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Computer Vision

Morphological image processing

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Preliminaries

Morphology offers a unified and powerful approach to numerous image processing problems

> The language of mathematical morphology is the **set theory**

We will consider thresholded images containing only **black** and **white** pixels.

The "set of white pixels" contains the complete morphological description of the image.

Such sets are subsets of Z^2 where each vector express coordinates of white (or black) pixels

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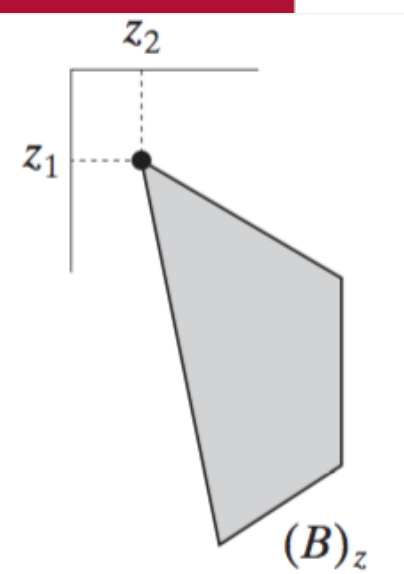
Set translation

The translation of a set B by point z = (z1, z2), denoted $(B)_z$, is defined as:

$$(B)_z = \{c \mid c = b + z \ \text{ for } \ b \in B\}$$



if B is the set of pixels representing an object in an image, then $(B)_z$ is the set of points in B whose (x,y) coordinates have been replaced by

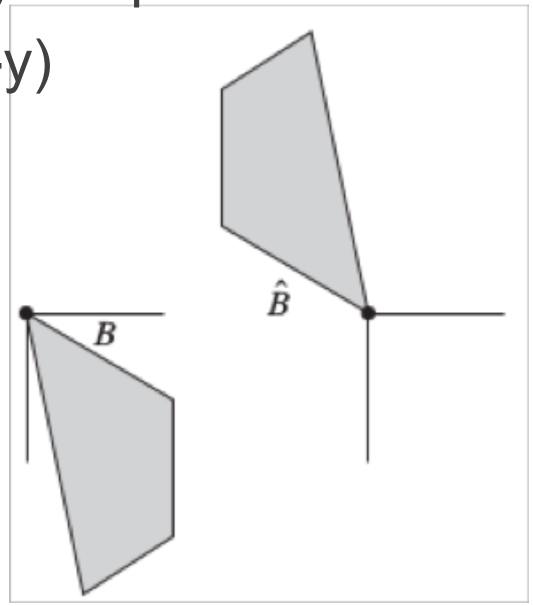
(x + z1, y + z2)

Set reflection

The reflection of a set B, denoted $\hat{B}$ is defined as:

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

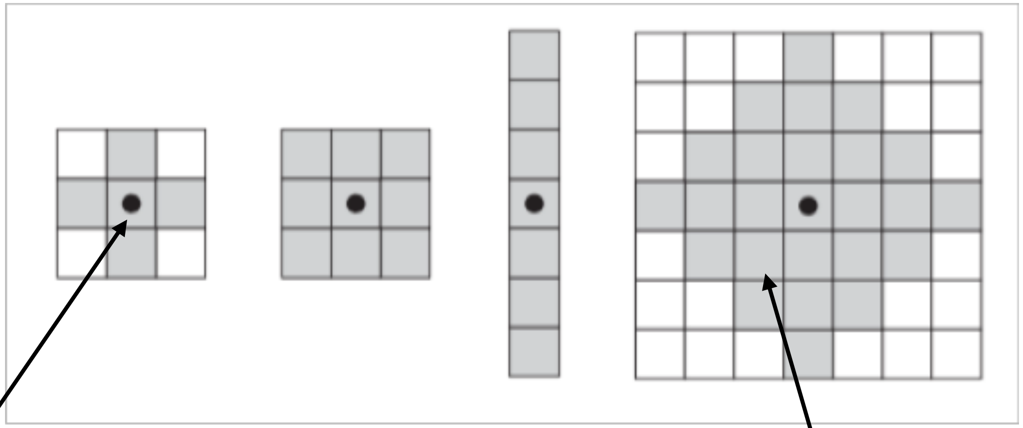
The reflection of B is composed by the points whose coordinates are replaced with (-x,-y)



Structuring elements

Morphological operations are based on so-called structuring elements (SEs):

SEs: small sets or subimages used to probe an image under study for properties of interest



Center of the SE

Only the "grey pixels" are part of the set defining the structuring element

Erosion

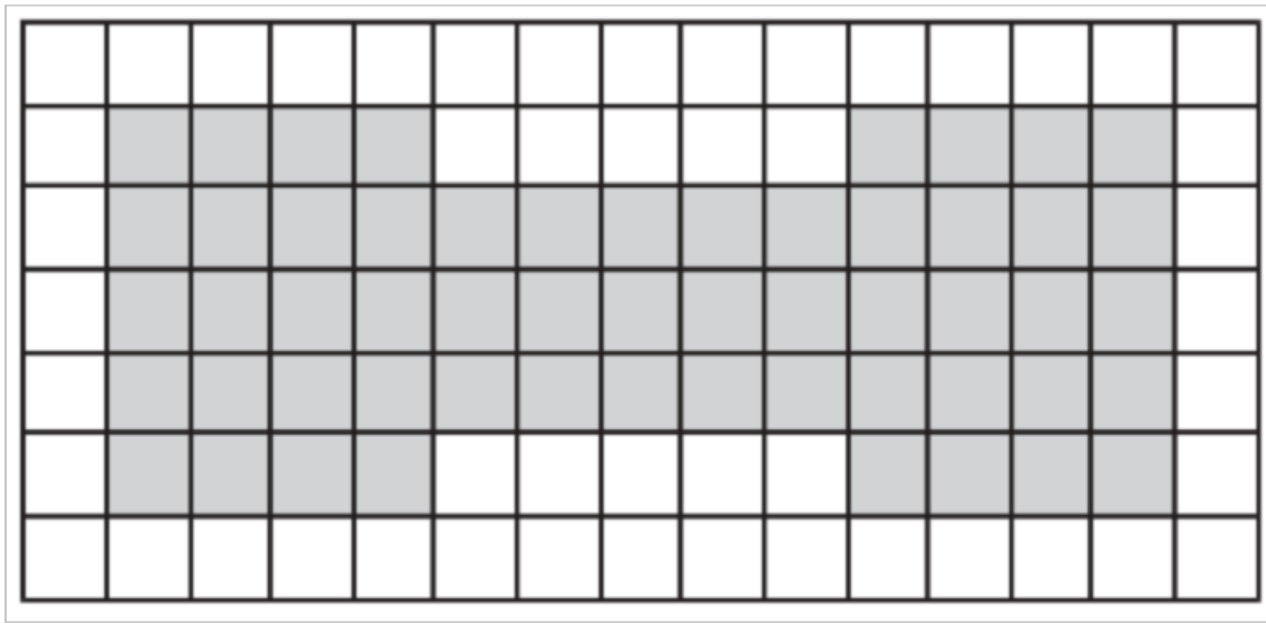
With A and B as sets in Z^2 , the erosion of A by B, denoted $A \ominus B$, is defined as:

$$A \ominus B = \{z \mid (B)_z \cap \bar{A} = \emptyset\}$$

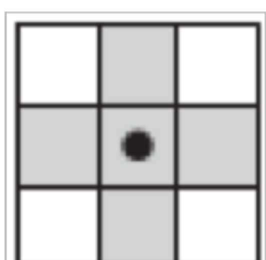
In words, the erosion of A by B is the set of all points z such that B, translated by z, is contained in A (the intersection between $(B)_z$ and the complement of A is empty)

Erosion example

A



B



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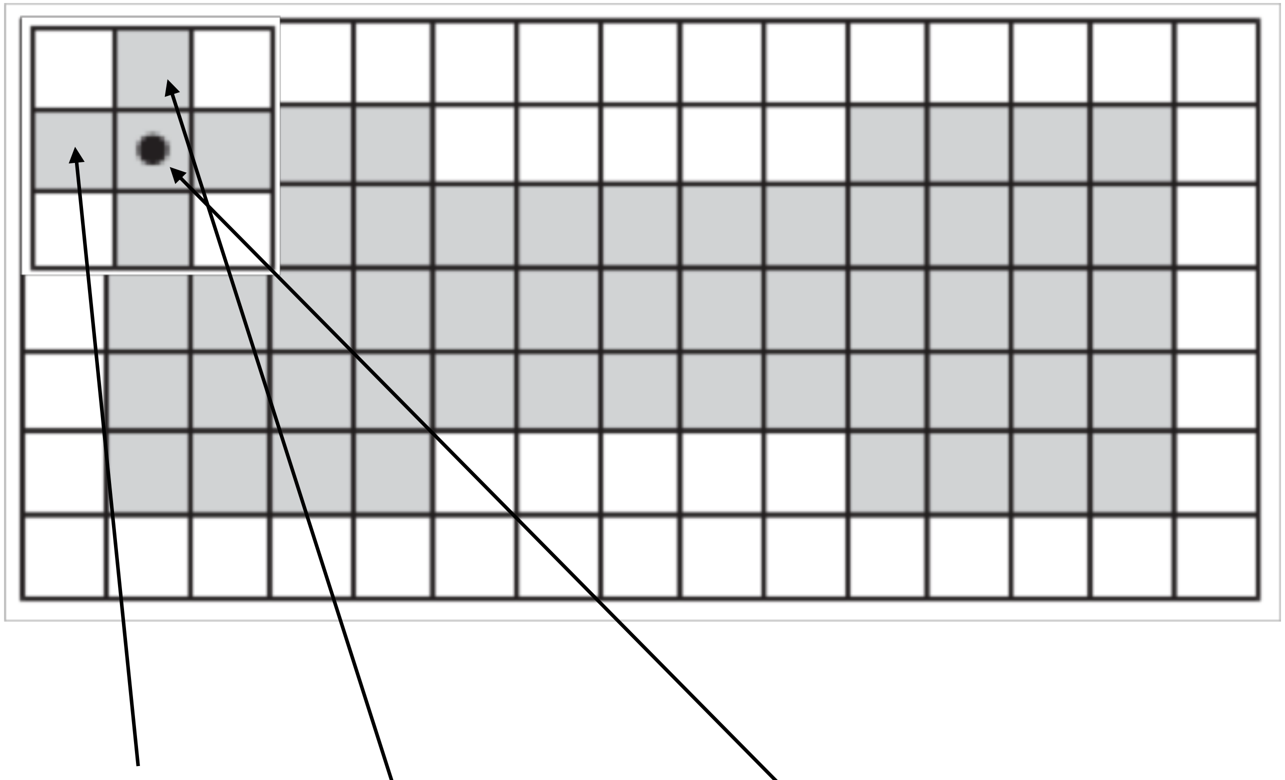
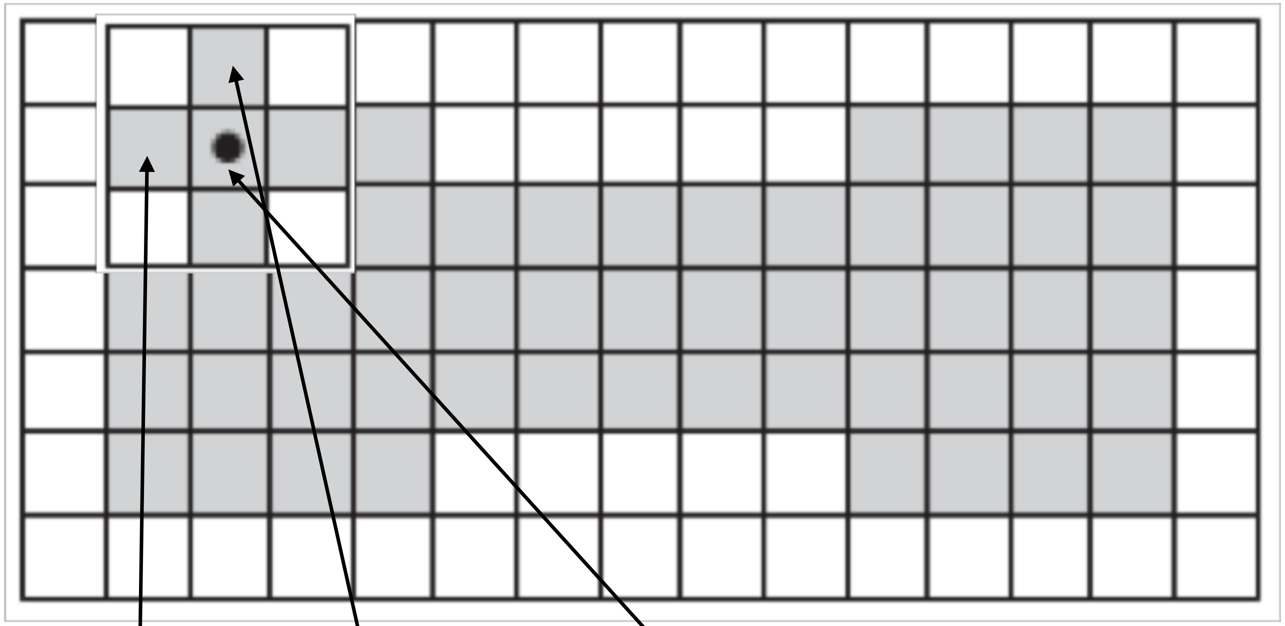
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be present in the output

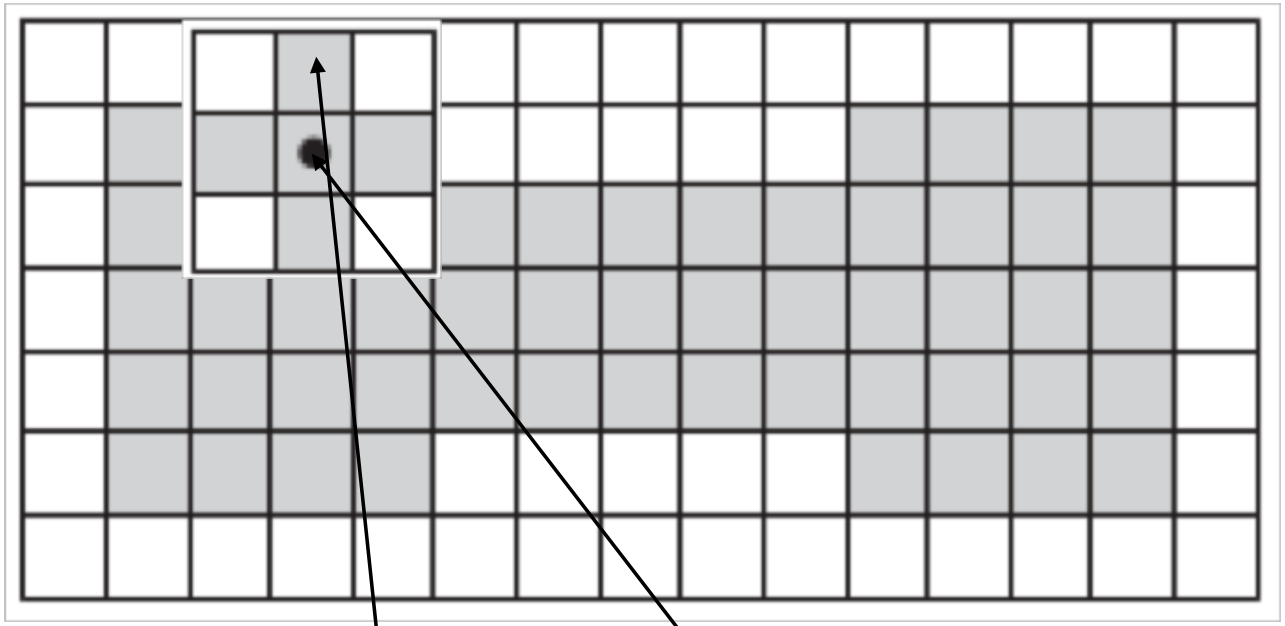
Erosion example



$(B)_z \cap \bar{A} \neq \emptyset$

This point will not be present in the output

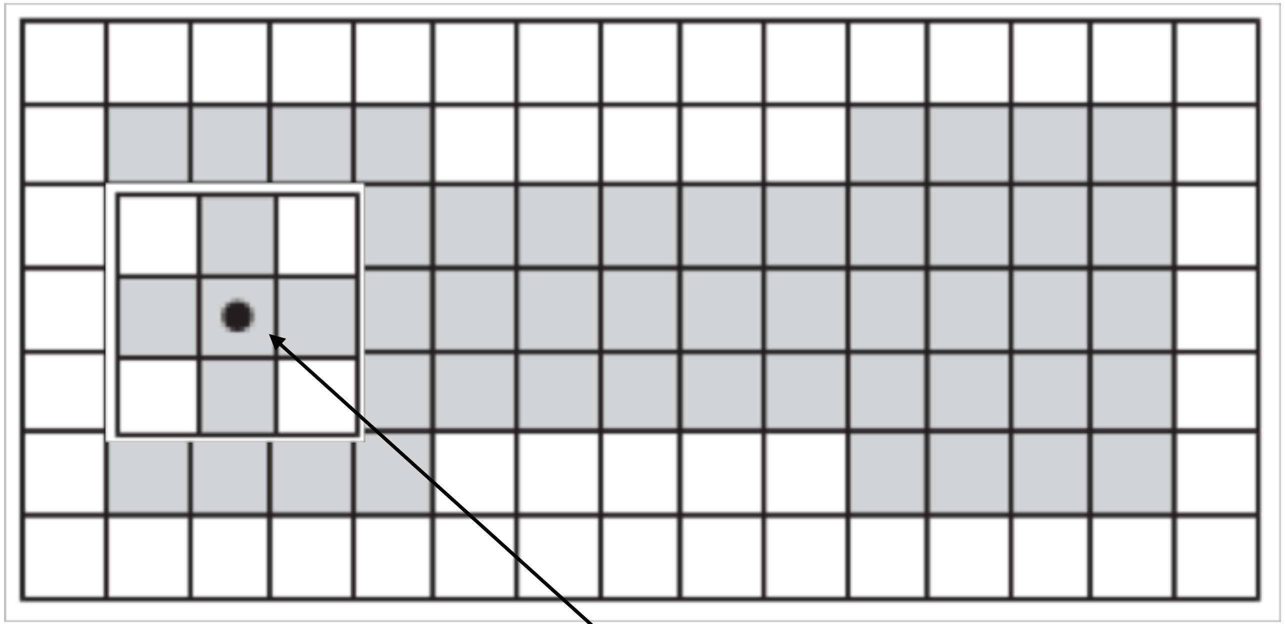
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be present in the output

Erosion example

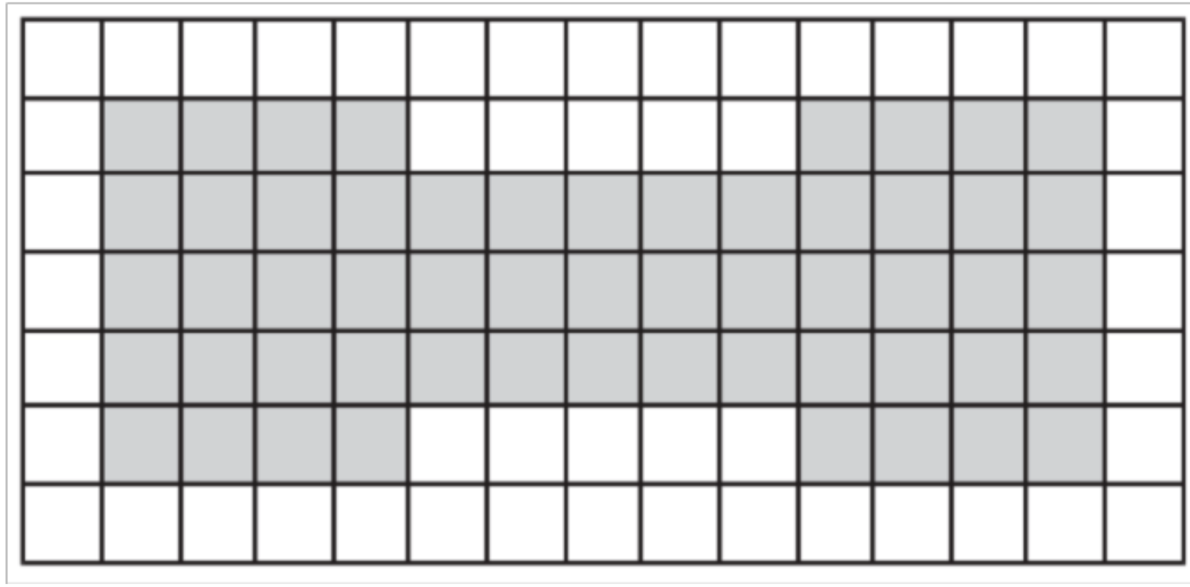


$$(B)_z \cap \bar{A} = \emptyset$$

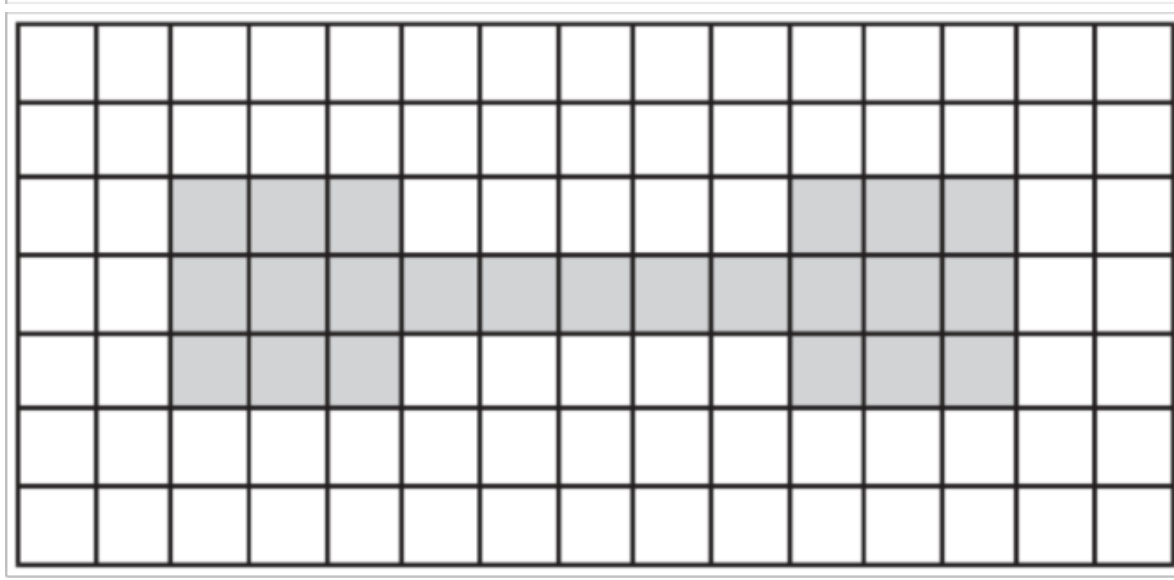
This point will be present in the output

Erosion example

A



$A \ominus B$



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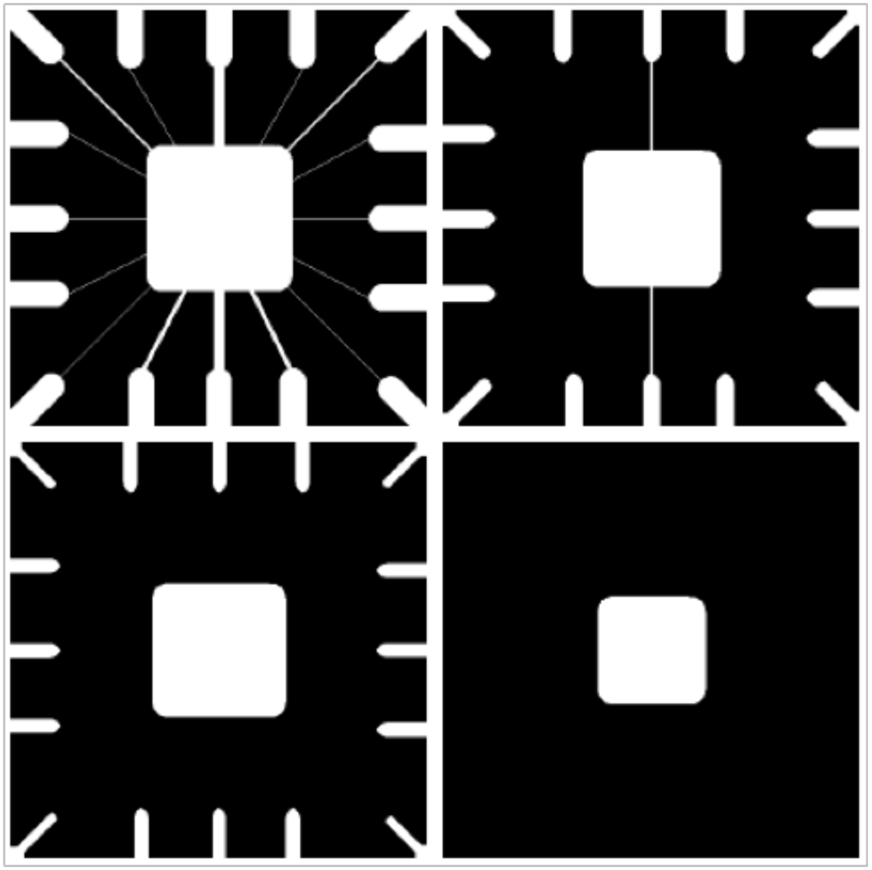
Erosion example



Original image

11x11 erosion mask

15x15 erosion mask

45x45 erosion mask

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Dilation

With A and B as sets in Z^2 , the dilation of A by B, denoted $A \oplus B$, is defined as:

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

The dilation of A by B is the set of all displacements such that the reflection of B and A overlap by at least one element.

The basic process of flipping B about its origin and then successively displacing it so that it slides over set (image) A is analogous to spatial convolution

Dilation

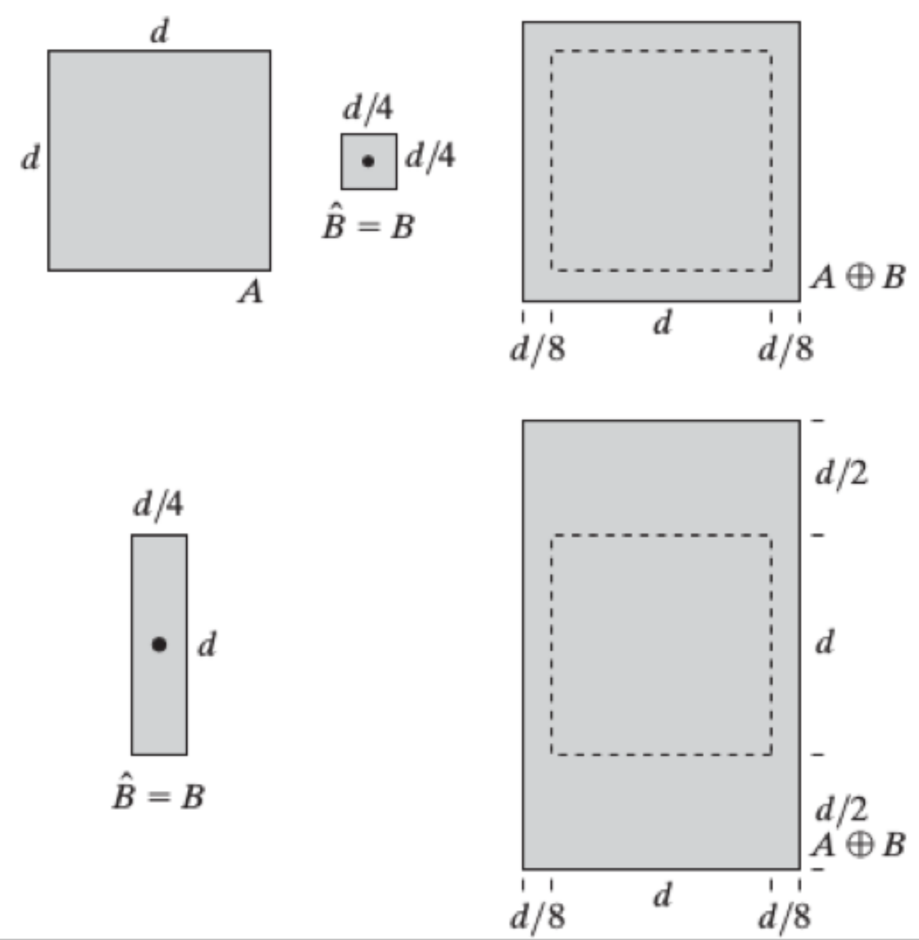

FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

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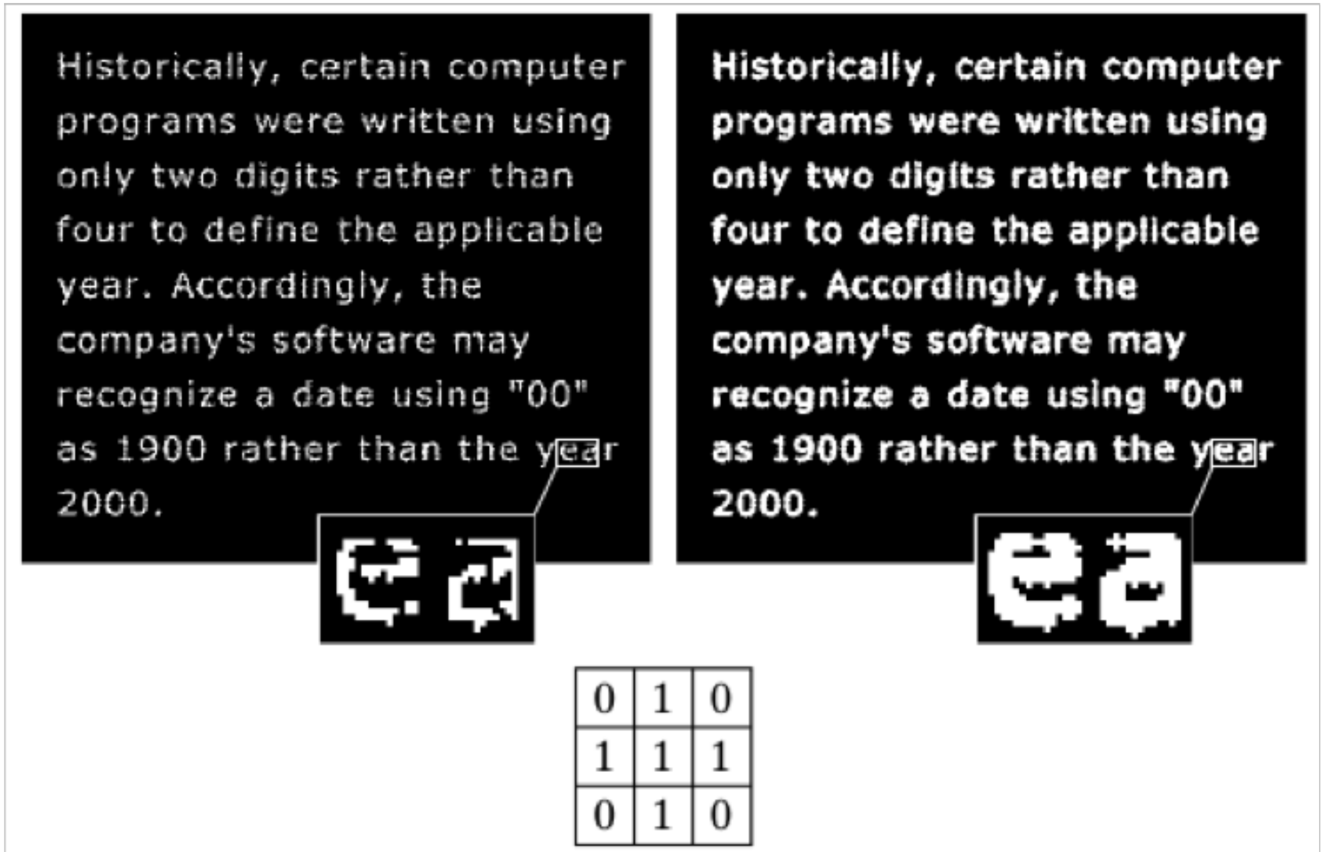
Dilation



One of the simplest applications of dilation is for bridging gaps

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Opening and Closing

Given an image A and a SE B:

Opening:

$$A \circ B = (A \ominus B) \oplus B$$

Erosion followed by a dilation

Closing:

$$A \bullet B = (A \oplus B) \ominus B$$

Dilation followed by an erosion

Opening and Closing

Opening: $A \circ B = (A \ominus B) \oplus B$

- smoothes the contour of an object
- breaks narrow isthmuses
- eliminates thin protrusions

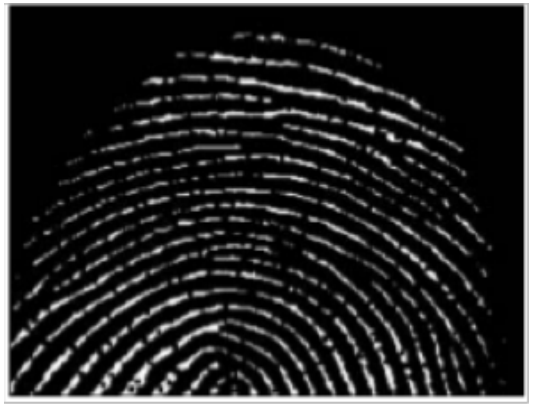
Closing: $A \bullet B = (A \oplus B) \ominus B$

- smooth sections of contours
- fuses narrow breaks and long thin gulfs
- eliminates small holes
- Fills gaps in the contour

Opening and Closing

Original noisy image

Erosion



Opening

Noise completely removed but some holes are created

Dilation partially closes the inner holes

Opening and Closing

Original image

Dilation

Closing

Most of the breaks are restored but ridges are thickened

Erosion reduces the thickness of the ridges

Opening and Closing

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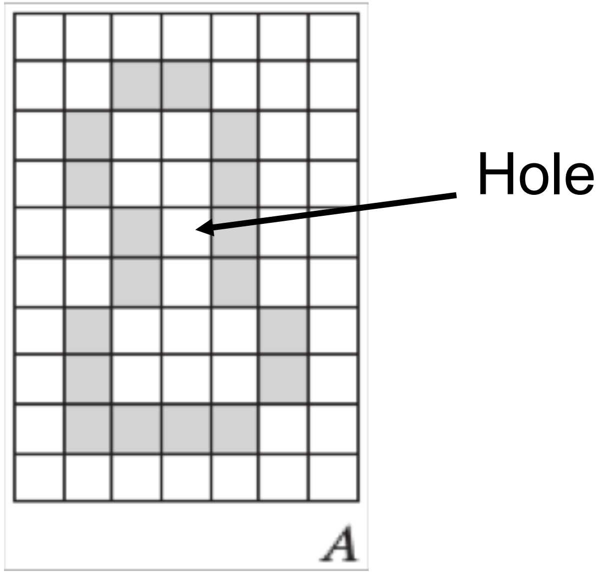
Original image

$$(A \circ B) \bullet B$$

A sequence of opening and closing both eliminate noise and close all the erroneous breaks due to an imperfect thresholding

Hole filling

A hole may be defined as a background region surrounded by a connected border of foreground pixels.



Given an initial point inside an hole, the goal is to fill the hole with 1s

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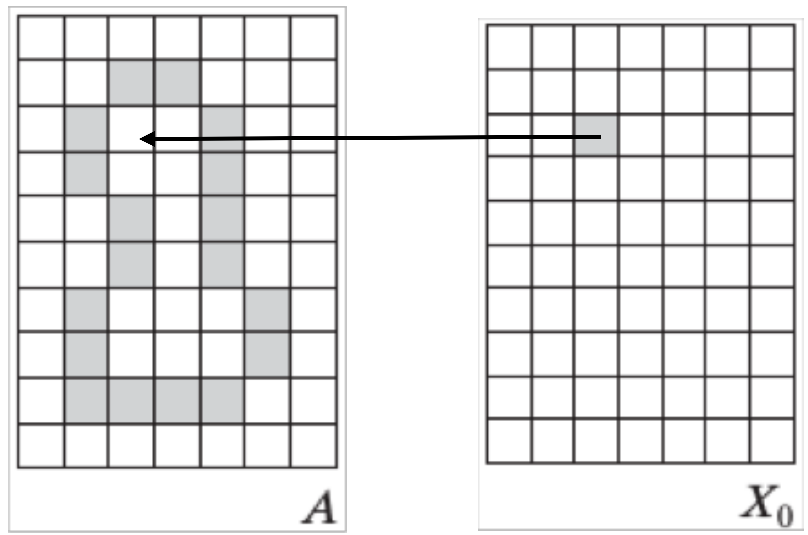
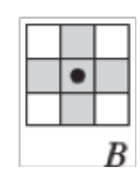
Hole filling

Let X_0 be the initial array with the same size of A, filled with 0s except at the location to a given point in each hole.

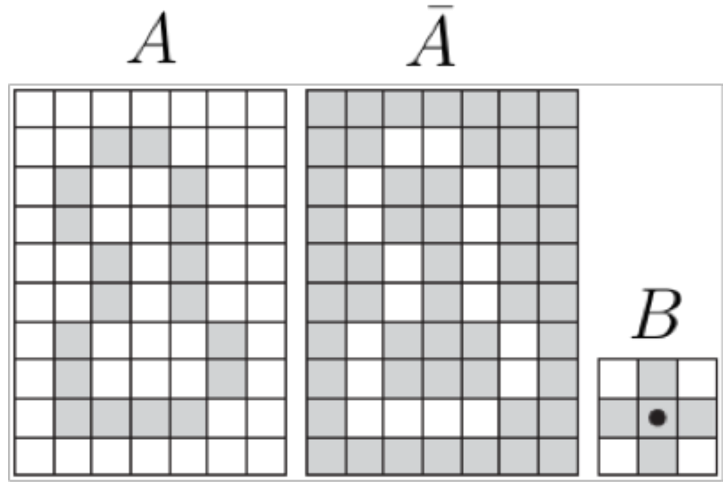


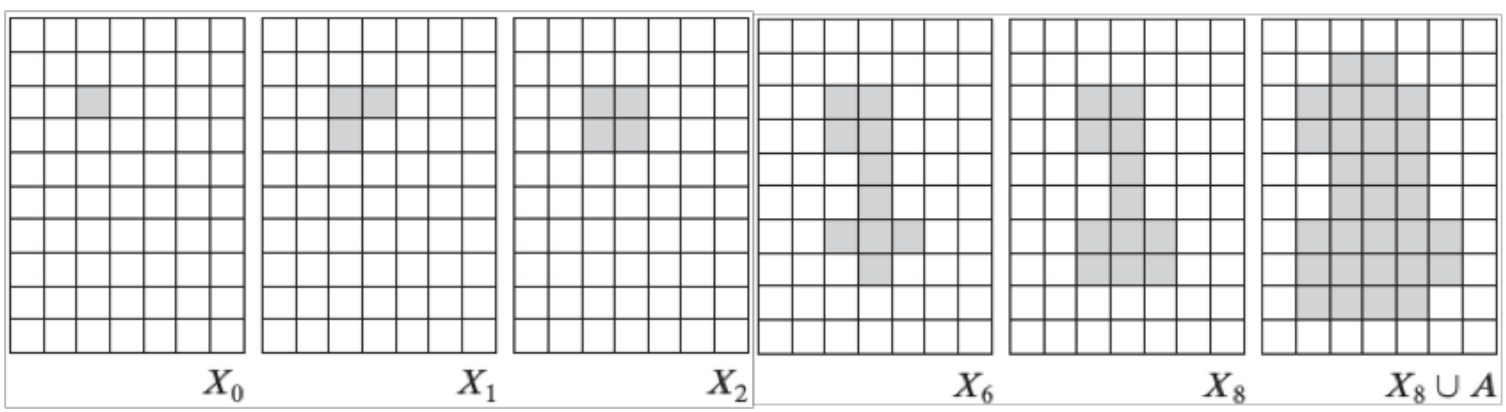
The following iterative procedure fills all holes with 1s:

$$X_k = (X_{k-1} \oplus B) \cap \bar{A}$$



Hole filling





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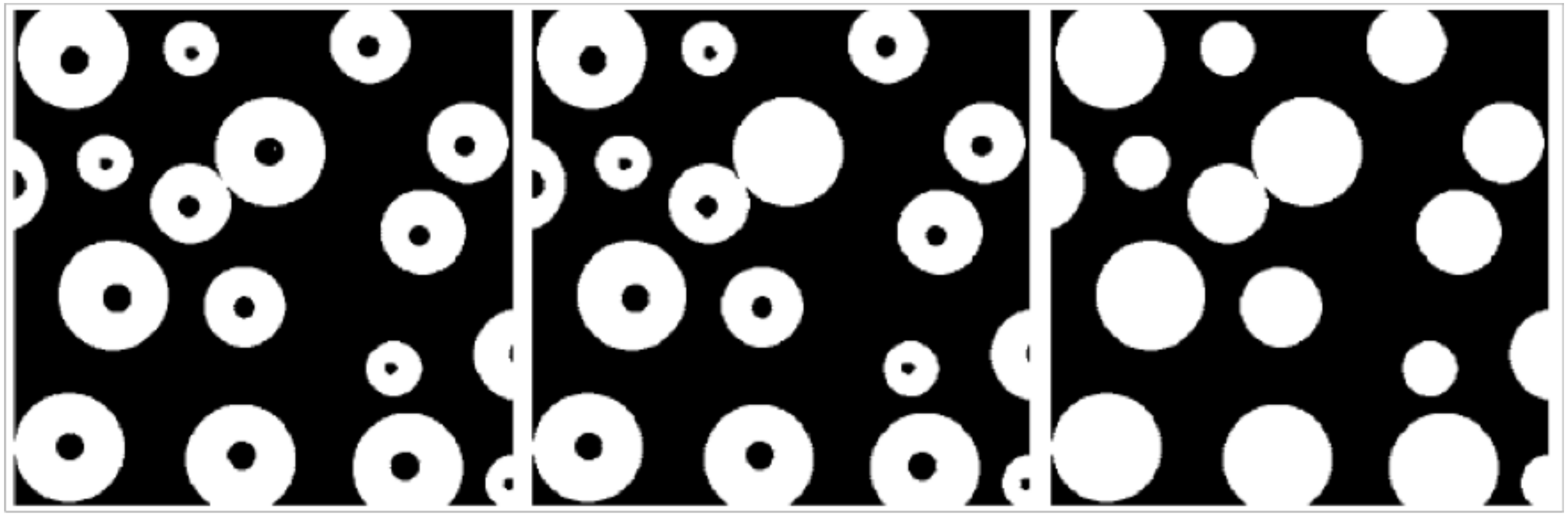
Hole Filling



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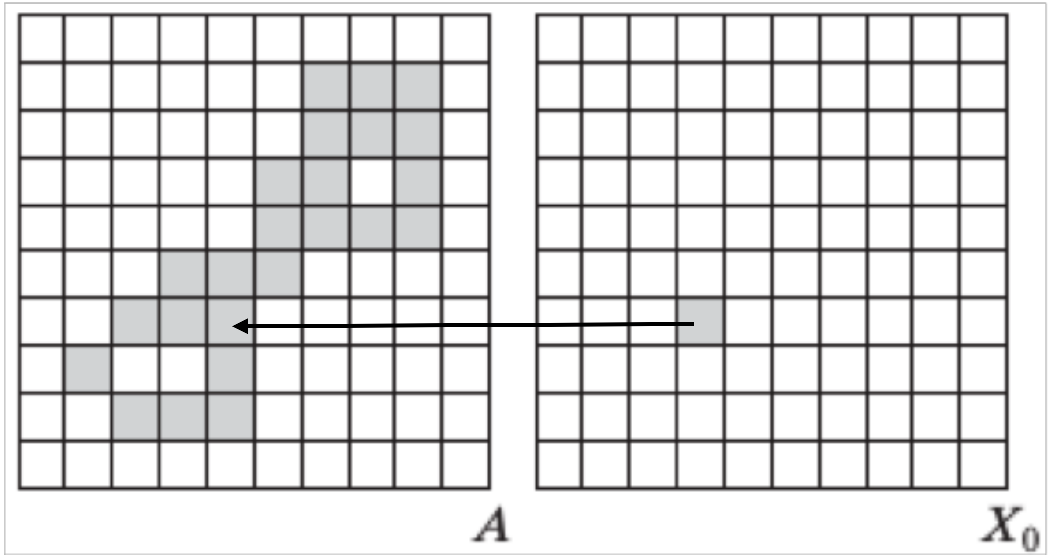
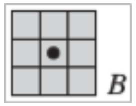
Connected components

Let X_0 be the initial array with the same size of A, filled with 0s except at the location to a known point in each region.

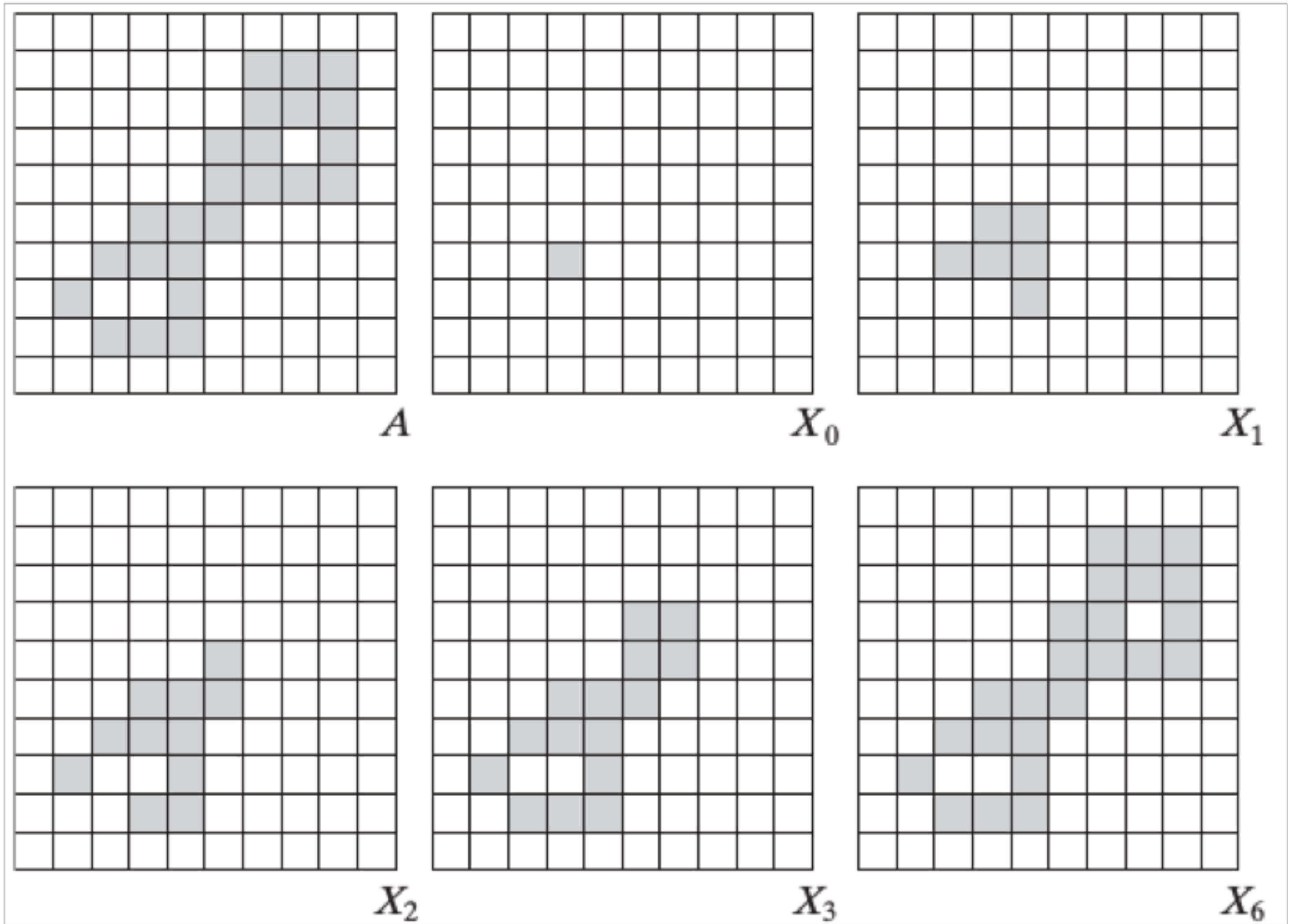


The following iterative procedure fills the connected component with 1s:

$$X_k = (X_{k-1} \oplus B) \cap A$$



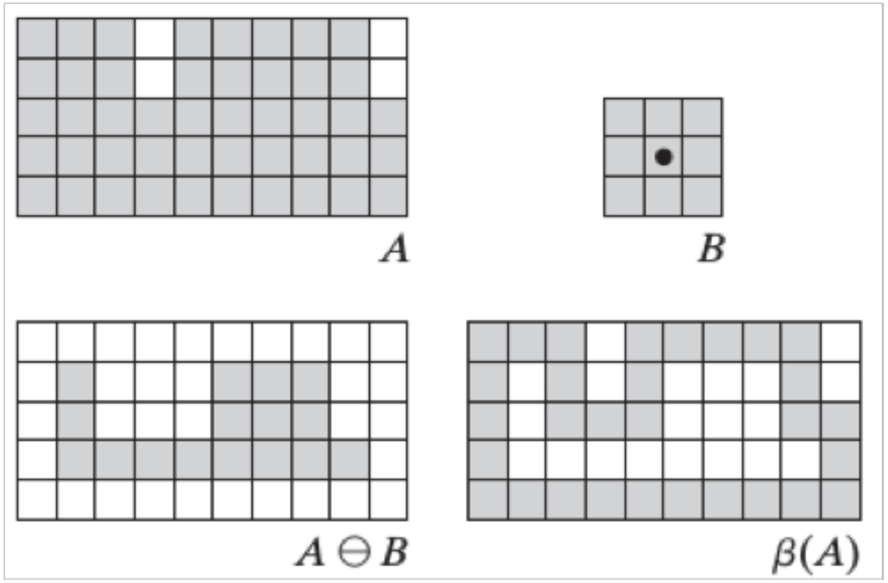
Connected components



Boundary extraction

The boundary of a set A can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

$$\beta(A) = A - (A \ominus B)$$



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Moore boundary tracking

Several algorithms require to extract an ordered sequence of foreground boundary points from a region

Assumptions:

1. We are working with binary thresholded images: 0:background 1:foreground
2. Images are padded with a border of 0s so that no foreground region touches the image border

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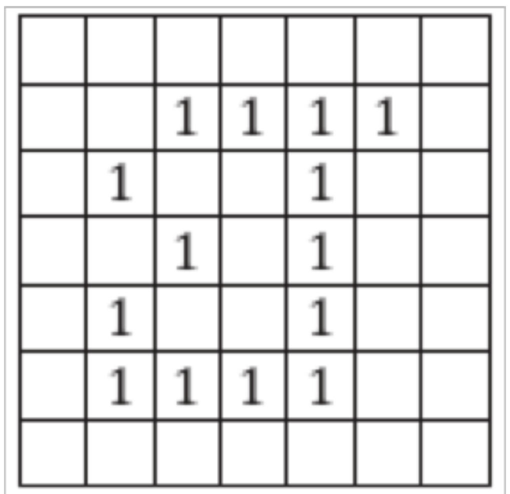
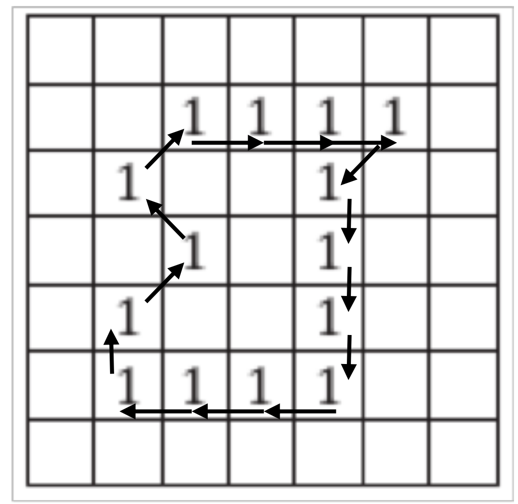
Moore boundary tracking





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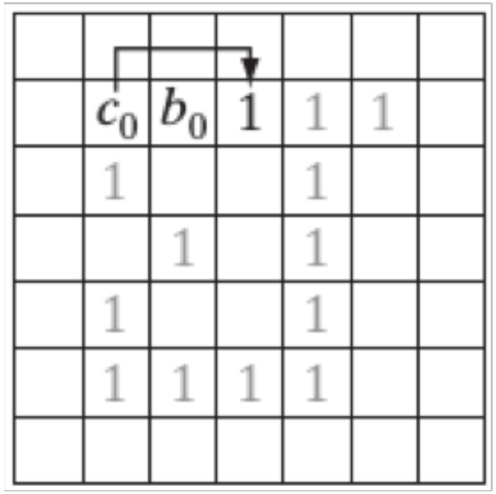
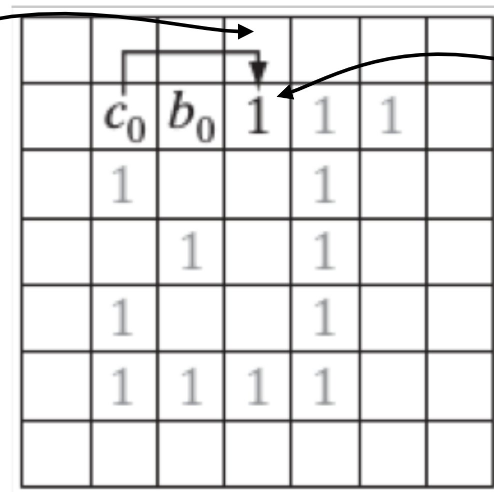
Moore boundary tracking



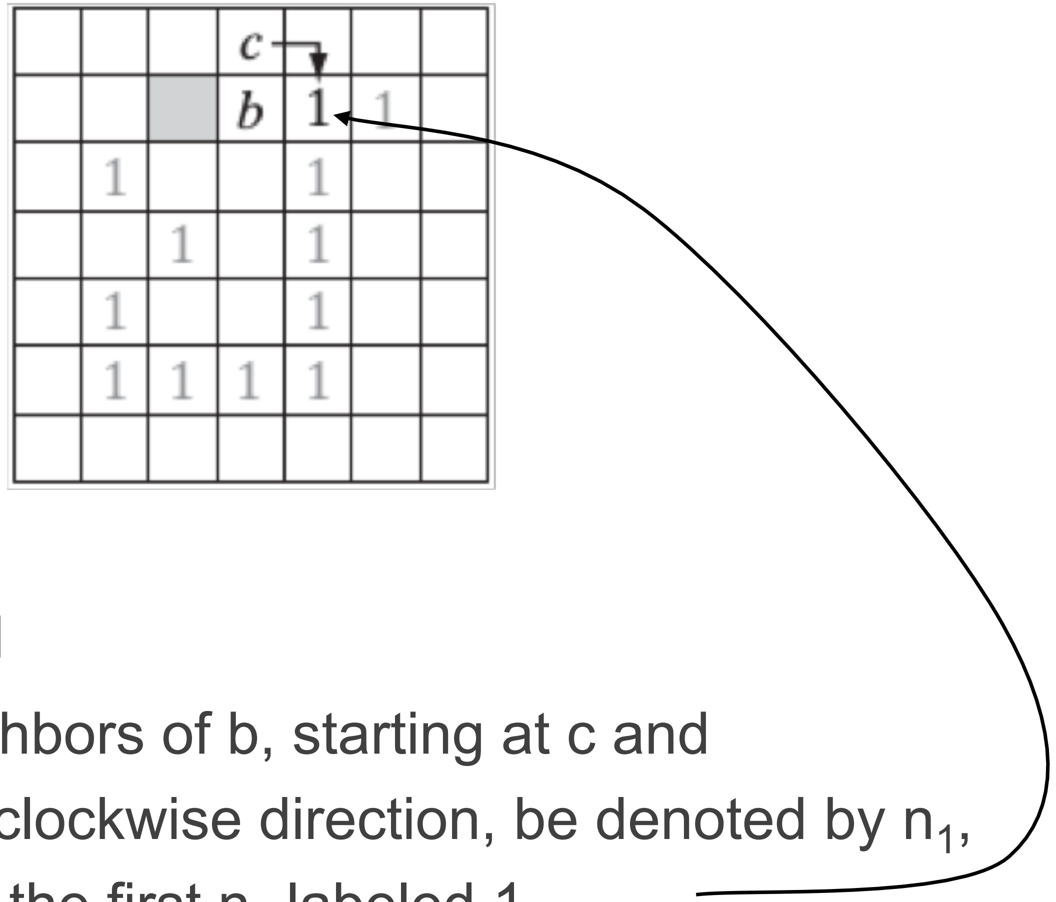
1. Let the starting point, b0 be the uppermost, leftmost point in the image that is labeled 1
   a. Let c0 be the west neighbor of b0

Moore boundary tracking



1. (initialization) Examine the 8-neighbors of b0, starting at c0 and proceeding in a clockwise direction.
   a. Let b1 denote the first neighbor encountered whose value is 1
   b. let c1 be the (background) point immediately preceding b1 in the sequence.

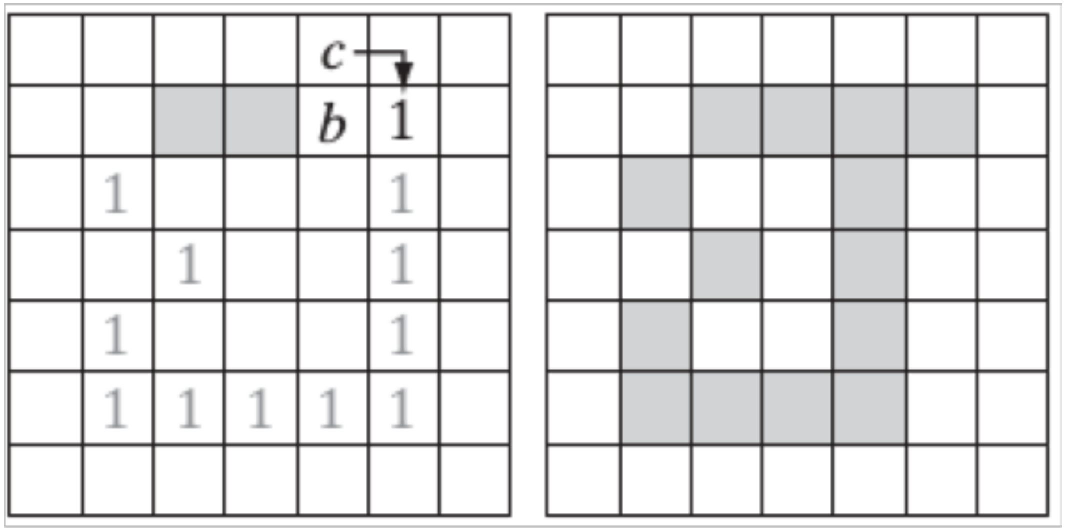
Moore boundary tracking



2. Let b=b1 c=c1

3. Let the 8-neighbors of b, starting at c and proceeding in a clockwise direction, be denoted by n_1, n_2 , ... , n_8 . Find the first n_k labeled 1.

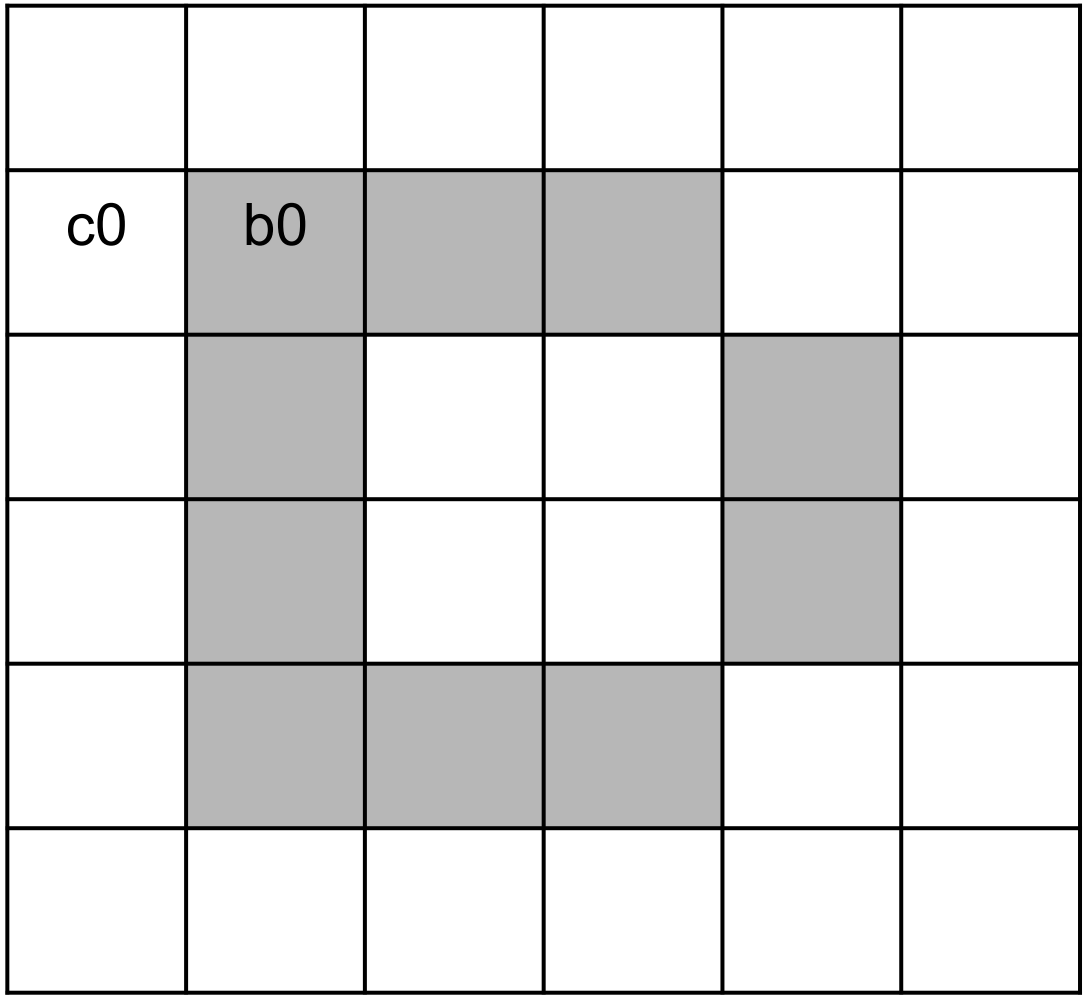
Moore boundary tracking



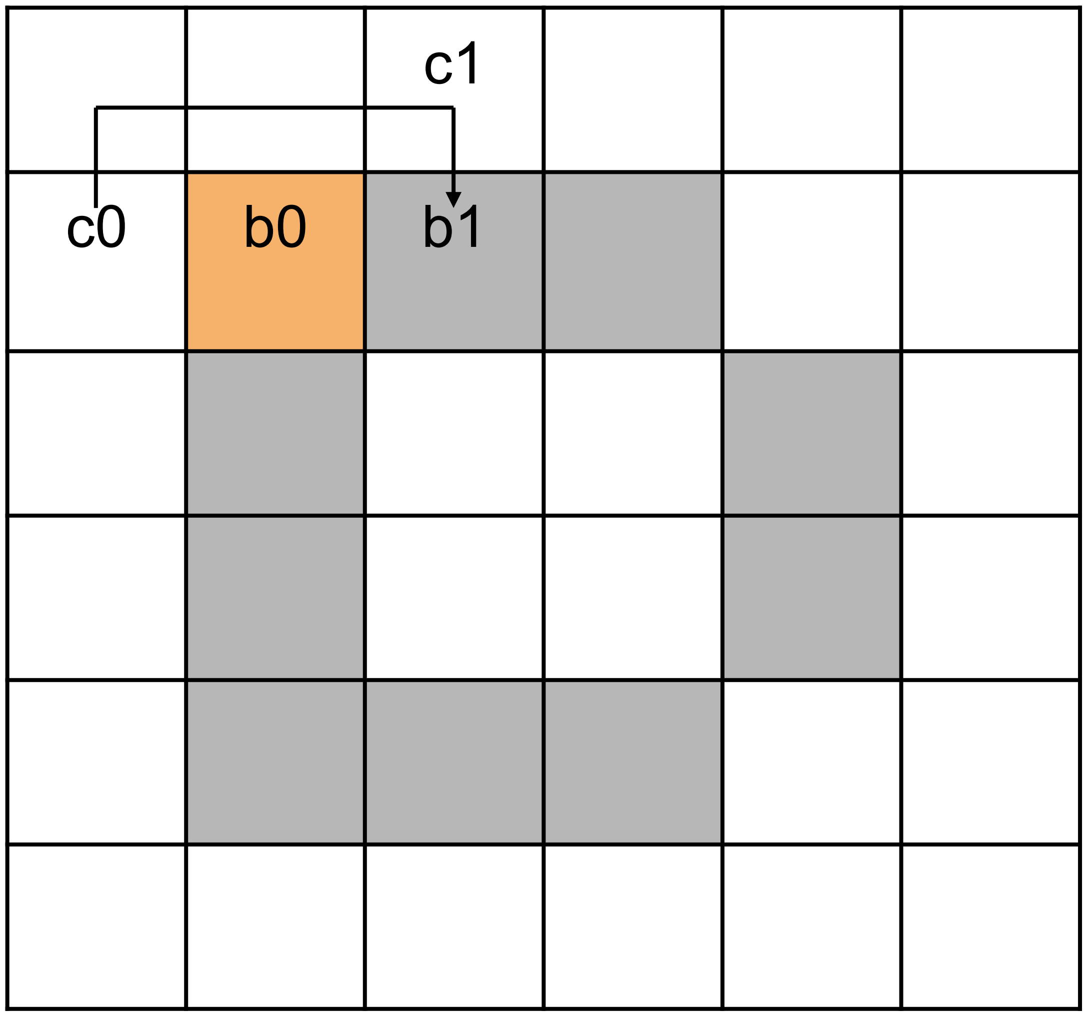
4. Let $b=n_k$, $c=n_{k-1}$

Repeat Steps 3 and 4 until $b = b_0$ and the next boundary point found is b_1.The sequence of b points found when the algorithm stops constitutes the set of ordered boundary points.

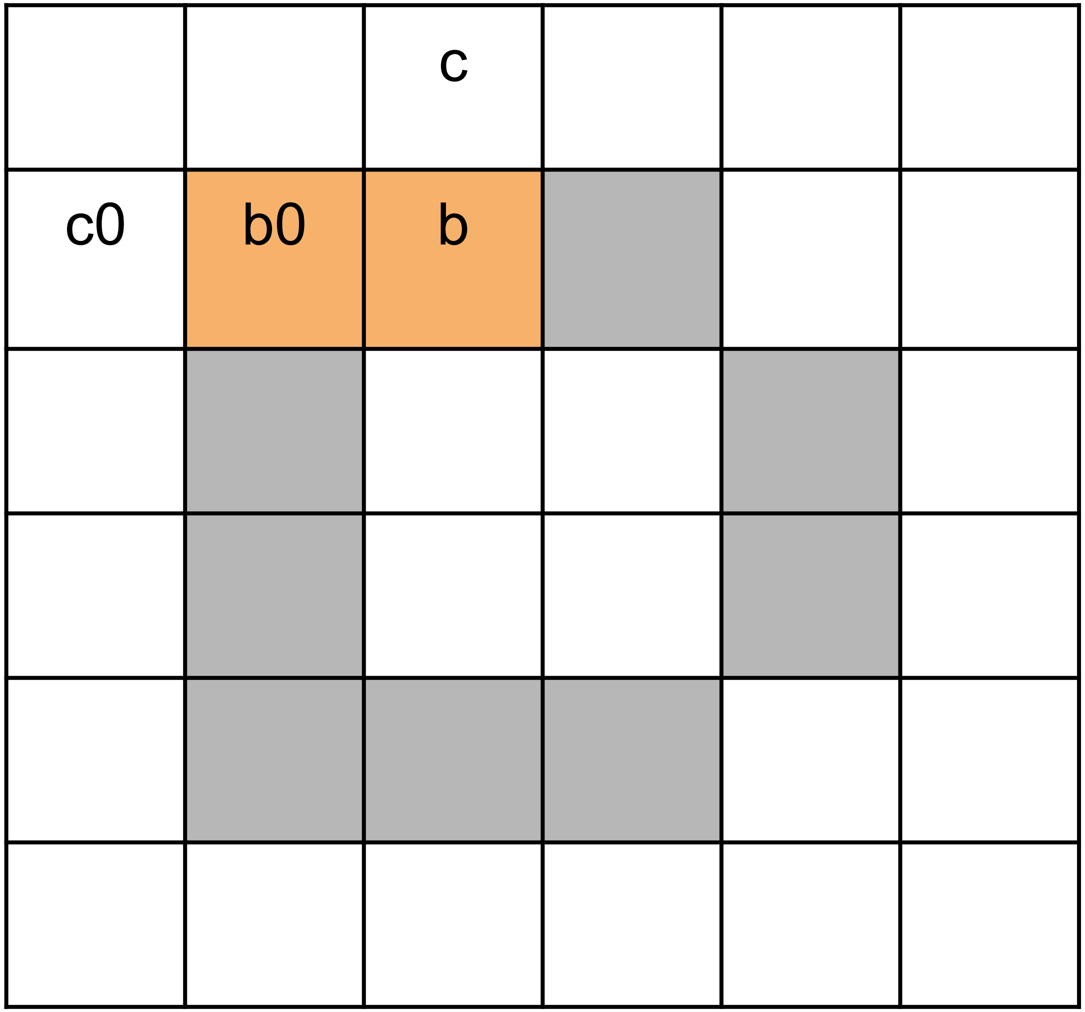
Moore boundary tracking



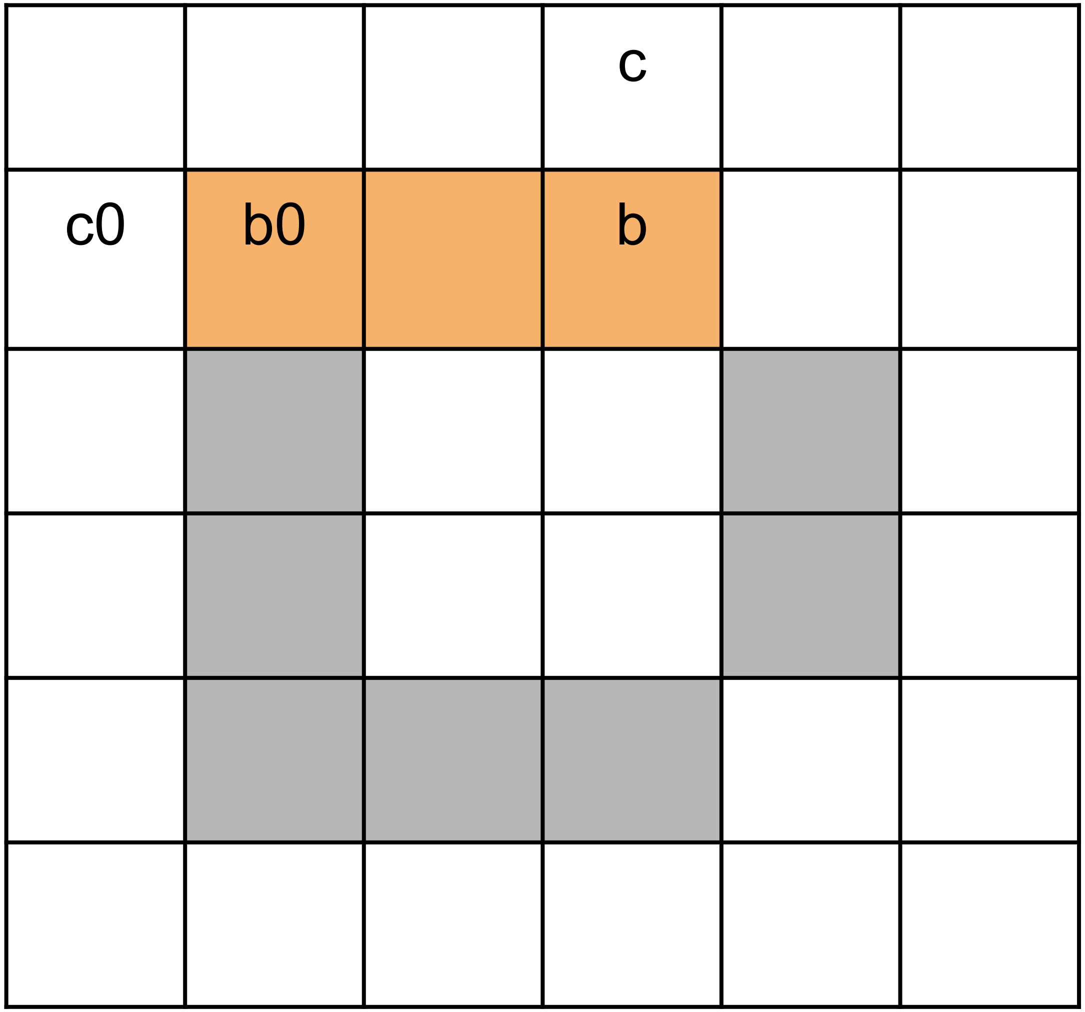
Moore boundary tracking



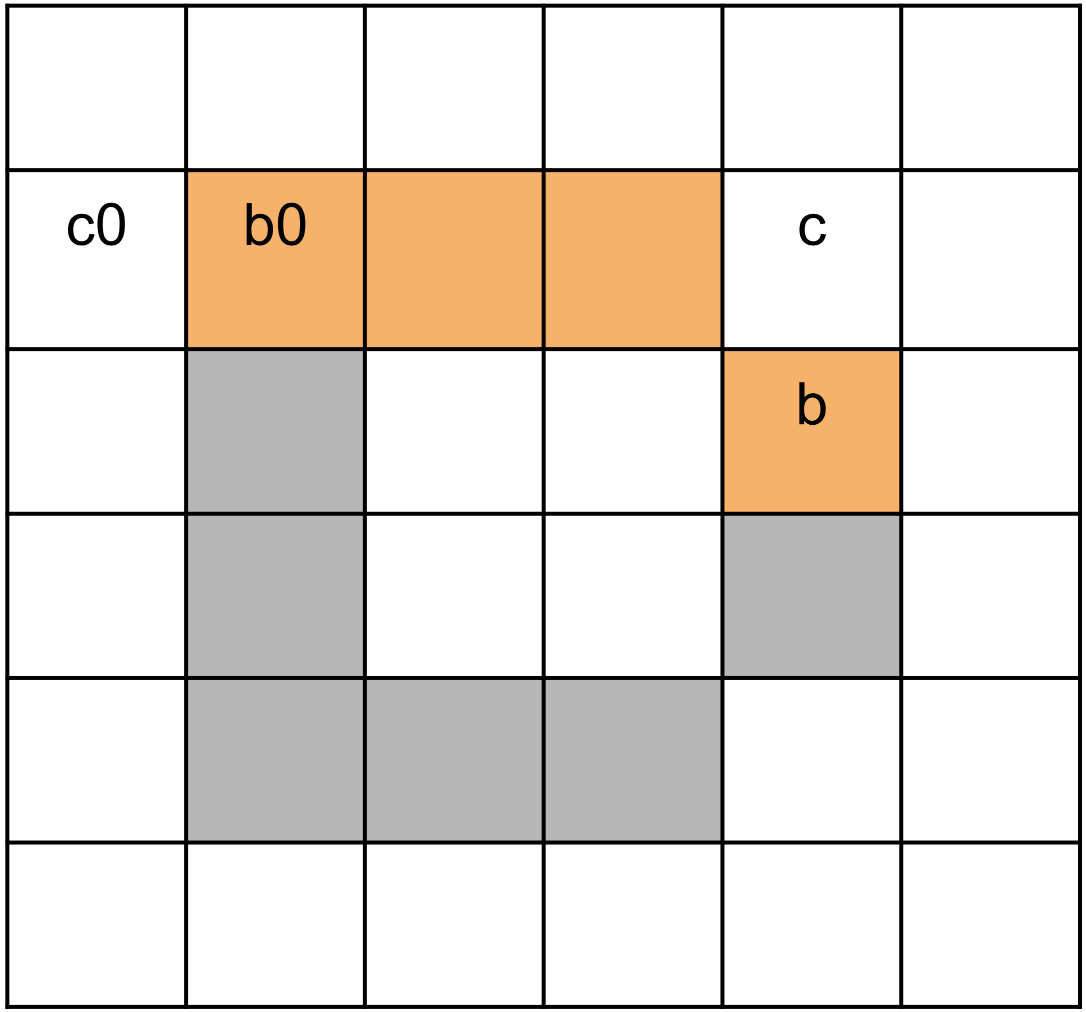
Moore boundary tracking



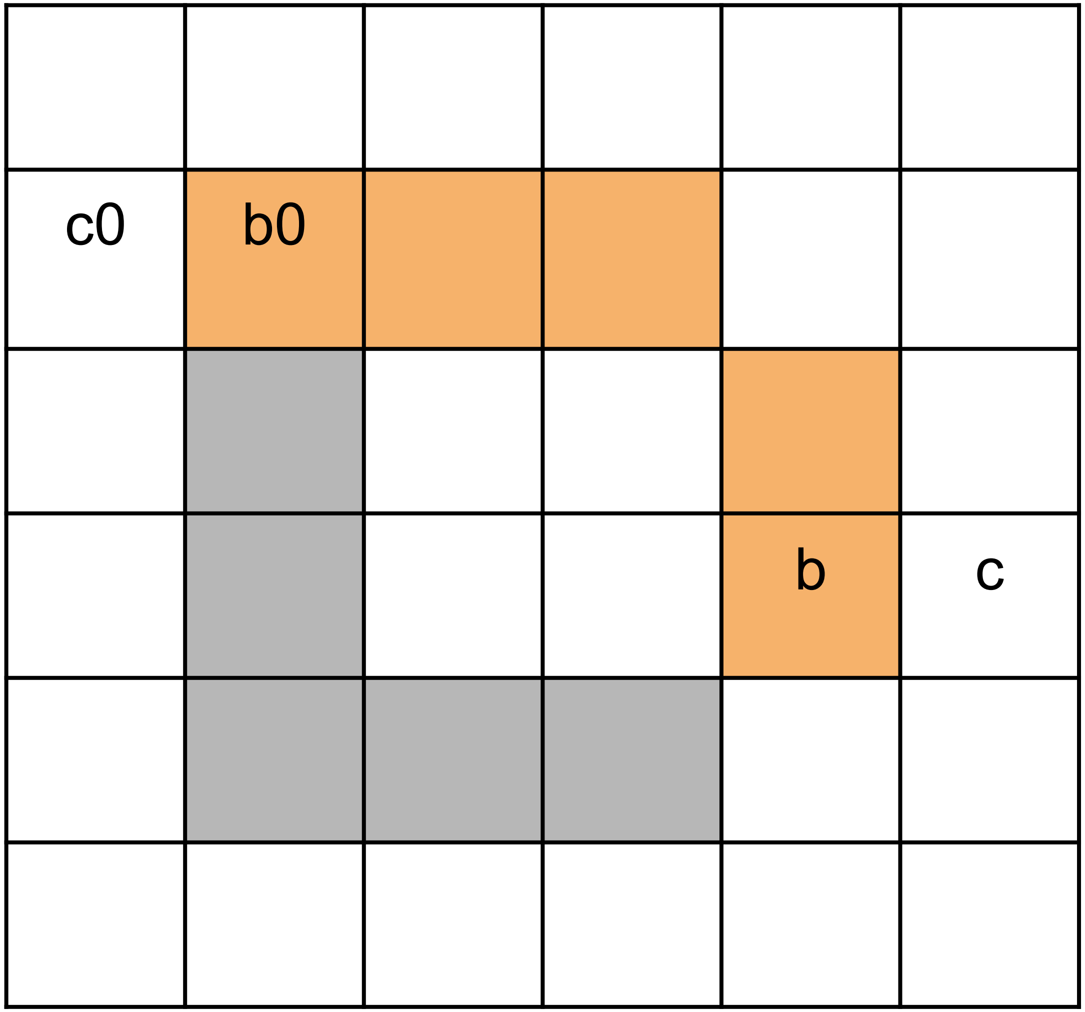
Moore boundary tracking



Moore boundary tracking



Moore boundary tracking



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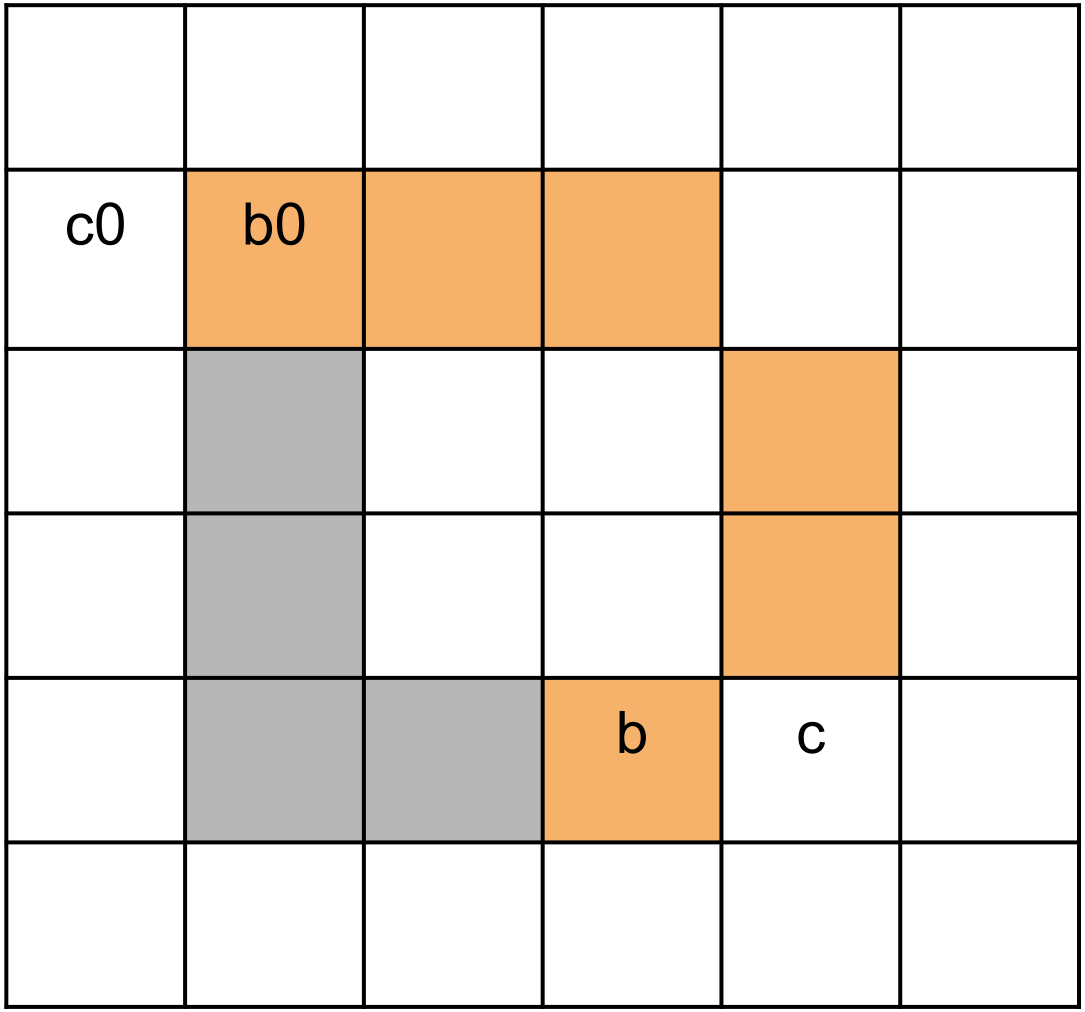
Moore boundary tracking



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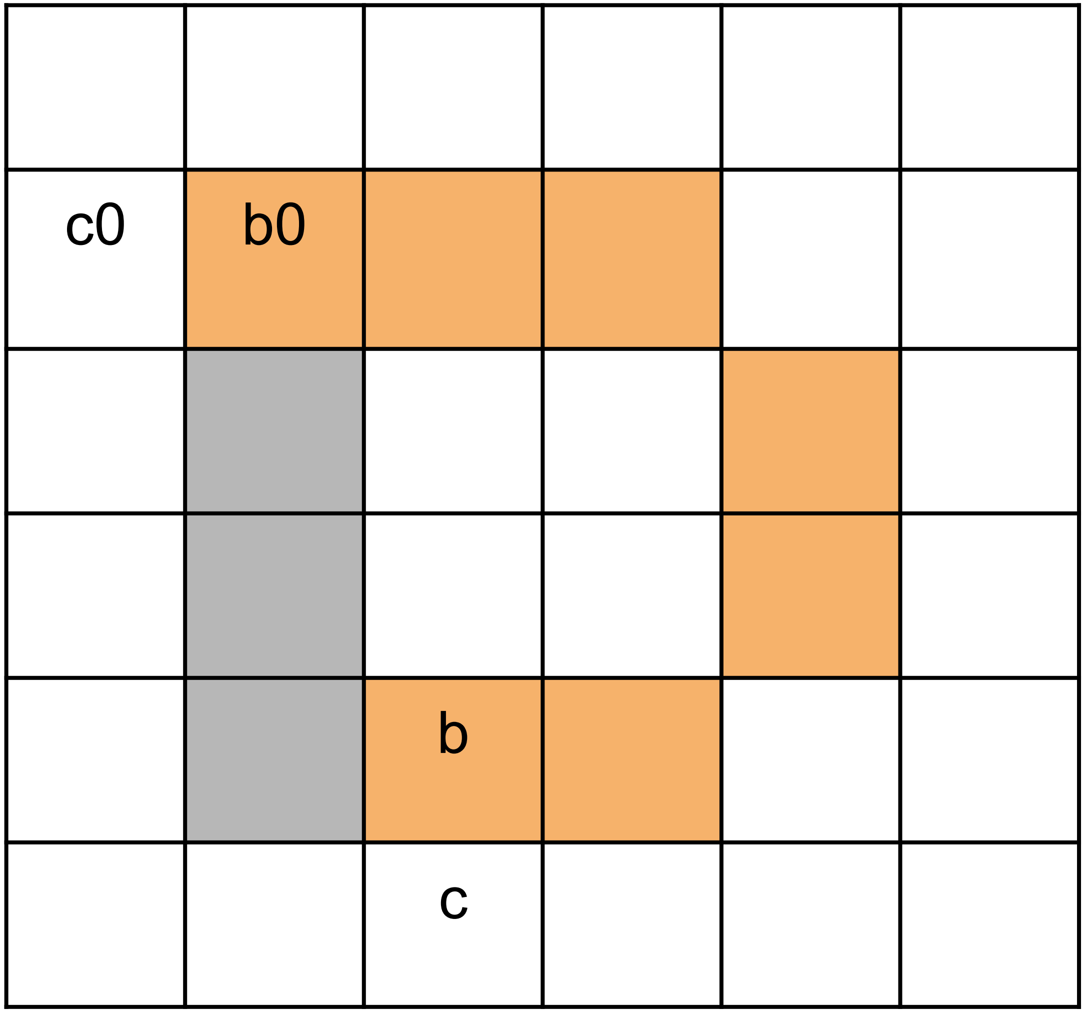
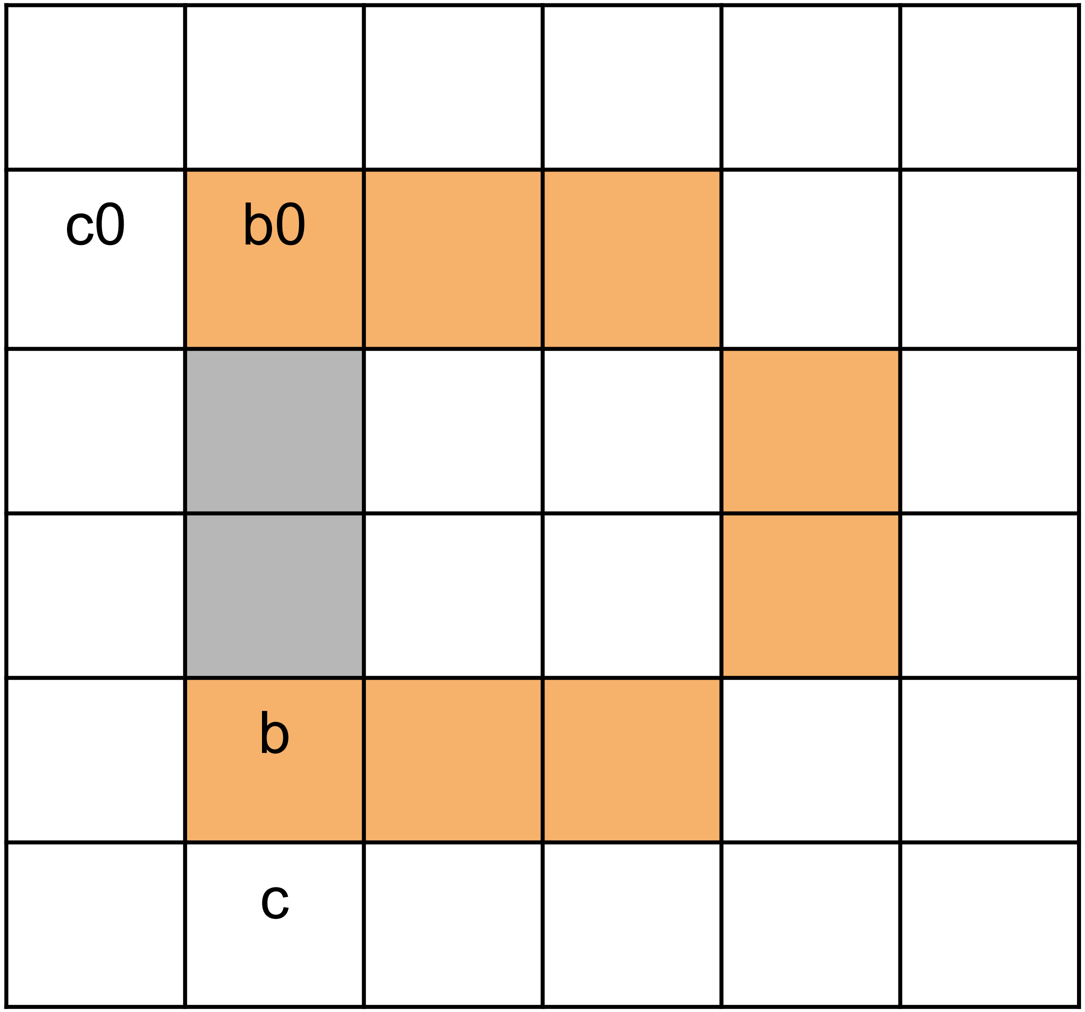
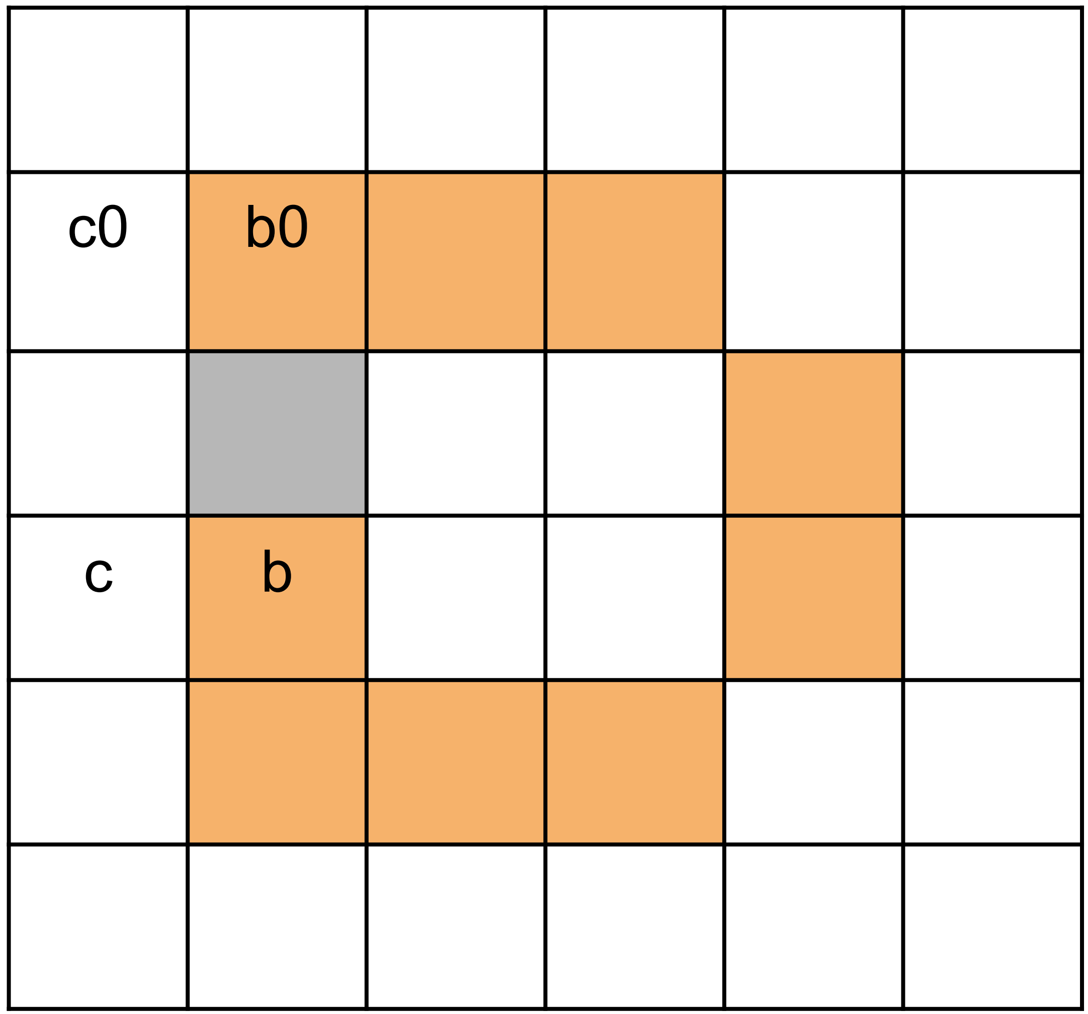
Moore boundary tracking



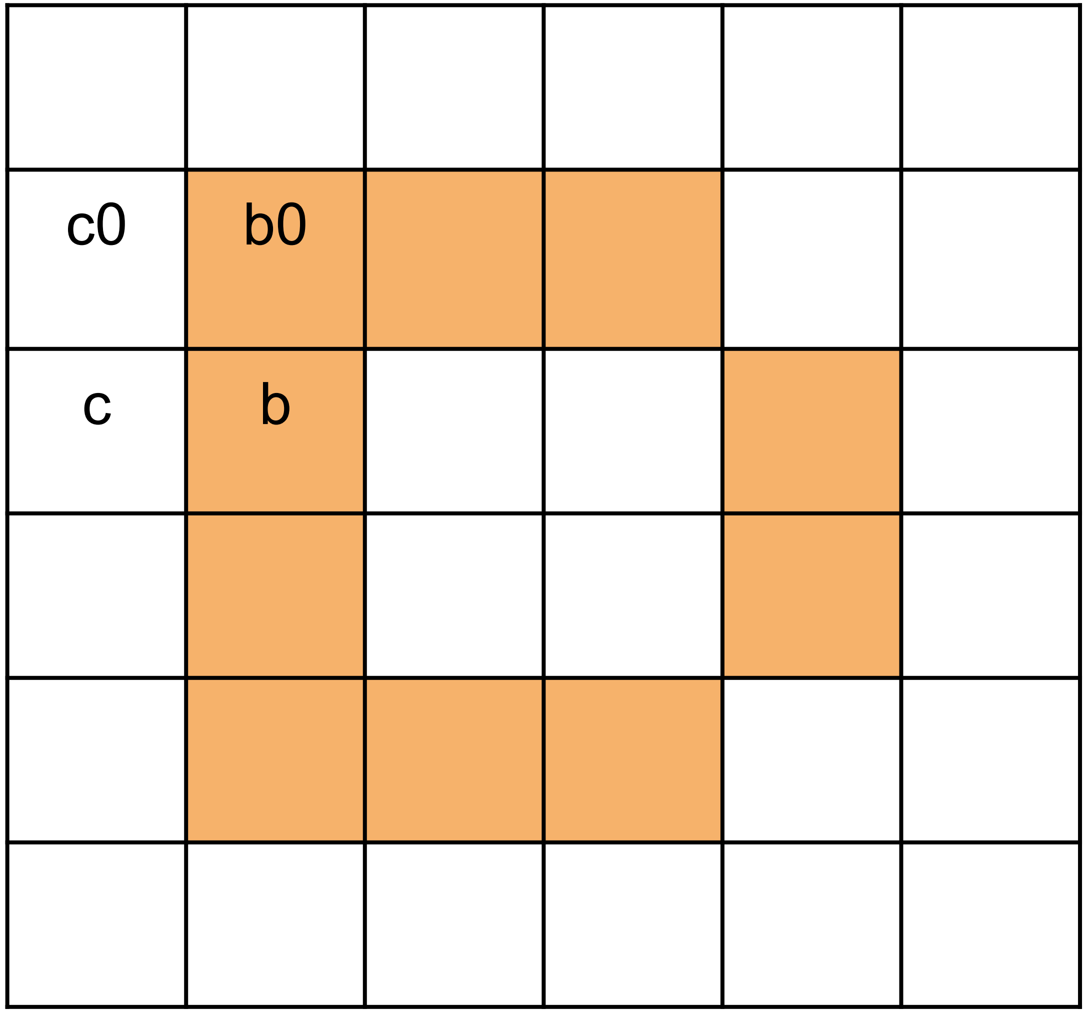
Moore boundary tracking



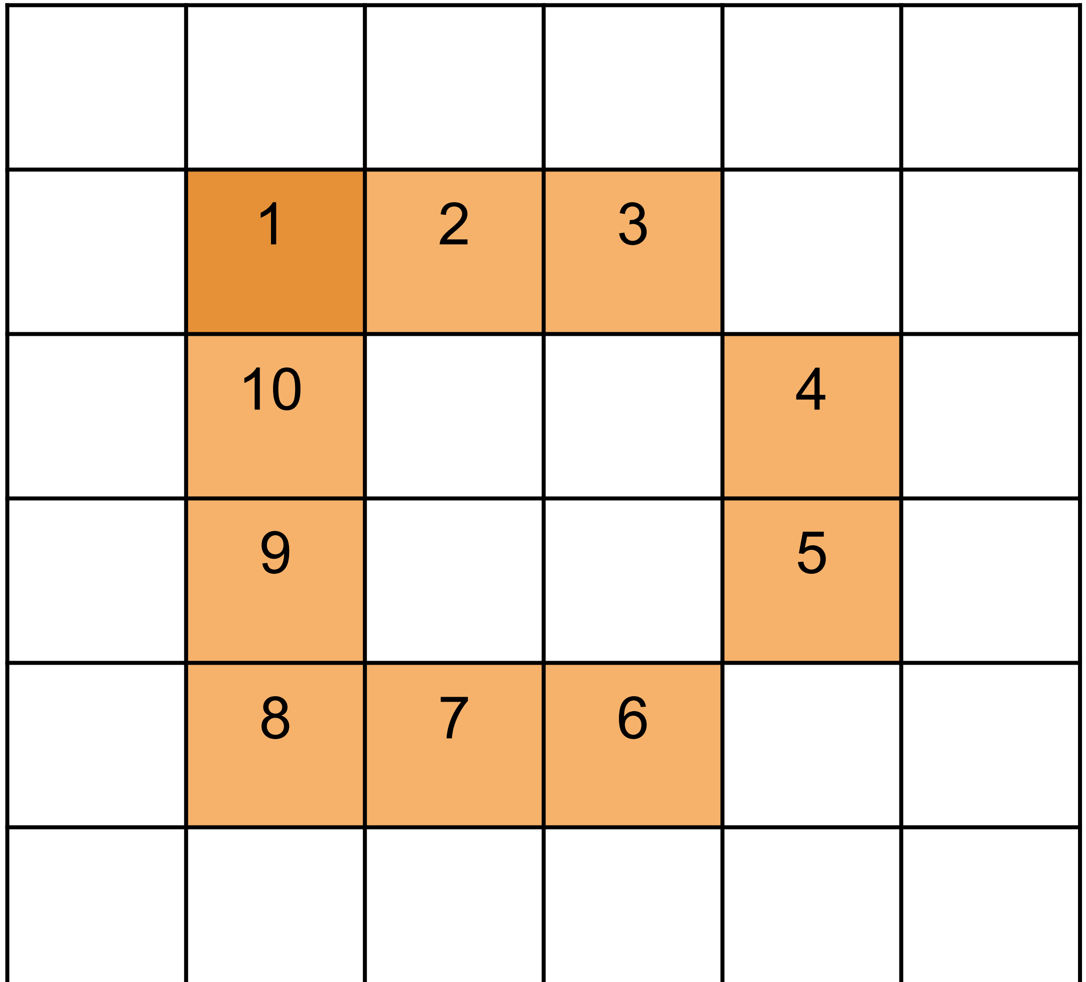
Moore boundary tracking



Moore boundary tracking



Moore boundary tracking



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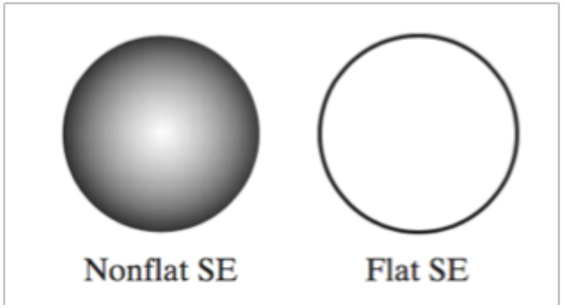
Grayscale morphology

So far, we considered thresholded images to derive a set of morphological operations using set theory.

Erosion, dilation, opening and closing can also be defined for grayscale images.

$$f(x,y) : \mathbb{Z}^2 \rightarrow \mathbb{R}$$
$$b(x,y) : \mathbb{Z}^2 \rightarrow \mathbb{R}$$

The structuring element b is used as "probe" to examine a given image for specific properties. Can be non-flat or flat (more common)



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Erosion

The erosion of f by a flat structuring element b at any location (x,y) is defined as the *minimum value* of the image in the region defined by b when the origin of b is at (x,y):

$$[f \ominus b](x, y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

The operation is conceptually similar to convolution. The structuring element b is shifted at every pixel location of the image and the minimum operation is performed among all the pixels in the region coincident with b

Dilation

The dilation of f by a flat structuring element b at any location (x,y) is defined as the *maximum value* of the image in the region defined by the reflection of b when the origin of it is at (x,y):

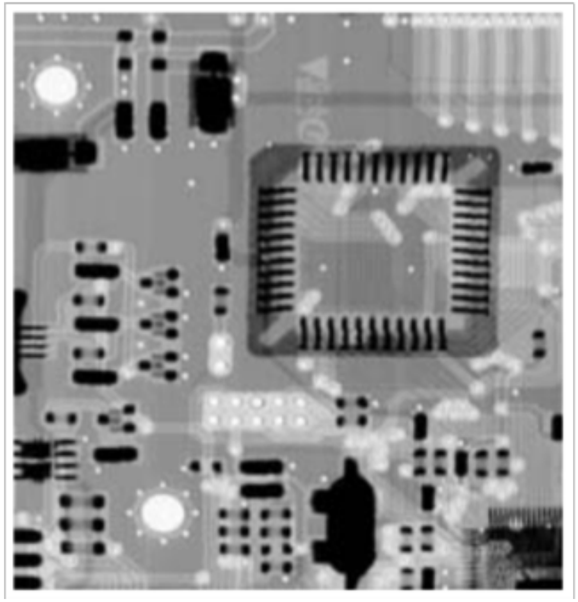
$$[f \oplus b](x, y) = \max_{(s,t)\in b} \{f(x - s, y - t)\}$$

Note the negative sign of the displacement resulting from the relation

$$\hat{b} = b(-x, -y)$$

Effect of erosion

Original



Eroded

Since erosion computes minimum intensity values over every neighborhood of (x,y), the resulting image is darker and the size of bright features is in general reduced.

Effect of dilation

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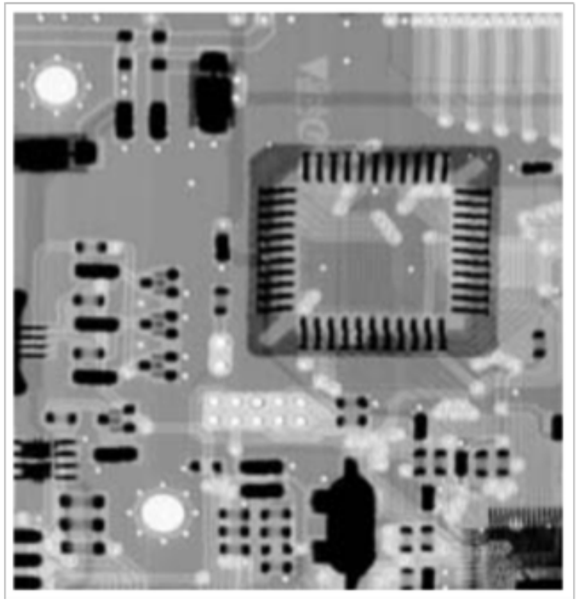
Original



Dilated

Dilation give opposite results with respect to erosion. The resulting image is brighter, bright features are thickened and the intensities of the dark features is reduced

Opening and Closing

The expressions for opening and closing gray-scale images have the same form as their binary counterparts:

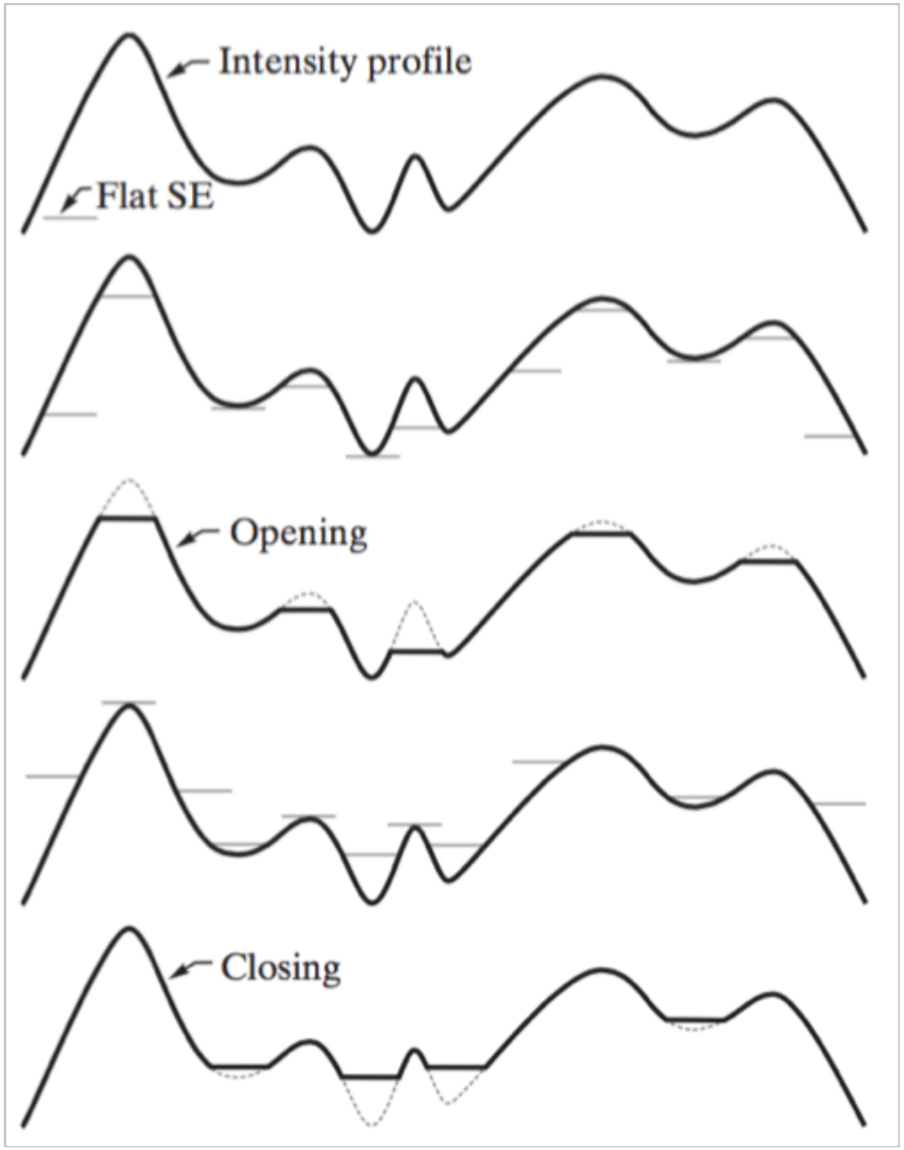
$$A \circ B = (A \ominus B) \oplus B \qquad A \bullet B = (A \oplus B) \ominus B$$

Geometric interpretation:

Suppose that f and b are 3D surfaces,

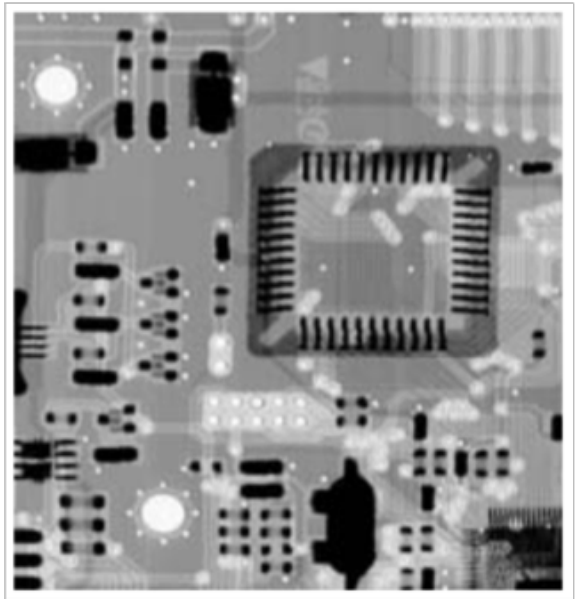
The opening of f by b is like pushing the structuring element up from below against the under-surface of f. The closing operation is like pushing down the structuring element on top of the curve while being translated to all locations

Opening and Closing

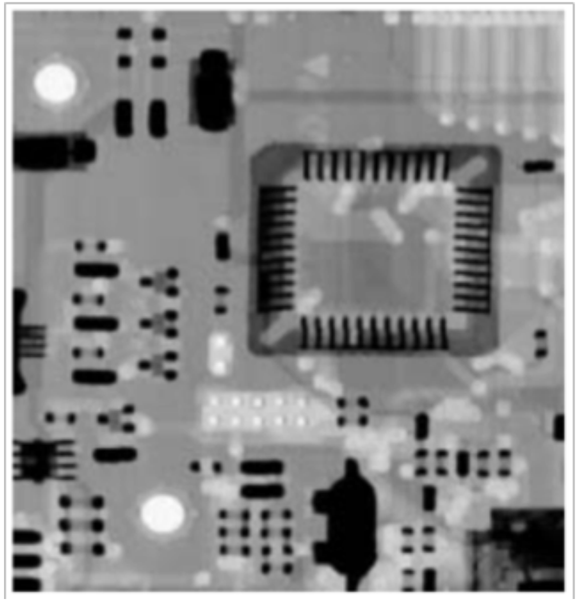


Effect of opening

Original



Opening



Opening **attenuate bright features** and has negligible effect on the dark features and the background of the image

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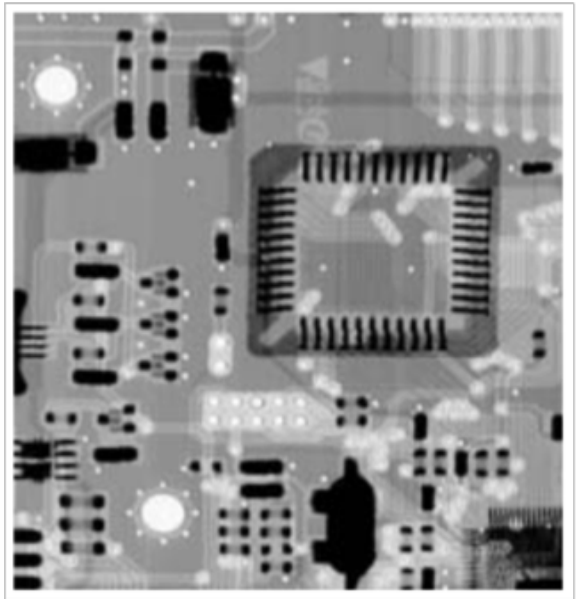
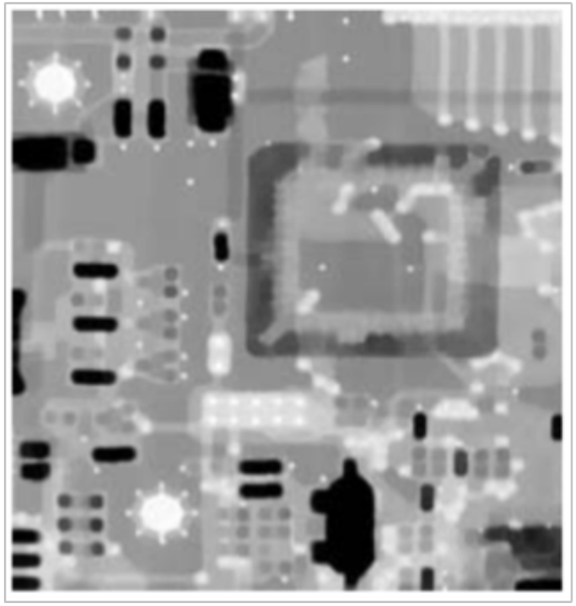
Effect of closing

Original



Closing



Closing **attenuate dark features** and has negligible effect on the bright features and the background of the image

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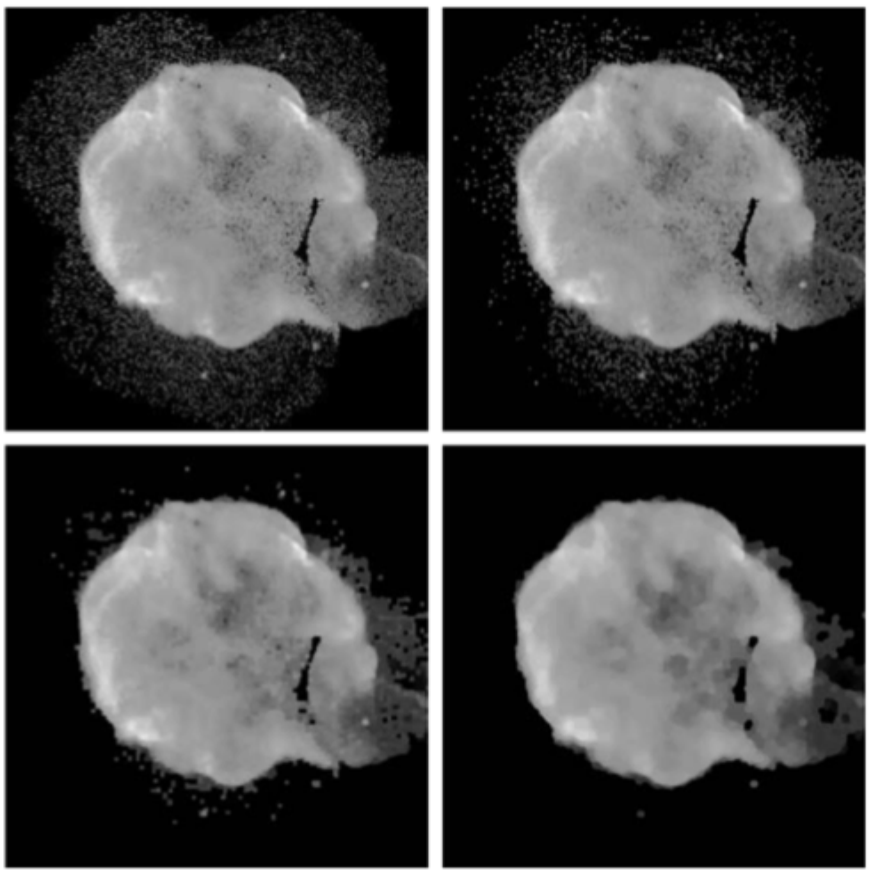
Morphological smoothing

Because opening suppresses bright details, and closing suppresses dark details, they are used often in combination for image smoothing and noise removal



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

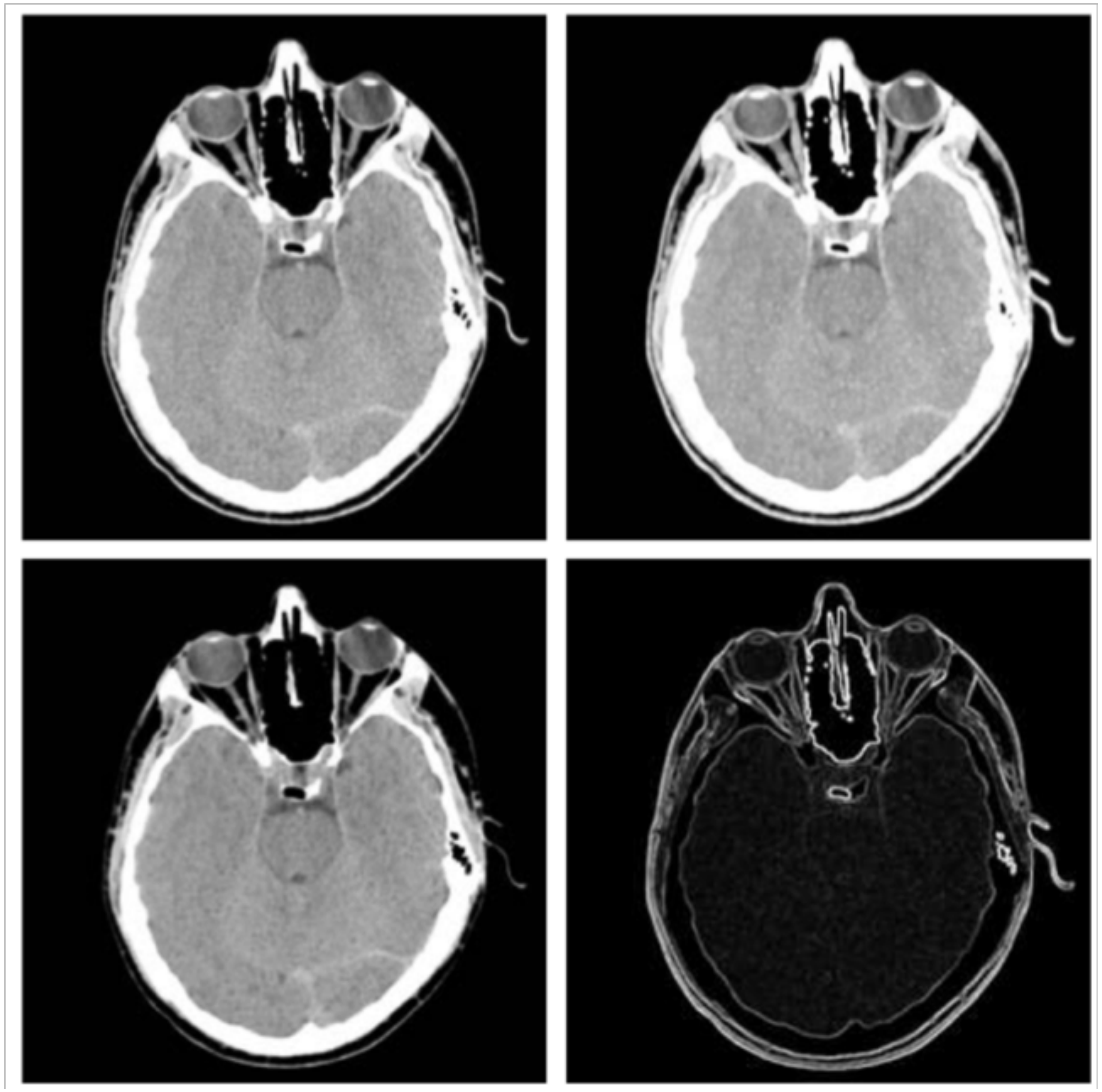
Morphological gradient

The dilation thickens regions in an image and the erosion shrinks them. Their difference is an operation with the effect of emphasize the boundaries between regions

$$g = (f \oplus b) - (f \ominus b)$$

The net result is an image in which the edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect

Morphological gradient



Top-hat, Bottom-hat

The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

the bottom-hat transformation of f is defined as the closing of f minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

Goal: select objects from an image by using a structuring element in the opening or closing operation that does not fit the objects to be removed. The difference then removes just the selected objects

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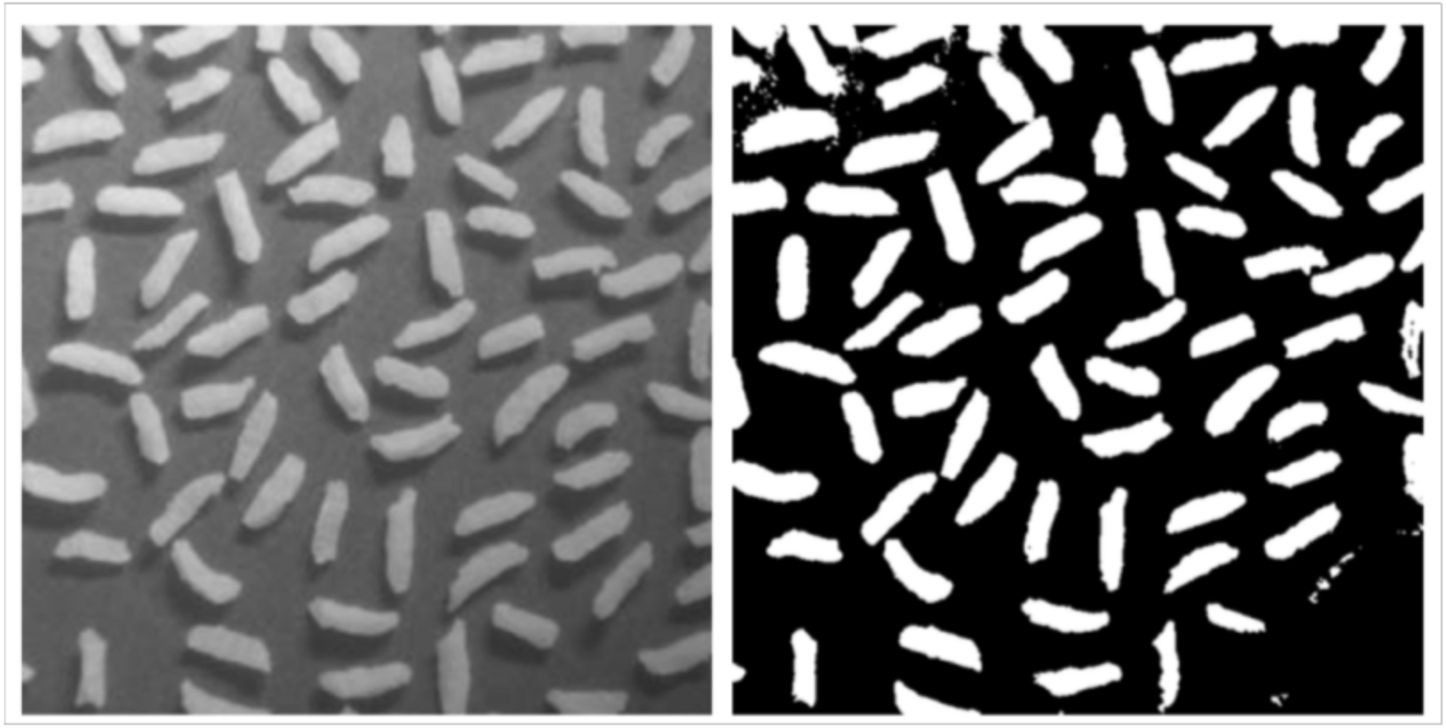
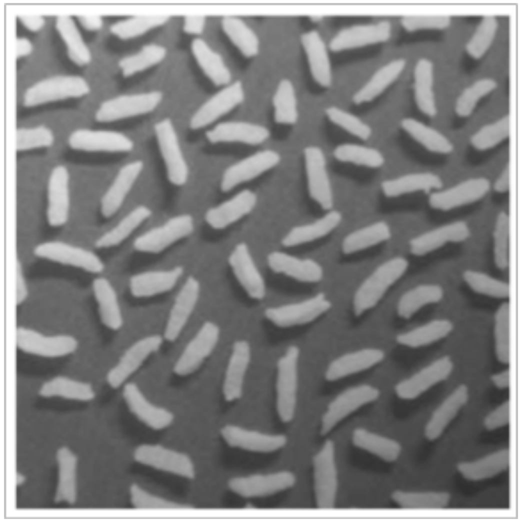
Top-hat to correct illumination

If an images exhibit uneven illumination, a global thresholding operation may fail on some areas

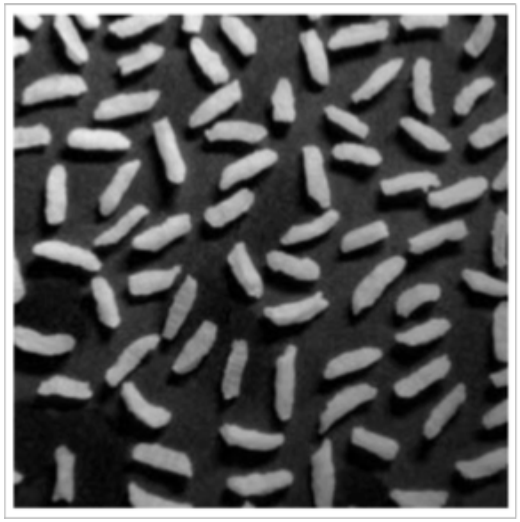


Top-hat to correct illumination



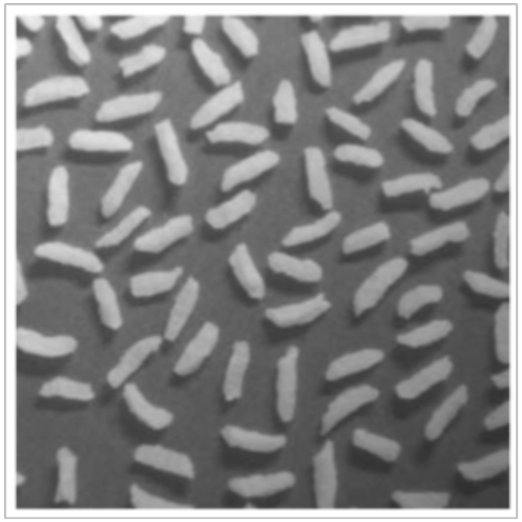
Original

Opening using a 40x40 disc



Top-hat

Top-hat to correct illumination



Original

Otsu

Otsu after Top-hat