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Computer Vision

Intensity transformations

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Introduction

We will discuss techniques that modify the intensity of pixels implemented in the **spatial domain** (ie. the image plane containing the pixels of the image)

A spatial domain process can be described by the expression:

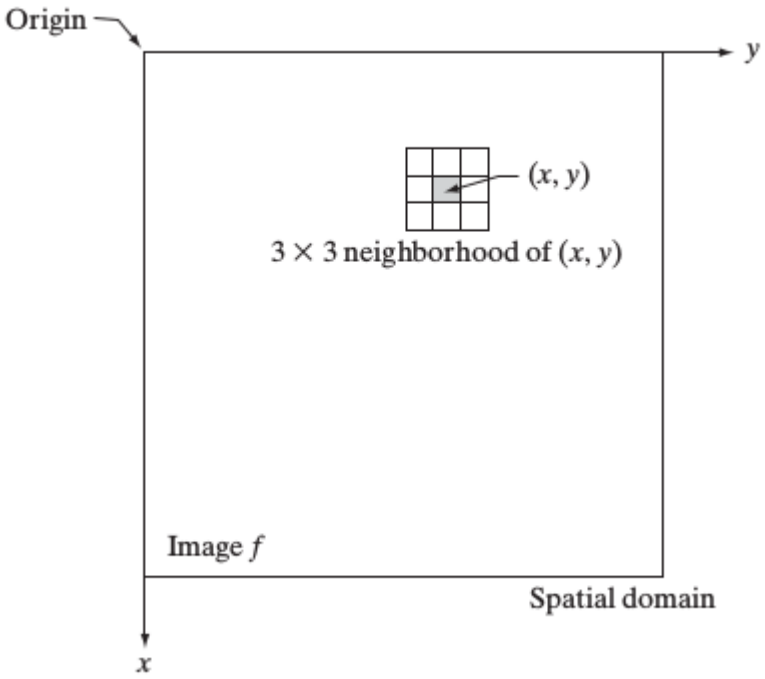
$$g(x,y) = T[f(x,y)]$$

Output image

Operator on f defined over a neighborhood of (x,y)

Input image

Introduction



Typically, the neighborhood of (x,y) is:

- Rectangular
- Centered on (x,y)
- Much smaller than the size of the image

Intensity transformations

When the neighborhood has size 1x1, g(x,y) depends only on the value of f at (x,y)

T is an **intensity transformation function**

$$s = T(r)$$

Where s and r are the intensity of g() and f() at a generic point (x,y)

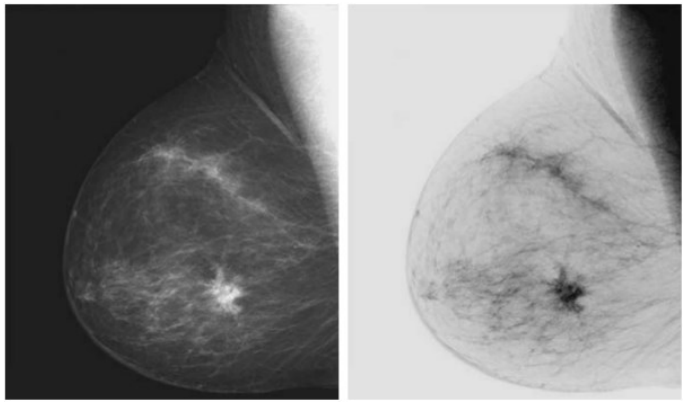
Note: not to be confused with **spatial transformations:** $g(\mathbf{x}) = f(s(\mathbf{x}))$

Negative

The negative of an image with intensity levels in the range [0...L-1] is obtained by the following expression:

$$s = L - 1 - r$$

This processing enhances white or gray details embedded in dark regions



Gain/Bias

Two commonly used point processes are multiplication and addition with a constant:

$$s = \alpha\, r + \beta$$

The two parameters α>0 and β are often called *gain* and *bias* and control **contrast** and **brightness** respectively.

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Gain/Bias

Original Image

$\alpha = 2$

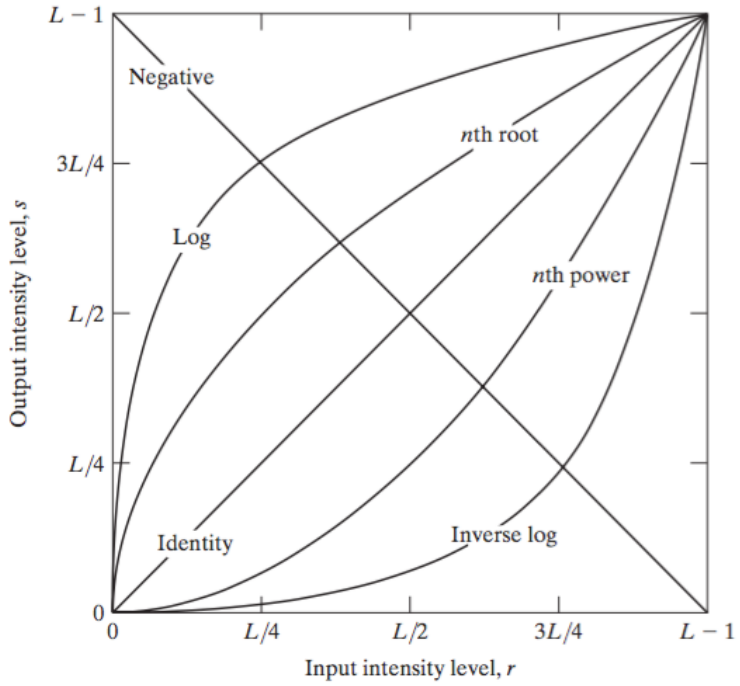
$\beta = 100$

Log Transformations

Log transformations are useful to **compress the dynamic range** for images with large variation in pixel values

$$s = c\log(1 + r)$$

Arbitrary constant



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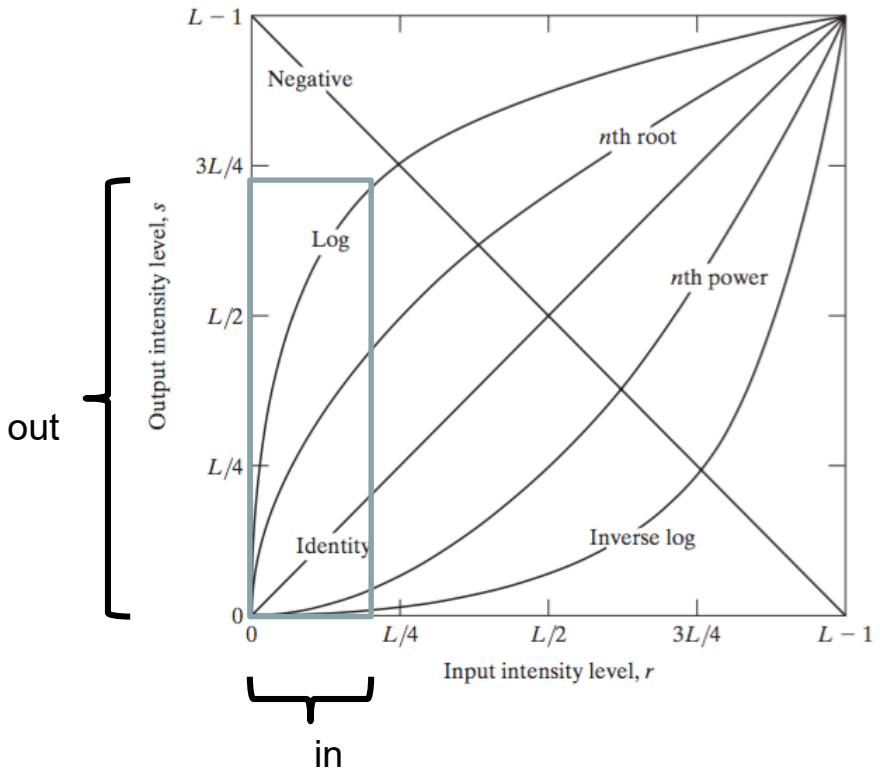
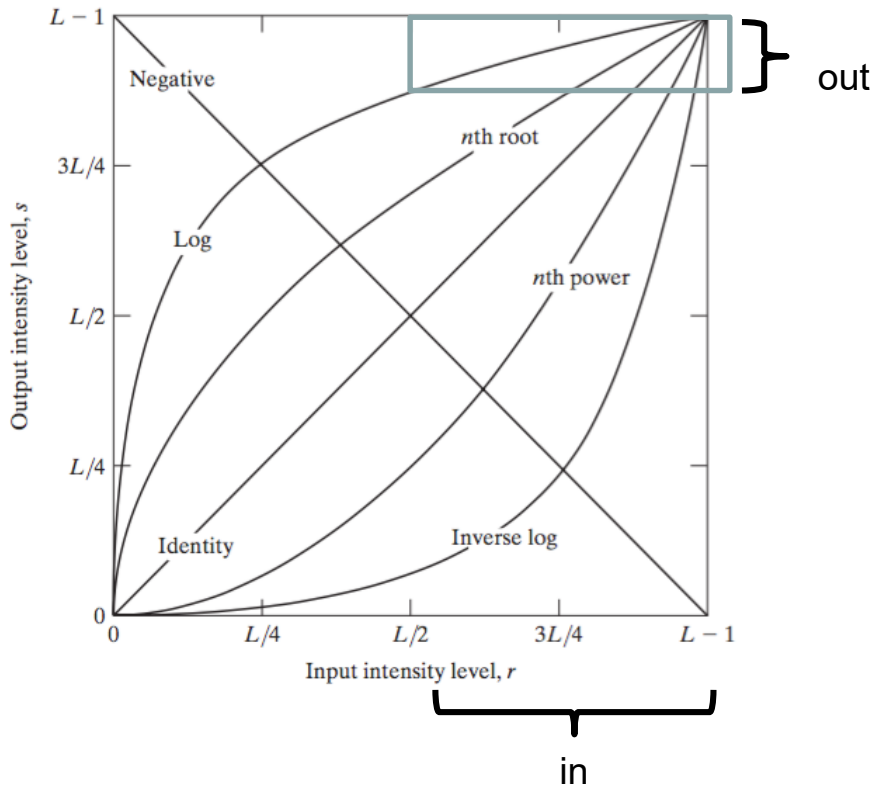
Log Transformations



Map a narrow-range of low intensity values in input to a wider range of output levels

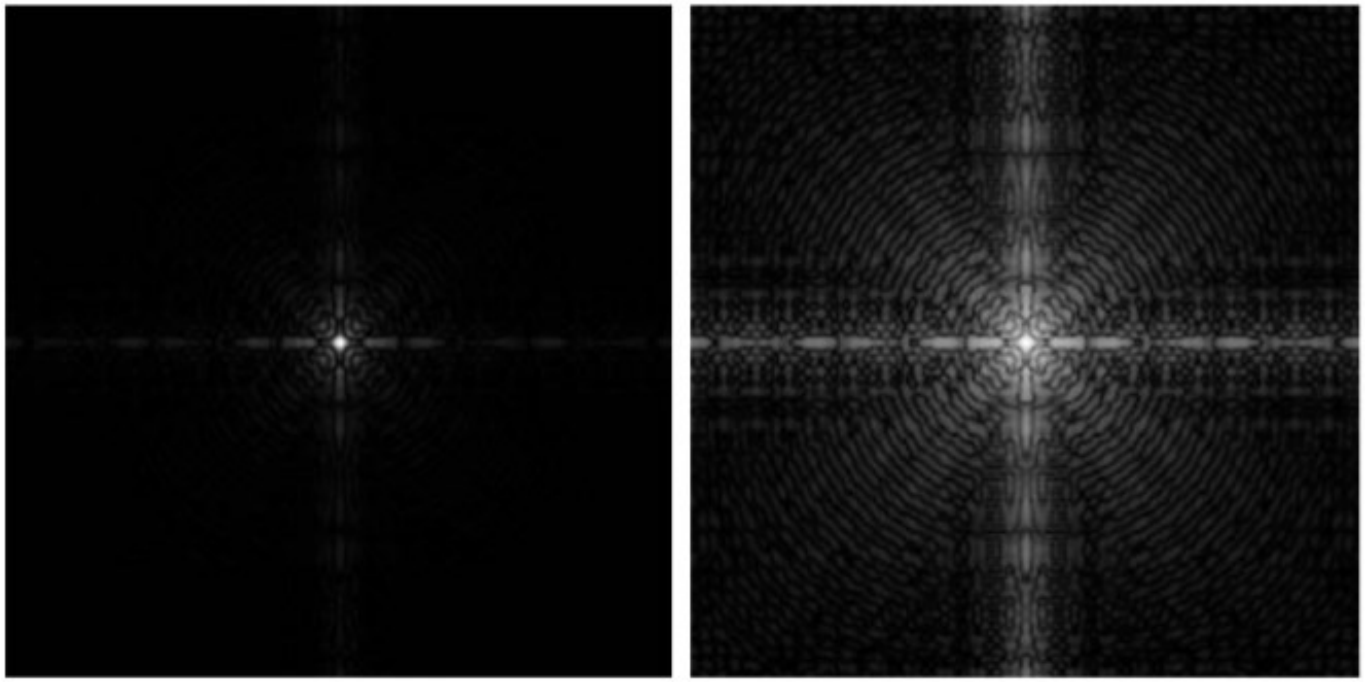
Log Transformations



Map a wide range of high intensity values in input to a narrow range of output levels

Log Transformations

$$s = c\log(1 + r)$$



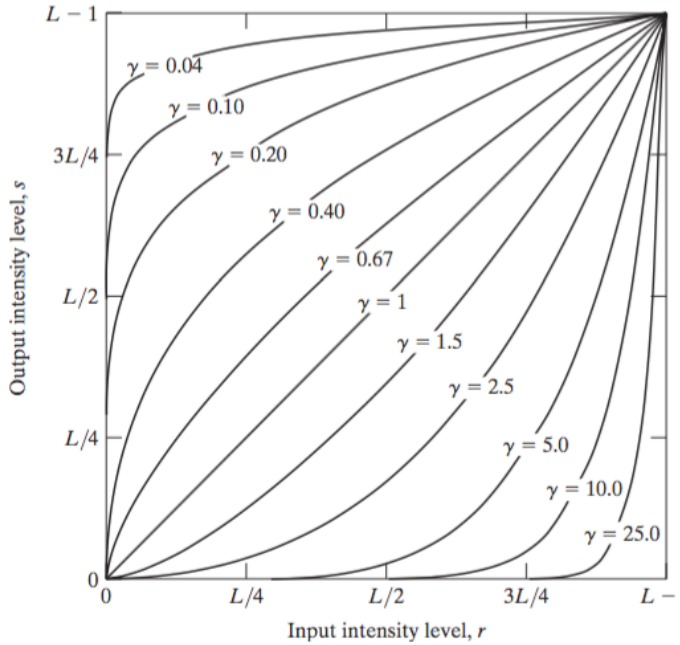
Fourier power spectrum is a nice example of a high dynamic range data

Gamma Transformations

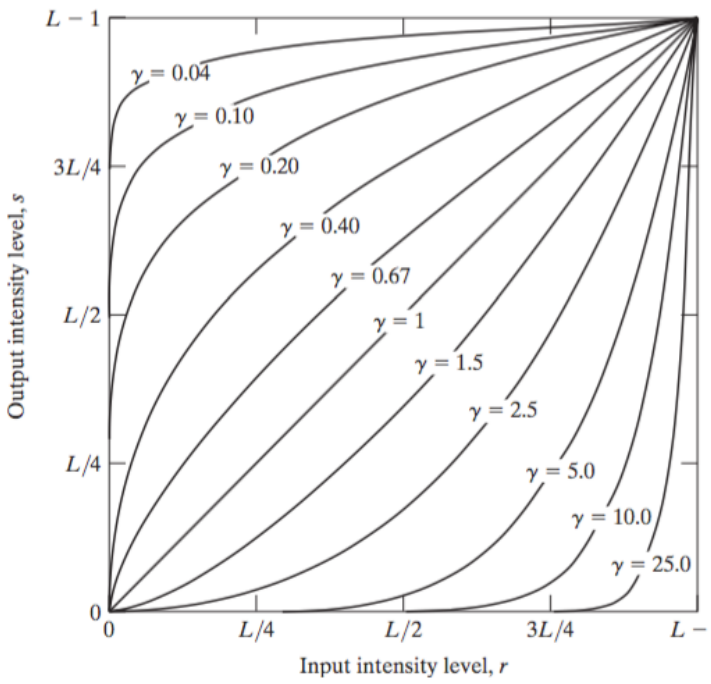
Gamma or power low transformations have the following basic form:

$$s = c\, r^{\gamma}$$

With c and γ positive constants.



Gamma Transformations



- Compress values similar to log transformation but more flexible due to the **γ** parameter
- Curves generated with a **γ**>1 have the opposite effect of those with **γ**<1
- Identity transformation when c=**γ**=1

Gamma Transformations

Gamma correction is useful because many image capture/printing/display devices have a power-law response (not linear!).

- For example, old CRT monitors or modern projectors have an intensity-to-voltage response which is a power-law with exponents varying from 1.8 to 2.5
- By using gamma correction we can remove this effect to obtain a response that is similar to the original image

Gamma Transformations

Monitor response is a power-law with $\gamma = 2.5$

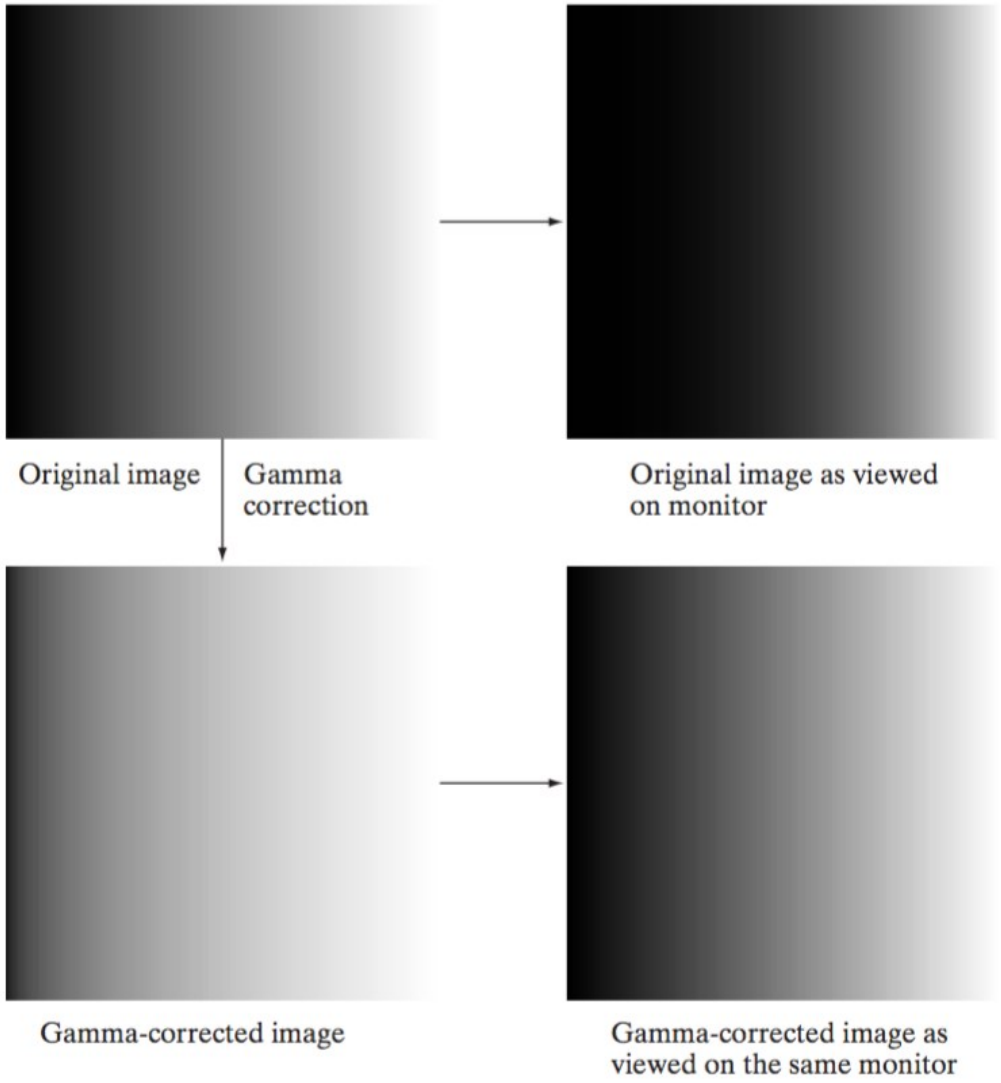
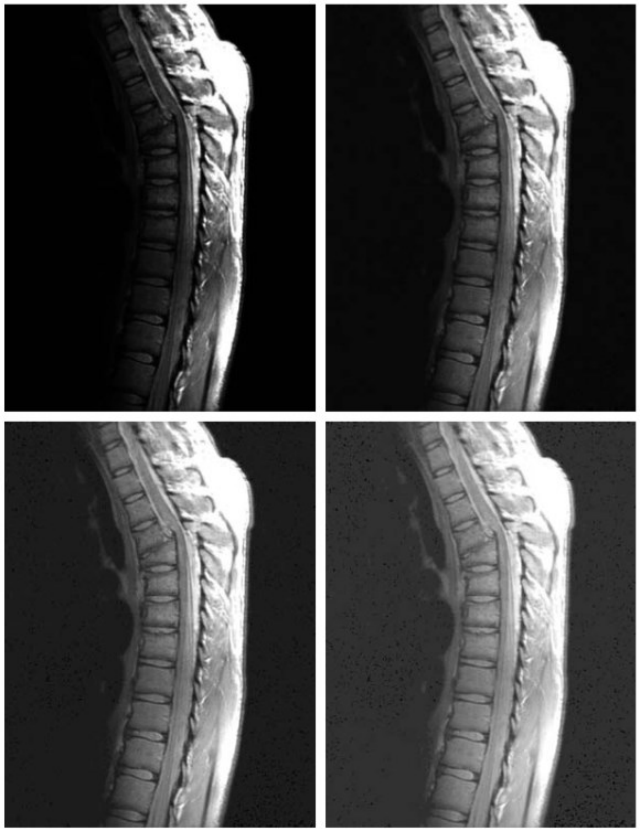


Image is pre-processed by applying a gamma transformation with $\gamma = 1/(2.5) = 0.4$

Gamma Transformations

In addition to gamma correction, gamma transformations are useful for general purpose contrast manipulation:

Image appears too dark



Corrected with $\gamma = 0.6$

Corrected with $\gamma = 0.4$

Corrected with $\gamma = 0.3$

Gamma Transformations

In addition to gamma correction, gamma transformations are useful for general purpose contrast manipulation:

Image is washed out

Corrected with $\gamma = 3$

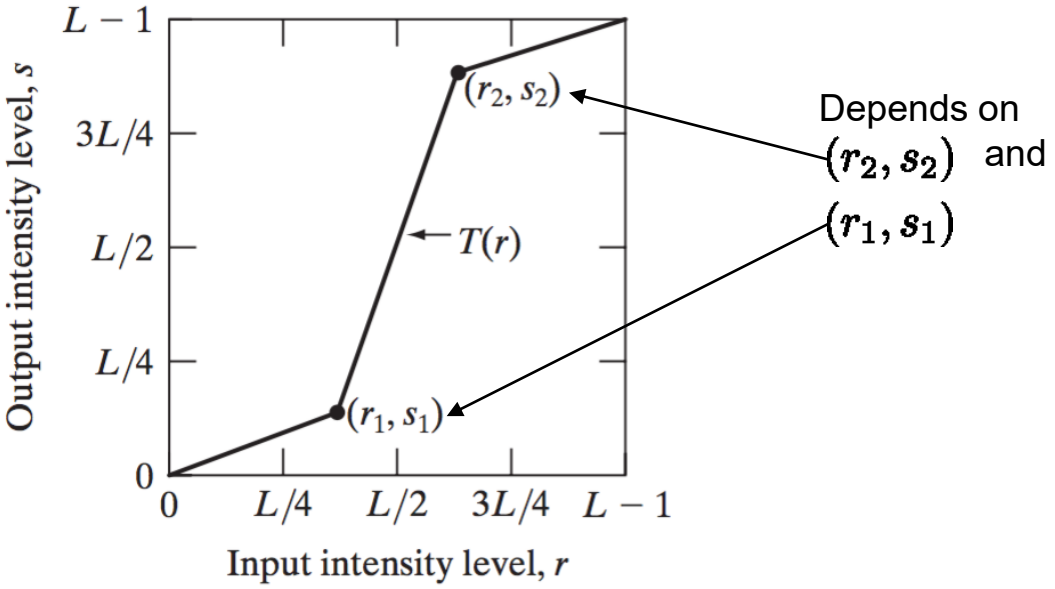
Corrected with $\gamma = 4$

Corrected with $\gamma = 5$

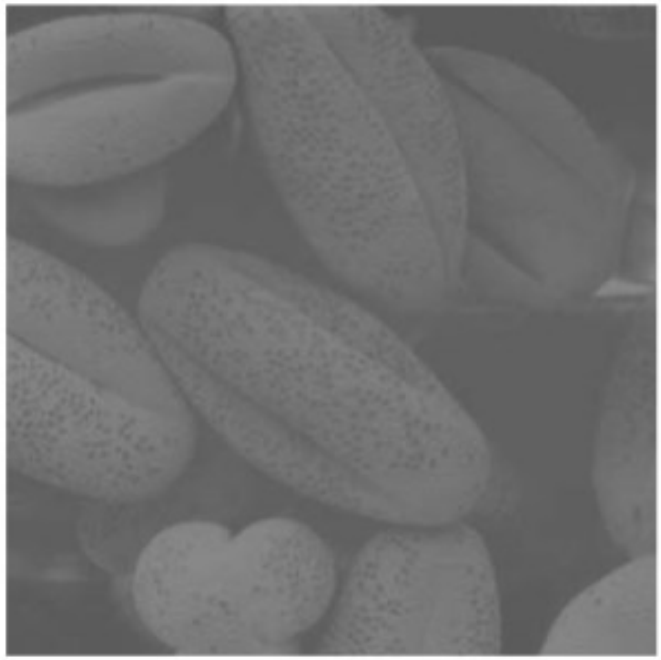
Contrast Enhancement

A whole family of transformations are defined using piecewise-linear functions.

One of the simplest and most useful piecewise-linear transformation is contrast enhancement (with a sigmoid shape)



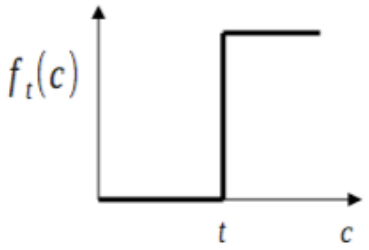
Contrast Enhancement



Thresholding

An extreme case of contrast enhancement is the following:

$$s = \begin{cases} 0 & \text{if } r \leq t \\ 1 & \text{if } r > t \end{cases}$$



Where t is a constant defined for the whole image. If t depends on the spatial coordinates it is often referred as adaptive thresholding

Image Histogram

All the function described so far can improve the appearance of an image by varying some *parameters*

How can we automatically determine their best values?

One effective tool is the Image Histogram that allows us to analyze problems in the intensity (or color) distribution of an image

Without spatial information, we can assimilate I(x,y) as a random intensity emitter.

The image histogram is the empirical distribution of image intensities

Image Histogram

Let [0 … L-1] be the intensity levels of an image.

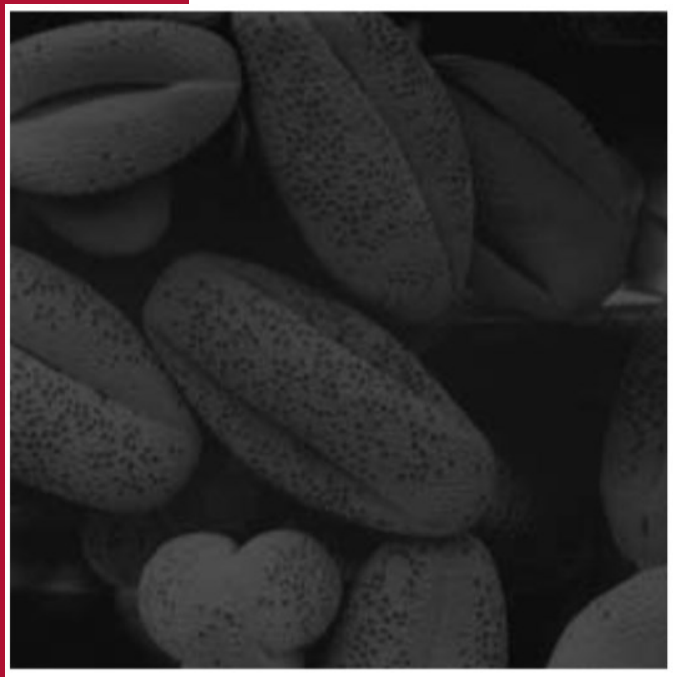
The image histogram is a discrete function

$$h(r_k) = n_k$$

Where r_k is the k^{th} intensity value and n_k is the number of pixels in the image with intensity r_k

Usually the histogram is normalized by dividing each component to the total number of pixels. This way **each histogram component is an estimate of the probability of the occurrence of the intensity r_k**

Image Histogram



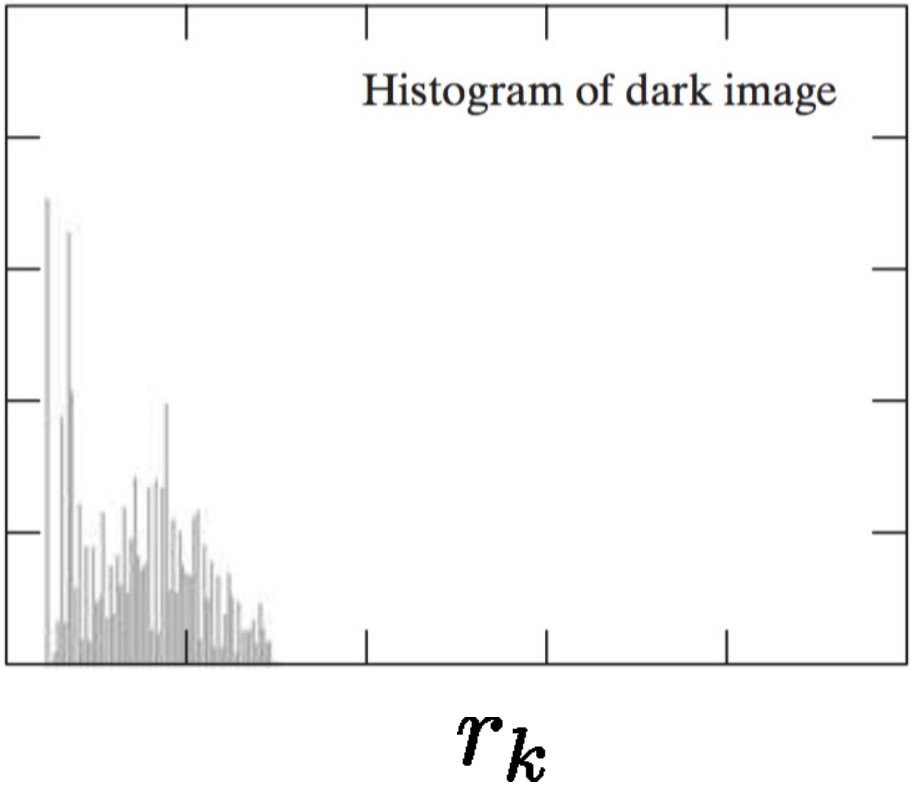


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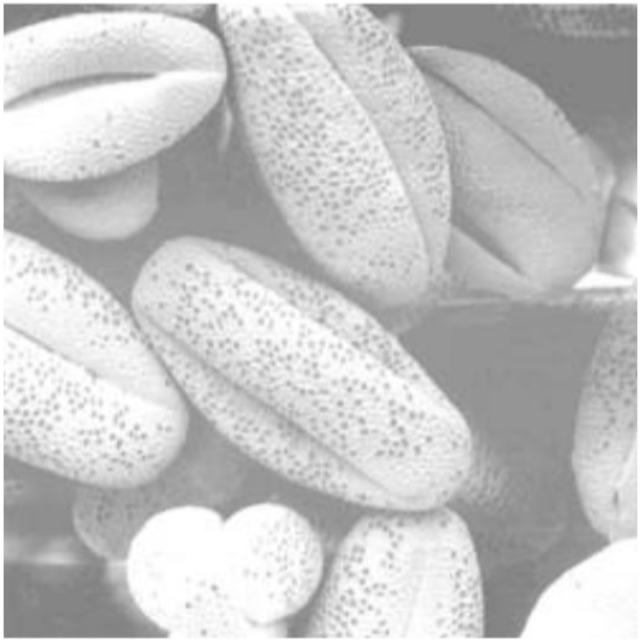
Image Histogram



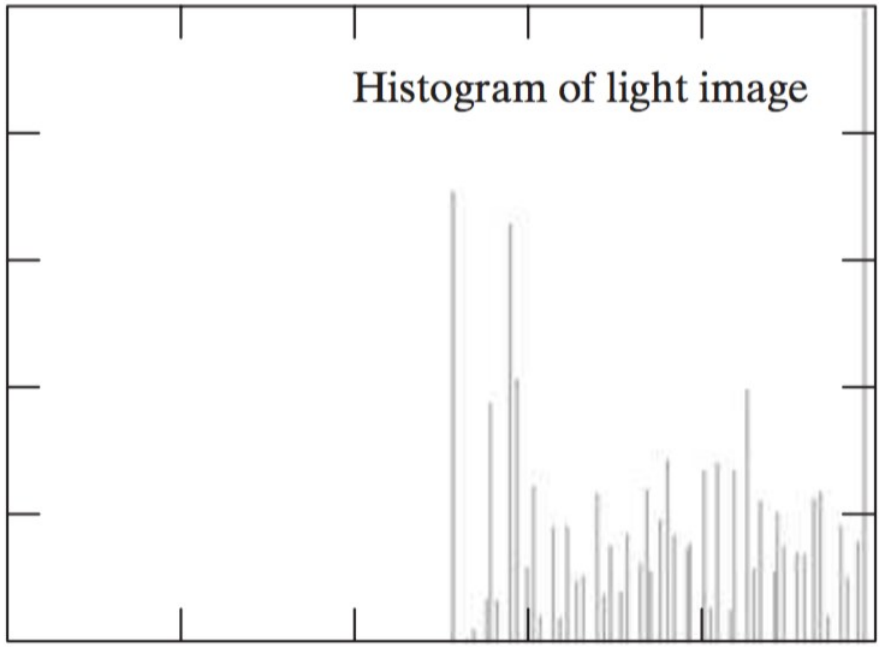


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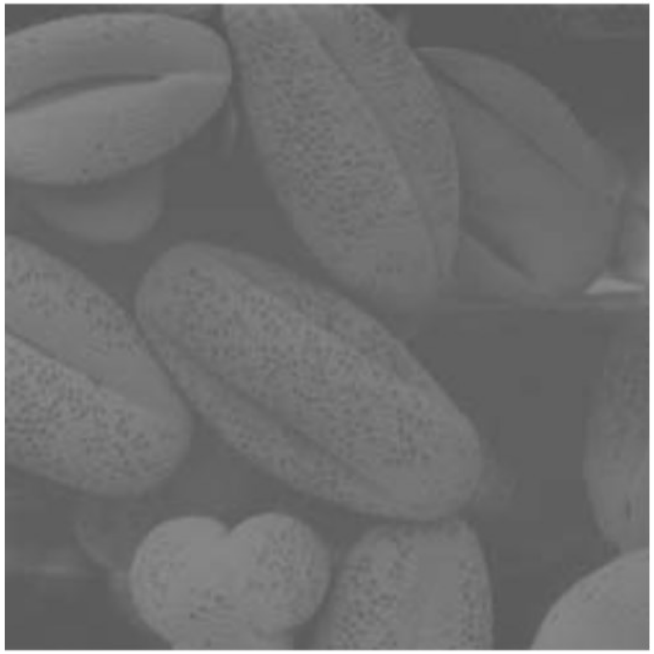
Image Histogram



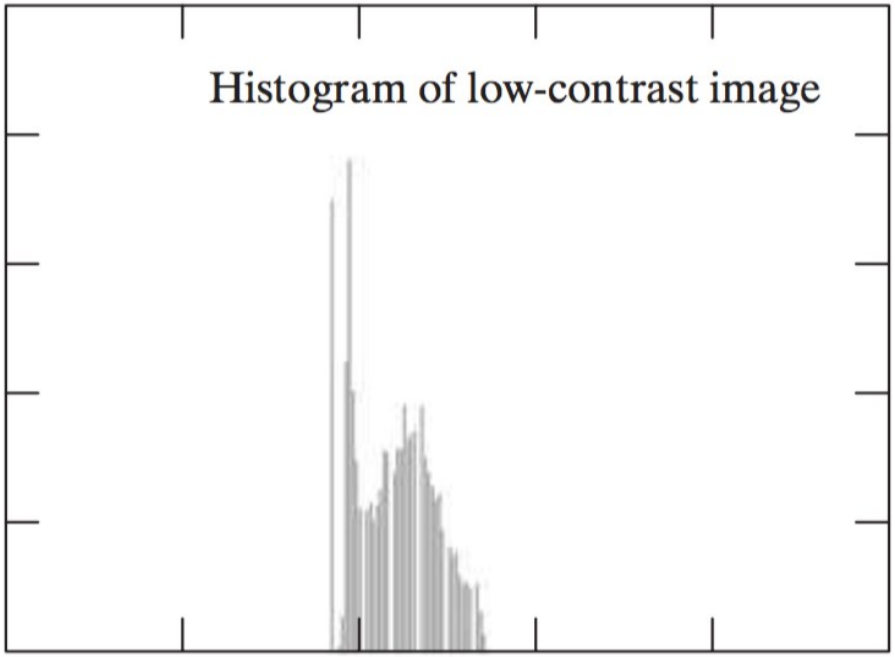
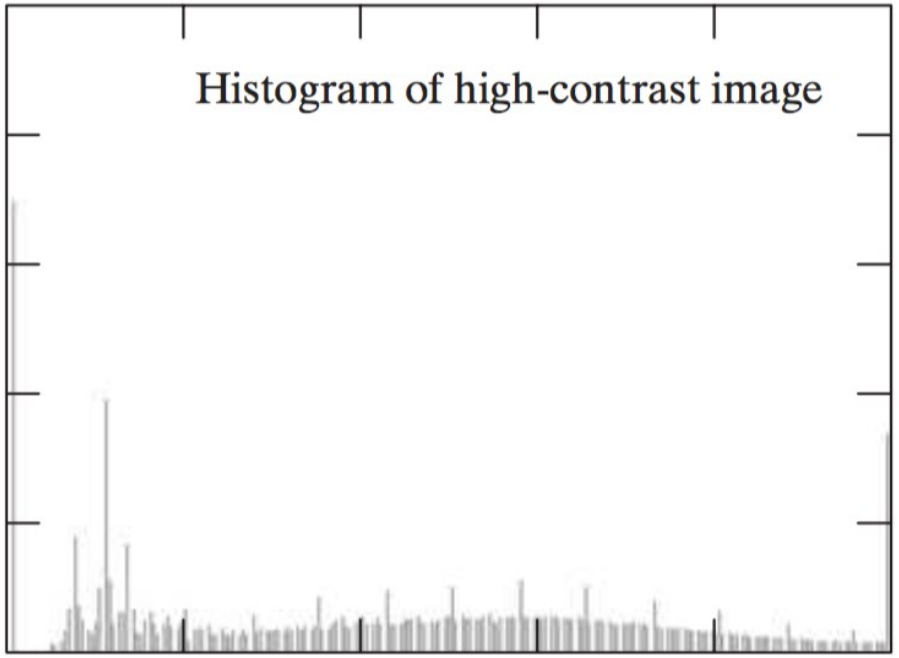


Image Histogram



Histogram equalization

When enhancing an image ideally we would like to maximize the dynamic range of the image to catch both the dark and bright details.

- Choosing the correct parameters requires human intervention… how can we automate the process?

A popular answer is to find a mapping function

$$s = T(r)$$

so that the resulting histogram of s is flat (uniform distribution).

Histogram equalization

More formally, intensity levels of an image may be viewed as random variables in interval [0 … L-1].

Image histogram of s is an estimate of the **PDF** of s ($p_s(s)$) and histogram of r is an estimate of $p_r(r)$.

From probability theory, when we apply a function s=T(r) to a random variable r we got:

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

Note: T must be continuous, differentiable and strictly monotonic

Histogram equalization

If we use the function:

CDF of r

$$T(r) = (L-1)\int_0^r p_r(w)dw$$

We have:

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = (L-1)p_r(r)$$

$$p_s(s) = p_r(r)\frac{dr}{ds} = p_r(r)\frac{1}{(L-1)p_r(r)} = \frac{1}{L-1}$$

Uniform distribution!

Histogram equalization

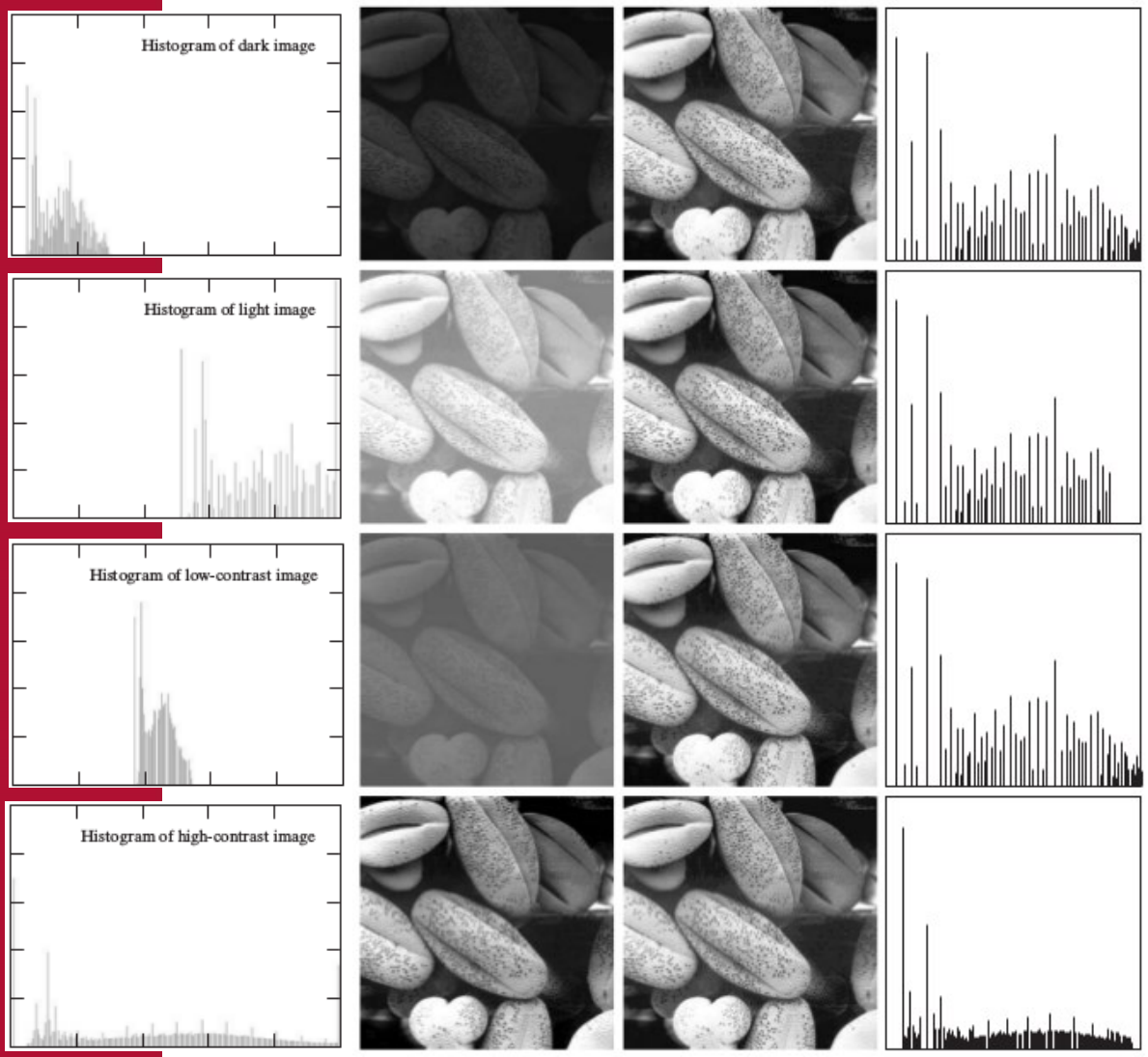
In the discrete case, if we apply the function

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{j=0}^{k} h(r_j)$$

Sum of the first k components of the input image histogram

We obtain a (quasi) flat histogram of the output image

Histogram equalization



Since histogram is a discrete approximation of a PDF, the resulting histogram is in general not perfectly flat.

It still remains a good approximation

Histogram matching

It is useful sometimes to be able to specify the shape of the histogram that we wish the processed image to have (instead of a simple flat one).

Histogram equalization discussed so far can be used also for histogram matching

Suppose that we have an input image with intensities described by r with PDF $p_r(r)$ and a *specified* PDF described by z with a given PDF $p_z(z)$

Histogram matching

Let s be a random variable with the following property:

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

(ie. s is the equalized version of r)

We define a function G(z) as following:

$$G(z) = (L-1)\int_0^z p_z(t)dt$$

Since G(z)=T(r), and both are monotonically increasing, we have $z = G^{-1}[T(r)] = G^{-1}(s)$

Histogram matching

Algorithm:

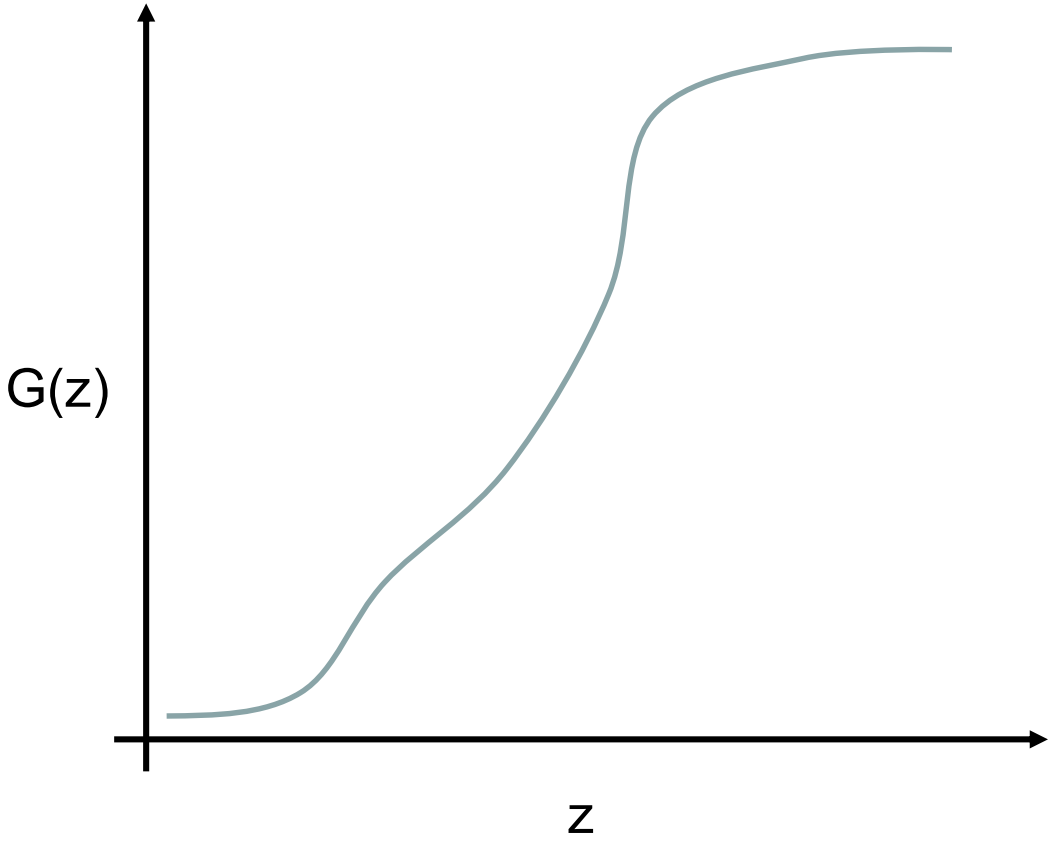
1. Compute the PDF (normalized histogram) of the input image $p_r(r)$
2. Use the *specified* PDF $p_z(z)$ to obtain the function $G(z)$
1. Obtain the inverse transformation $z = G^{-1}(s)$
2. Equalize the input image to obtain s. Apply the function $G^{-1}(s)$ to the equalized image s to obtain the corresponding output image.

When all pixels are processed, the PDF of the output image will be equal to the specified PDF

Discrete case

In the discrete case the function G is implemented as a lookup table with L entries

z	G(z)
0	1
1	3
2	3
…	…
254	250
255	255

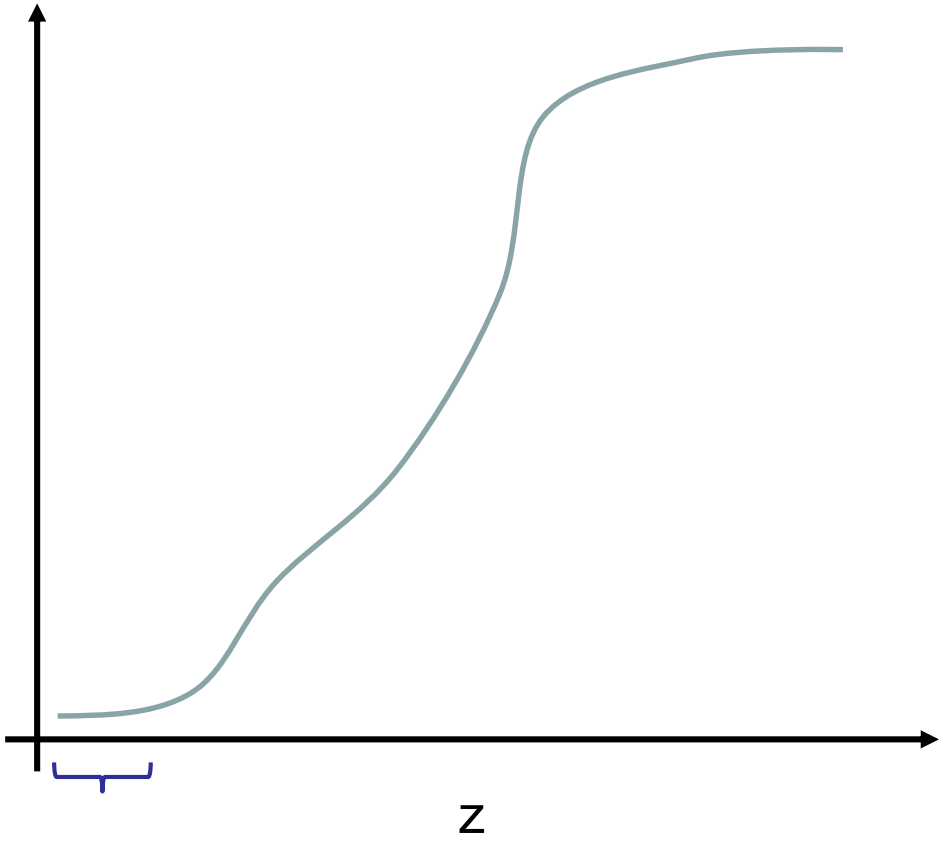


Problems with inversion

Duplicate entries (ie. constant G(z))

Solution: Choose the smallest one (by convention)

z	G(z)
0	1
1	3
2	3
…	…
254	250
255	255



Problems with inversion

One or more values in [0…L-1] missing from G(z)

Solution: Choose the nearest one (or interpolate)

z	G(z)
0	1
1	3
2	3
…	...
254	250
255	255

← 251 is missing.
Choose 250 so the mapping is 251->254

Histogram for thresholding

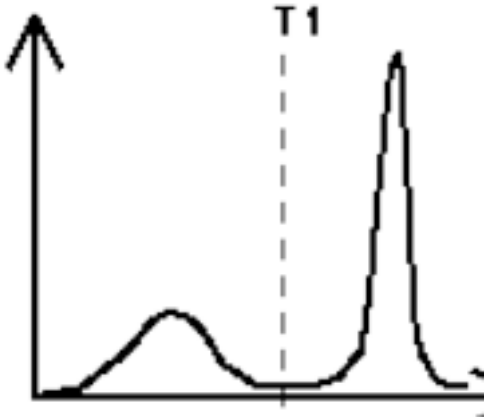
Let's go back to the thresholding operation, which is a common step in many cv applications:

When using global thresholding a common problem is to automatically find a good threshold t that separates well dark from bright areas

$$s = \begin{cases} 0 & \text{if } r \leq t \\ 1 & \text{if } r > t \end{cases}$$

Histogram for thresholding

Image histogram can give us useful clues on the threshold level



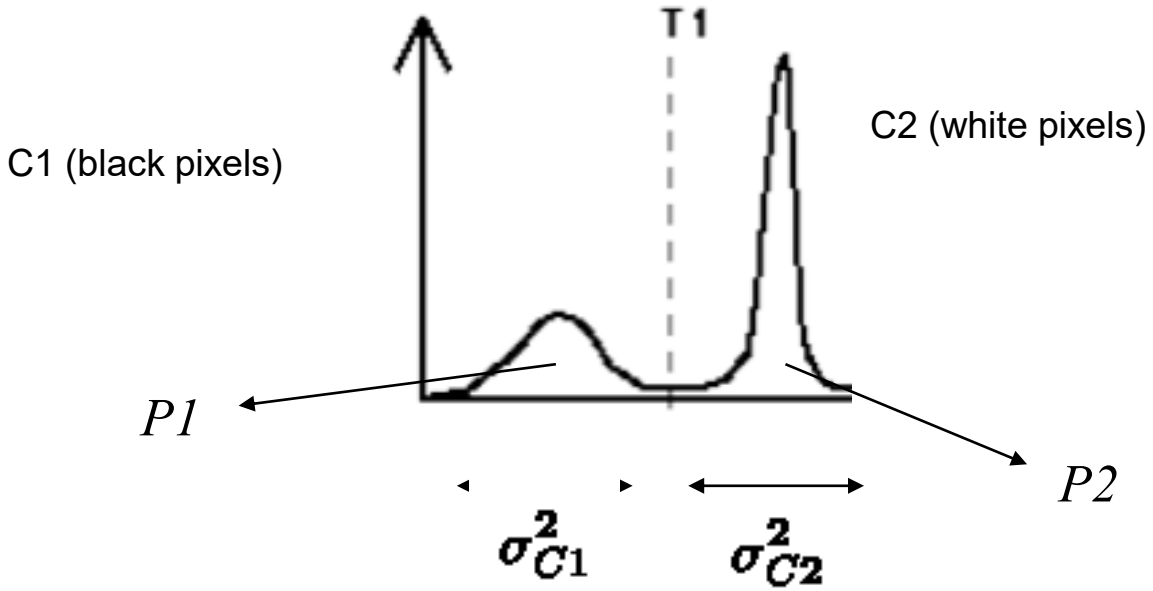
If an image is separable through thresholding there will be a range of intensity with low probability

Thresholding is essentially a **clustering problem** in which two clusters (black and white pixels) are sought

Otsu Thresholding

The idea is to find the optimum threshold so that the variance of each class (**within-class variance**) is minimized

$$\underset{T}{\operatorname{argmin}} \; P_1\sigma_{C1}^2 + P_2\sigma_{C2}^2$$



Otsu Thresholding

$$\underset{T}{\operatorname{argmin}} \; P_1\sigma_{C1}^2 + P_2\sigma_{C2}^2$$

Probability that a pixel is assigned to C1 given a threshold T

Probability that a pixel is assigned to C2 given a threshold T

$$P_1(T) = \sum_{i=0}^{T} p_i$$

$$P_2(T) = \sum_{i=T+1}^{L-1} p_i = 1 - P_1(T)$$

Otsu Thresholding

Mean intensity value for the pixels assigned to C1:

$$m_1(T) = \sum_{i=0}^{T} iP(i|C1) = \sum_{i=0}^{T} i\frac{P(C1|i)P(i)}{P(C1)} = \frac{1}{P_1(T)}\sum_{i=0}^{T} ip_i$$

Bayes rule

Always 1 because we are dealing only with values i from C1

Probability of class C1 (as computed before)

Mean intensity value for the pixels assigned to C2:

$$m_2(T) = \sum_{i=T+1}^{L-1} iP(i|C2) = \frac{1}{P_2(T)}\sum_{i=T+1}^{L-1} ip_i$$

Otsu Thresholding

C1 class variance:

$$\sigma_{C1}^2(T) = \frac{1}{P_1(T)} \sum_{i=0}^{T} \big(i - m_1(T)\big)^2 p_i$$

C2 class variance:

$$\sigma_{C2}^2(T) = \frac{1}{P_2(T)} \sum_{i=T+1}^{L-1} \big(i - m_2(T)\big)^2 p_i$$

Otsu Thresholding

What is the best threshold?

Operatively, we can try all the possible T from 0 to L-1 and keep the threshold for which

$$P_1(T)\sigma_{C1}^2(T) + P_2(T)\sigma_{C2}^2(T)$$

Is minimum.

Problem: Is computationally expensive to compute

$$\sigma_{C1}^2(T), \sigma_{C2}^2(T)$$

Solution: Otsu demonstrated that the optimal T that minimizes the *within-class-variance* also **maximizes** the *between-class-variance*

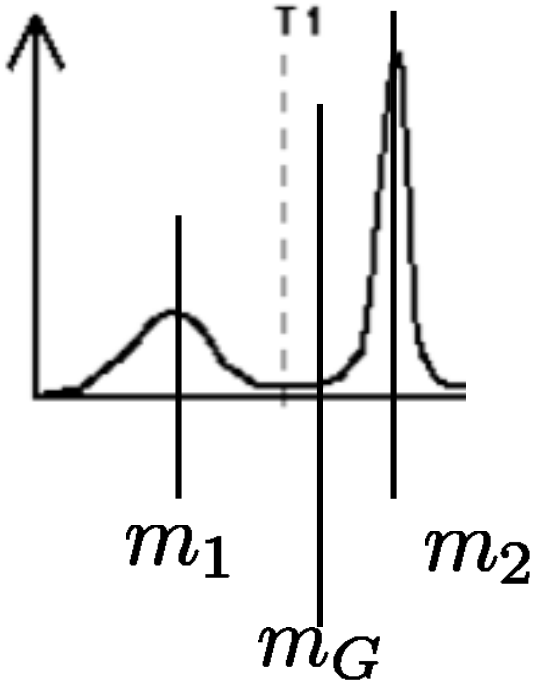
Otsu Thresholding

Between-class-variance:

$$\sigma_B^2(T) = P_1(T)\big(m_1(T) - m_G\big)^2 + P_2(T)\big(m_2(T) - m_G\big)^2$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

Global mean



Otsu Thresholding

Between-class-variance can be rewritten as:

$$\sigma_B^2(T) = \frac{[m_G P_1(T) - m(T)]^2}{P_1(T)[1 - P_1(T)]}$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

Global mean

$$m(T) = \sum_{i=0}^{T} i p_i$$

Cumulative mean

Optimum T can be now easily computed efficiently

Otsu Thresholding

Otsu Algorithm for optimal global thresholding:

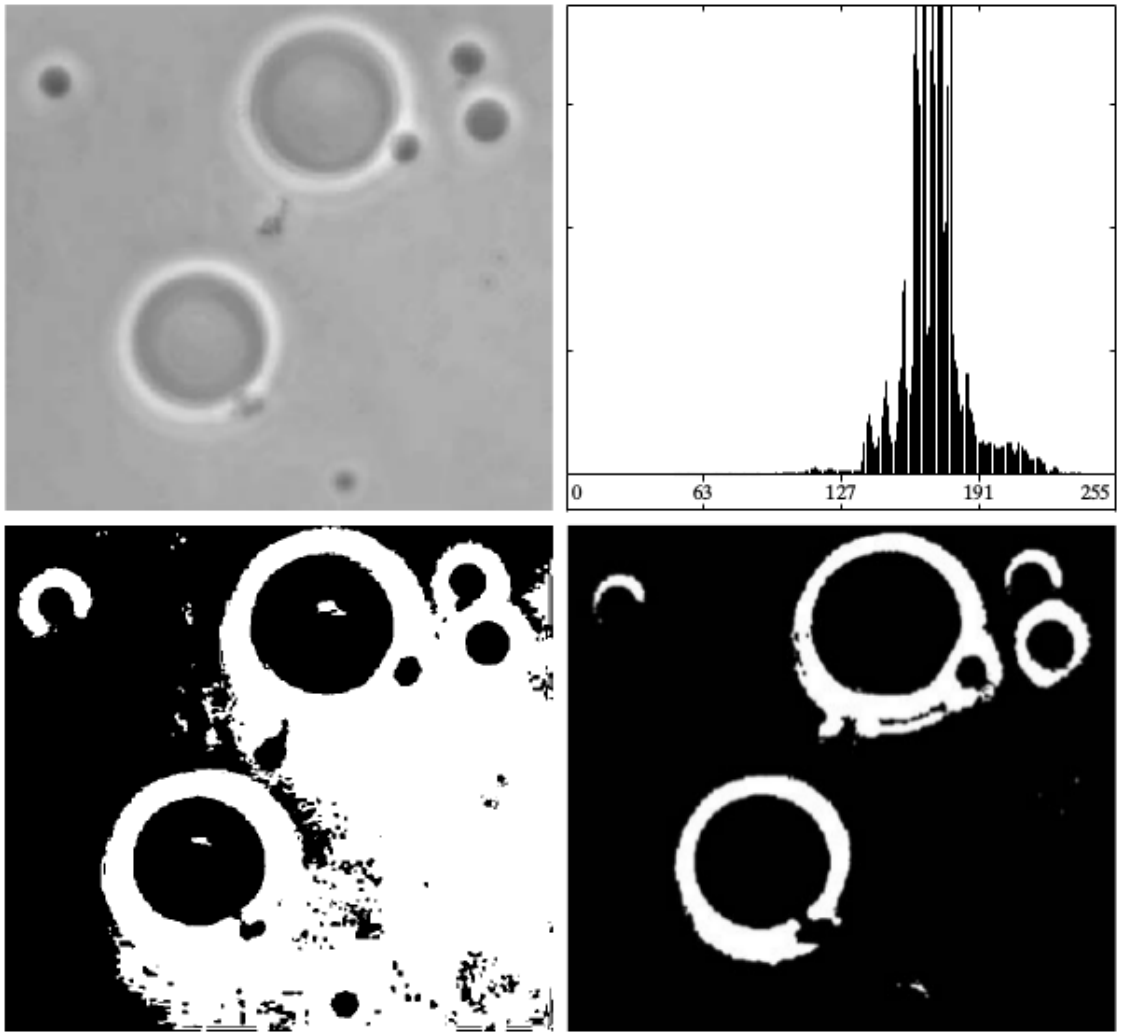
1. Compute the normalized histogram of the input image. Denote each component of the histogram as

$$p_i \ , i = 0, 1, \ldots, L-1$$

1. Compute the cumulative sums $P_1(T) \ \forall T = 0 \ldots L-1$
2. Compute the cumulative means $m(T) \ \forall T = 0 \ldots L-1$
3. Compute the global intensity mean m_G
4. Compute the between class variance $\sigma_B^2(T) \ \forall T = 0 \ldots L-1$
5. Apply threshold with a value of T for which $\sigma_B^2(T)$ is maximum

Otsu thresholding iterates on image histogram and not on image pixels as other global methods (like k-means)!

Otsu Thresholding



K-means based

Otsu