion, "possibly P" is equivalent to "not necessarily P and not necessarily not P".

Aristotle builds his treatment of modal syllogisms on his account of assertoric syllogisms: he works his way through the syllogisms he has already proved and considers the consequences of adding a modal qualification to one or both premises

Aristotelian Sciences

The subject of the *Posterior Analytics* is *epistêmê*.

This is one of several Greek words that can reasonably be translated "scientific knowledge". We have scientific knowledge, according to Aristotle, when we know:

«the cause why the thing is, that it is the cause of this, and that this cannot be otherwise.»

(*Posterior Analytics* I.2)

This implies two strong conditions on what can be the object of scientific knowledge:

1. Only what is necessarily the case can be known scientifically
2. Scientific knowledge is knowledge of causes

Is “Scientific Knowledge” Possible? The agnostic view

In *Posterior Analytics* I.3, Aristotle considers two challenges to the possibility of scientific knowledge.

One party (dubbed the "agnostics" by Jonathan Barnes) argued that demonstration is impossible using the following argument:

1. If the premises of a demonstration are scientifically known, then they must be demonstrated.
2. The premises from which each premise are demonstrated must be scientifically known.
3. Either this process continues forever, creating an infinite regress of premises, or it comes to a stop at some point.
4. If it continues forever, then there are no first premises from which the subsequent ones are demonstrated, and so nothing is demonstrated.
5. On the other hand, if it comes to a stop at some point, then the premises at which it comes to a stop are undemonstrated and therefore not scientifically known; consequently, neither are any of the others deduced from them.
6. Therefore, nothing can be demonstrated.

The Alternative View

A second group accepted the agnostics' view that scientific knowledge comes only from demonstration but rejected their conclusion by rejecting the dilemma.

Instead, they maintained that demonstration "in a circle" is possible, so that it is possible for all premises also to be conclusions and therefore demonstrated.

Aristotle does not give us much information about how circular demonstration was supposed to work, but the most plausible interpretation would be supposing that at least for some set of fundamental principles, each principle could be deduced from the others.

Some modern interpreters have compared this position to a coherence theory of knowledge.

Aristotle's Solution:
“It Eventually Comes to a Stop”

Aristotle rejects circular demonstration as an incoherent notion on the grounds that the premises of any demonstration must be prior (in an appropriate sense) to the conclusion, whereas a circular demonstration would make the same premises both prior and posterior to one another (and indeed every premise prior and posterior to itself).

He agrees with the agnostics' analysis of the regress problem: the only plausible options are that it continues indefinitely or that it "comes to a stop" at some point.

However, he thinks both the agnostics and the circular demosntrators are wrong in maintaining that scientific knowledge is only possible by demonstration from premises scientifically known: instead, he claims, there is another form of knowledge possible for the first premises, and this provides the starting points for demonstrations.

Knowledge of First Principles: Nous

Aristotle's account of knowledge of the indemonstrable first premises of sciences is found in *Posterior Analytics* II.19, long regarded as a difficult text to interpret.

Briefly, what he says there is that it is another cognitive state, **nous** (translated variously as "insight", "intuition", "intelligence"), which knows them.

There is wide disagreement among commentators about the interpretation of his account of how this state is reached, but it acan be regarded as a sort of inductive process.

The Principles of Logic

In his metaphysical writings, Aristotle espoused two principles of great importance in propositional logic:

- ***Principle of Non-Contradiction*** [no statement is both true and false]
- ***Principle of Excluded Middle*** [every statement is either true or false]

These are cornerstones of classical propositional logic.

There is some evidence that Aristotle, or at least his successor at the Lyceum, Theophrastus (d. 287 BC), did recognize a need for the development of a doctrine of "complex" or "hypothetical" propositions, i.e., those involving conjunctions (statements joined by "and"), disjunctions (statements joined by "or") and conditionals (statements joined by "if... then..."), but their investigations into this branch of logic seem to have been very minor.

Time and Necessity: The Sea-Battle

The passage in Aristotle's logical works which has received perhaps the most intense discussion in recent decades is *On Interpretation* 9, where Aristotle discusses the question whether every proposition about the future must be either true or false.

Consider these two propositions:

1. There will be a sea-battle tomorrow
2. There will not be a sea-battle tomorrow

It seems that exactly one of these must be true and the other false.

But if (1) is *now* true, then there *must* be a sea-battle tomorrow, and there *cannot* fail to be a sea-battle tomorrow.

The result, according to this puzzle, is that nothing is possible except what actually happens: there are no unactualized possibilities.

Aristotle's logic, especially his theory of the syllogism, has had an unparalleled influence on the history of Western thought.

It did not always hold this position: in the Hellenistic period, Stoic logic, and in particular the work of Chrysippus, was much more celebrated.

However, in later antiquity, following the work of Aristotelian Commentators, Aristotle's logic became dominant, and Aristotelian logic was what was transmitted to the Arabic and the Latin medieval traditions, while the works of Chrysippus have not survived.

During the rise of modern formal logic following Frege and Peirce, adherents of Traditional Logic (seen as the descendant of Aristotelian Logic) and the new mathematical logic tended to see one another as rivals, with incompatible notions of logic. More recent scholarship has often applied the very techniques of mathematical logic to Aristotle's theories, revealing (in the opinion of many) a number of similarities of approach and interest between Aristotle and modern logicians.

References and Further Readings

- *Aristotle's Logic* (Stanford Encyclopedia of Philosophy) http://plato.stanford.edu/entries/aristotle-logic/

- Aristotele. *Organon*. (Tr. it G. Colli). Adelphi.