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Computer Vision

Morphological image processing

Filippo Bergamasco (filippo.bergamasco@unive.it)

<http://www.dais.unive.it/~bergamasco>

DAIS, Ca' Foscari University of Venice

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Preliminaries

Morphology offers a unified and powerful approach to numerous image processing problems

> The language of mathematical morphology is **set theory**

We will consider thresholded images containing only black and white pixels.

The “set of white pixels” contains the complete morphological description of the image.

Such sets are subsets of \mathbb{Z}^2 where each vector express coordinates of white (or black) pixels

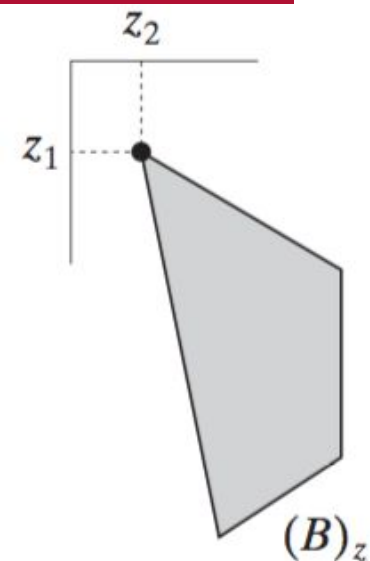


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Set translation

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_z$, is defined as:

$$(B)_z = \{c \mid c = b + z \text{ for } b \in B\}$$



if B is the set of pixels representing an object in an image, then $(B)_z$ is the set of points in B whose (x, y) coordinates have been replaced by $(x + z_1, y + z_2)$



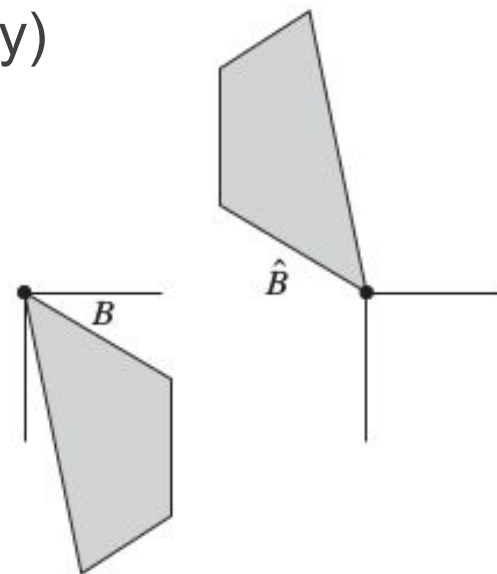
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Set reflection

The reflection of a set B , denoted \hat{B} is defined as:

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

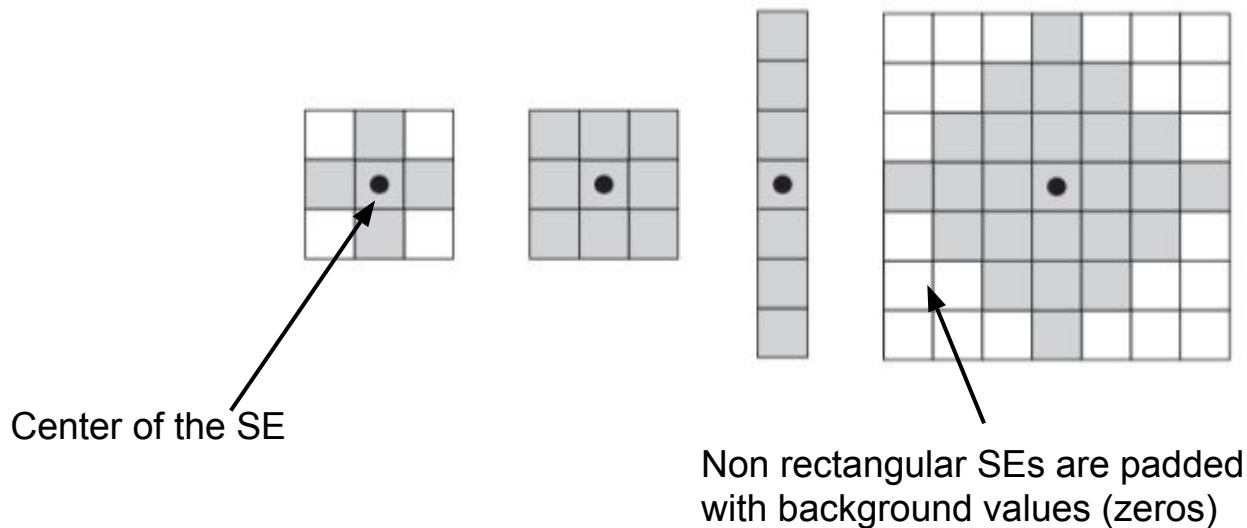
The reflection of B is composed by the points whose coordinates are replaced with $(-x, -y)$



Structuring elements

Morphological operations are based on so-called structuring elements (SEs):

SEs: small sets or subimages used to probe an image under study for properties of interest





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Erosion

With A and B as sets in Z^2 , the erosion of A by B , denoted $A \ominus B$, is defined as:

$$A \ominus B = \{z \mid (B)_z \cap \bar{A} = \emptyset\}$$

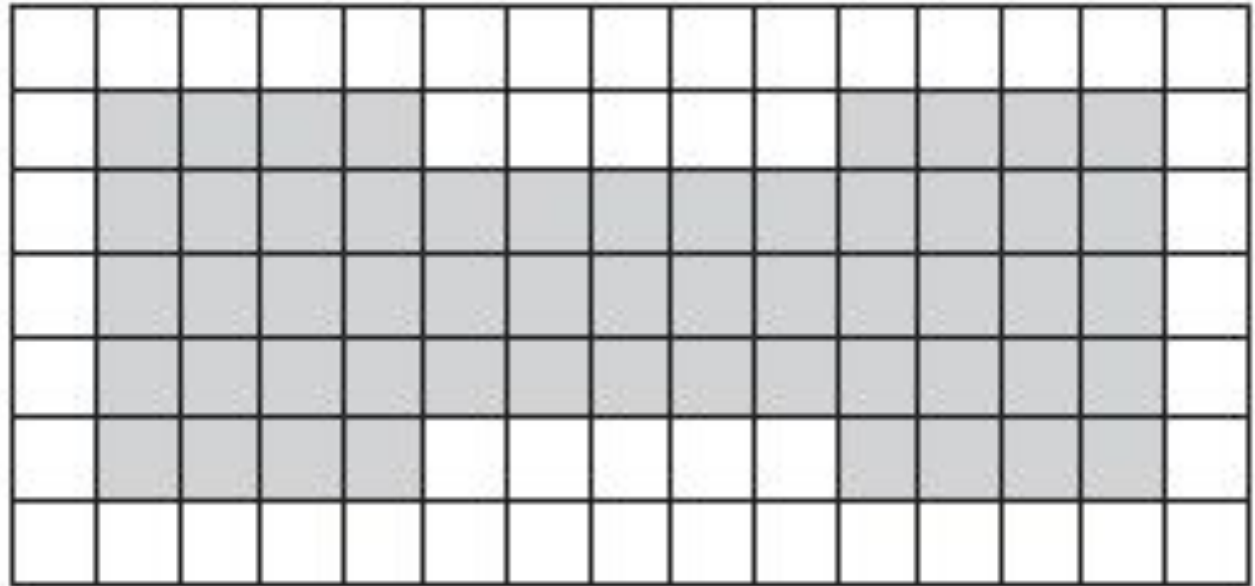
In words, the erosion of A by B is the set of all points z such that B , translated by z , is contained in A (the intersection between $(B)_z$ and the complement of A is empty)



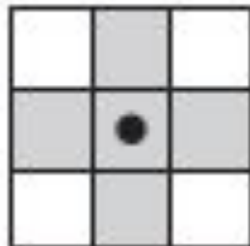
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Erosion example

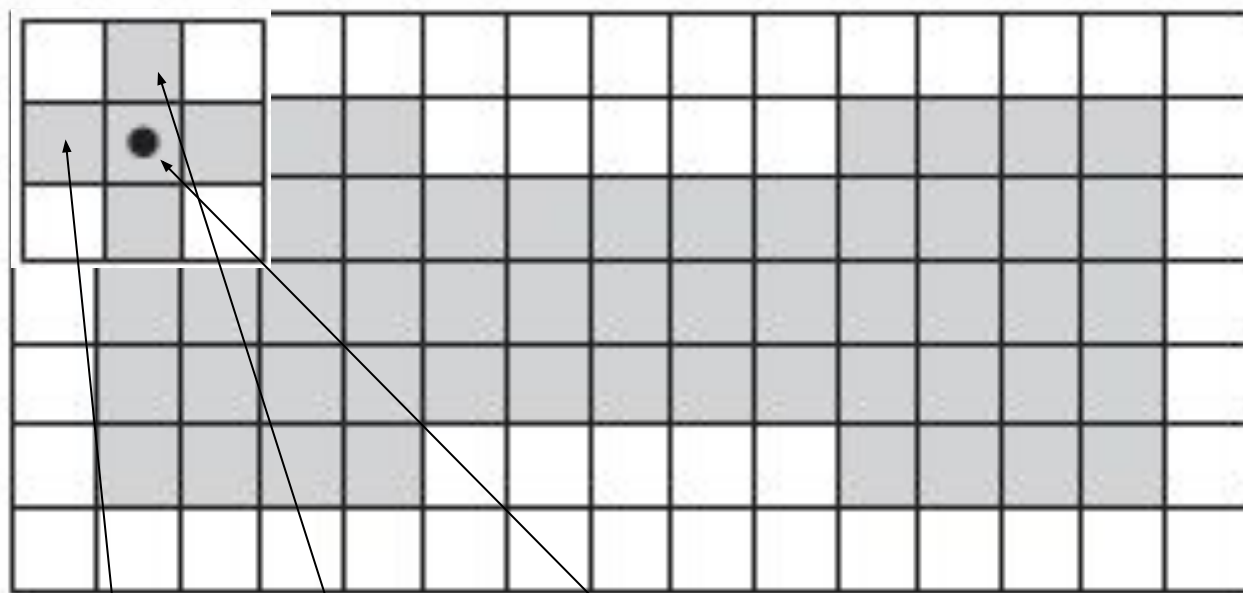
A



B



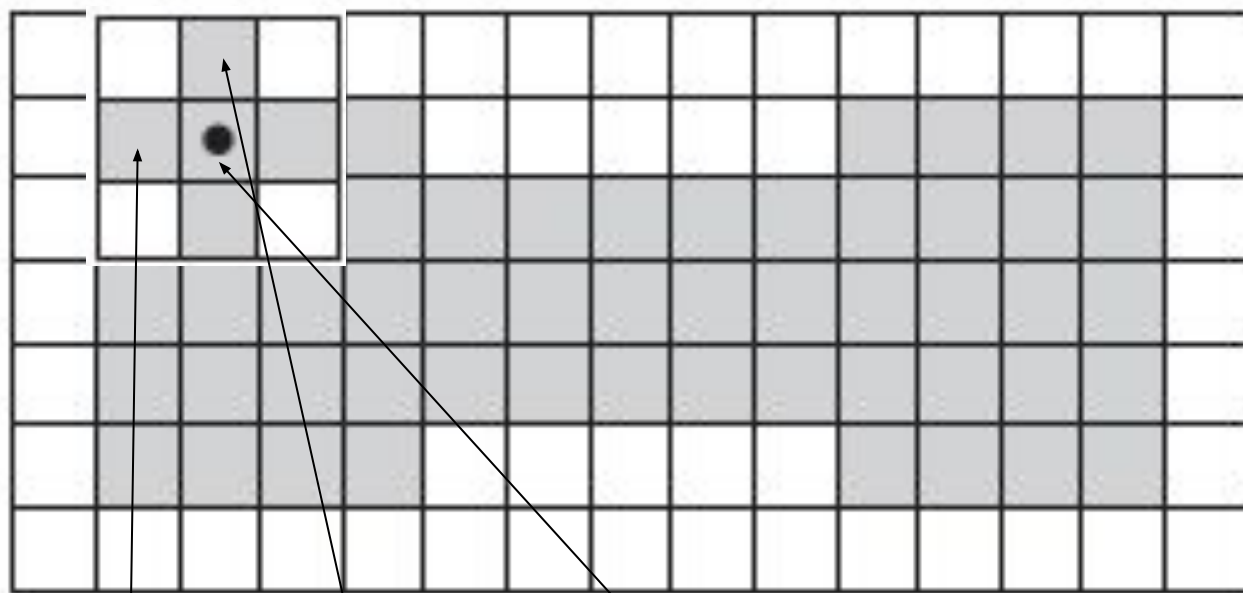
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be
present in the output

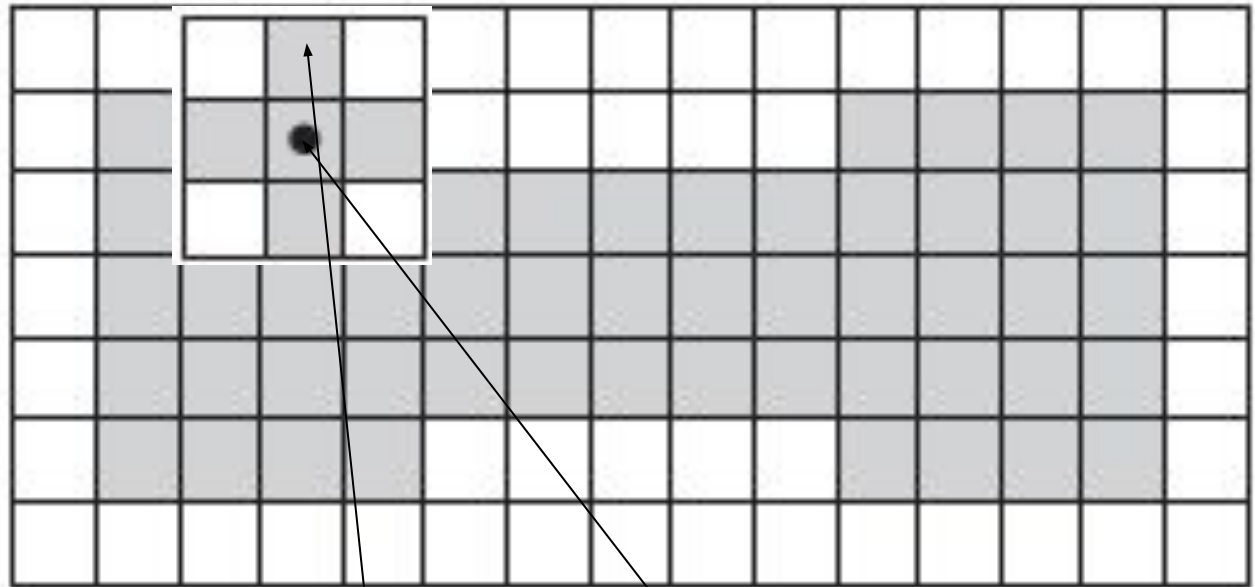
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be present in the output

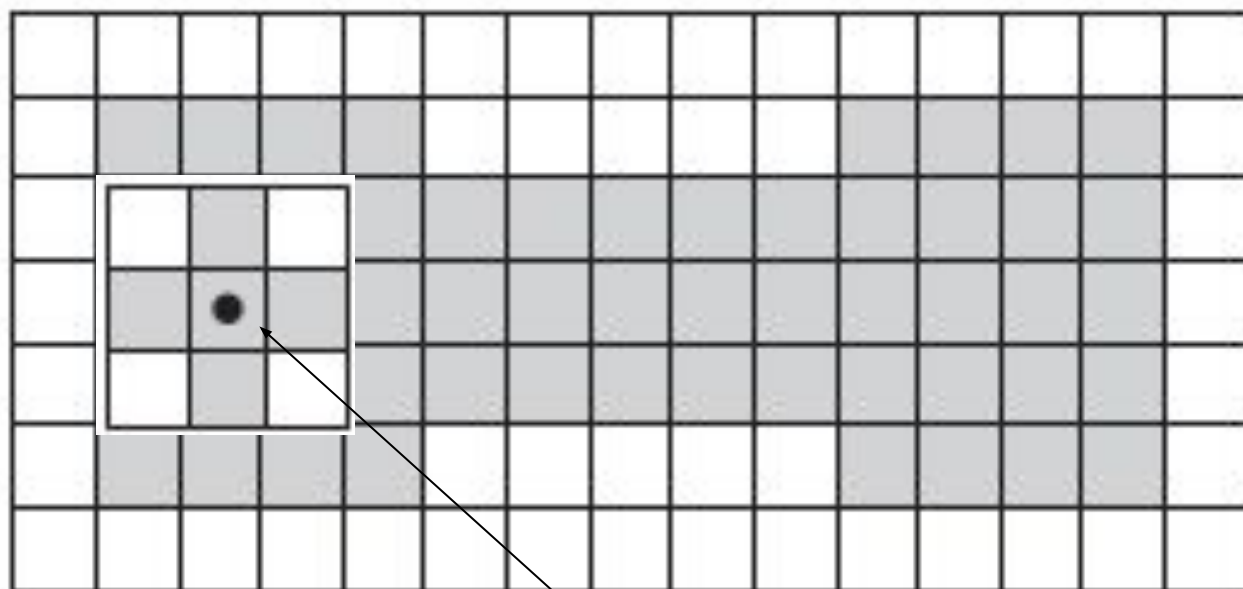
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be
present in the output

Erosion example

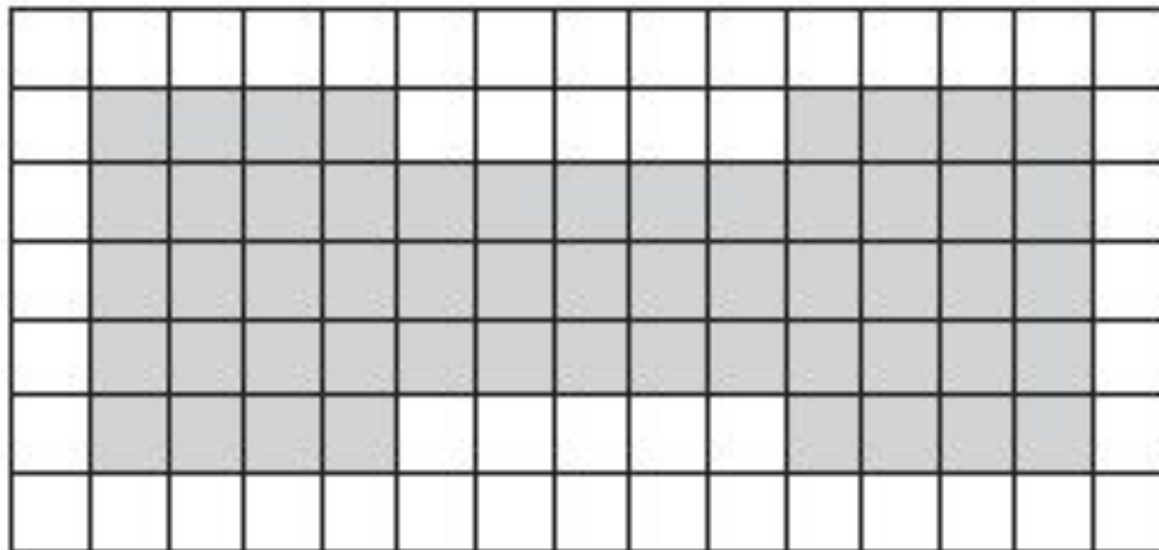


$$(B)_z \cap \bar{A} = \emptyset$$

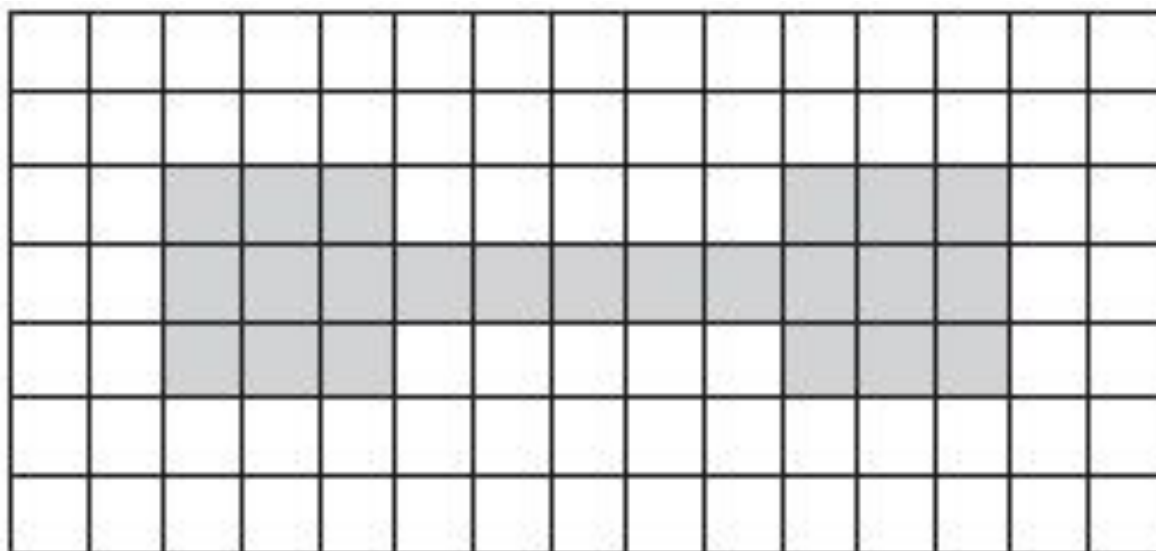
This point will be present in
the output

Erosion example

A



$A \ominus B$

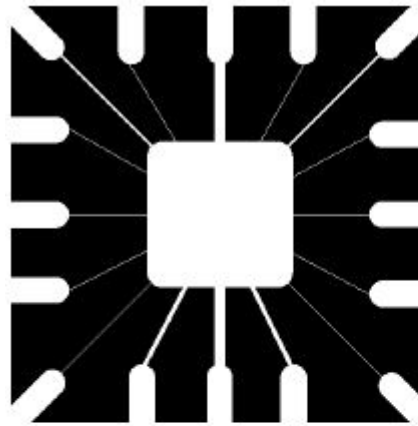




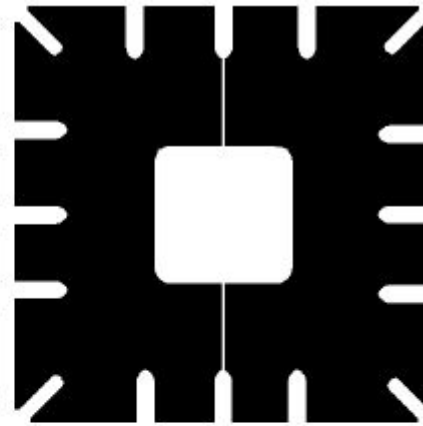
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Erosion example

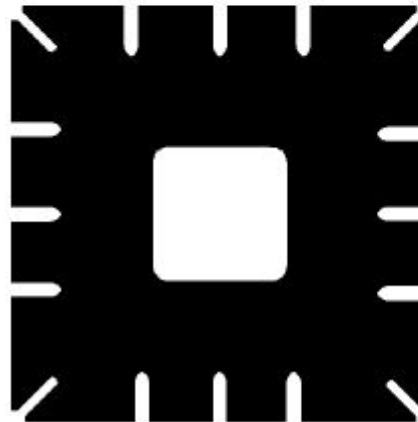
Original image



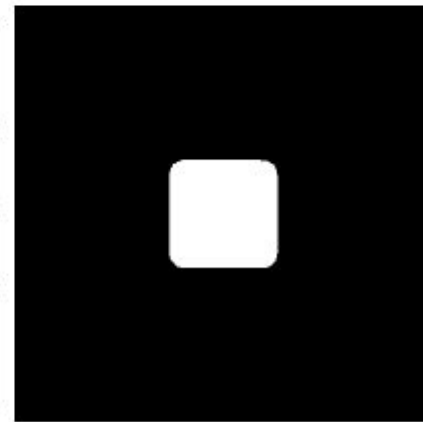
11x11 erosion mask



15x15 erosion mask



45x45 erosion mask





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Dilation

With A and B as sets in \mathbb{Z}^2 , the dilation of A by B , denoted $A \oplus B$, is defined as:

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

The dilation of A by B is the set of all displacements such that the reflection of B and A overlap by at least one element.

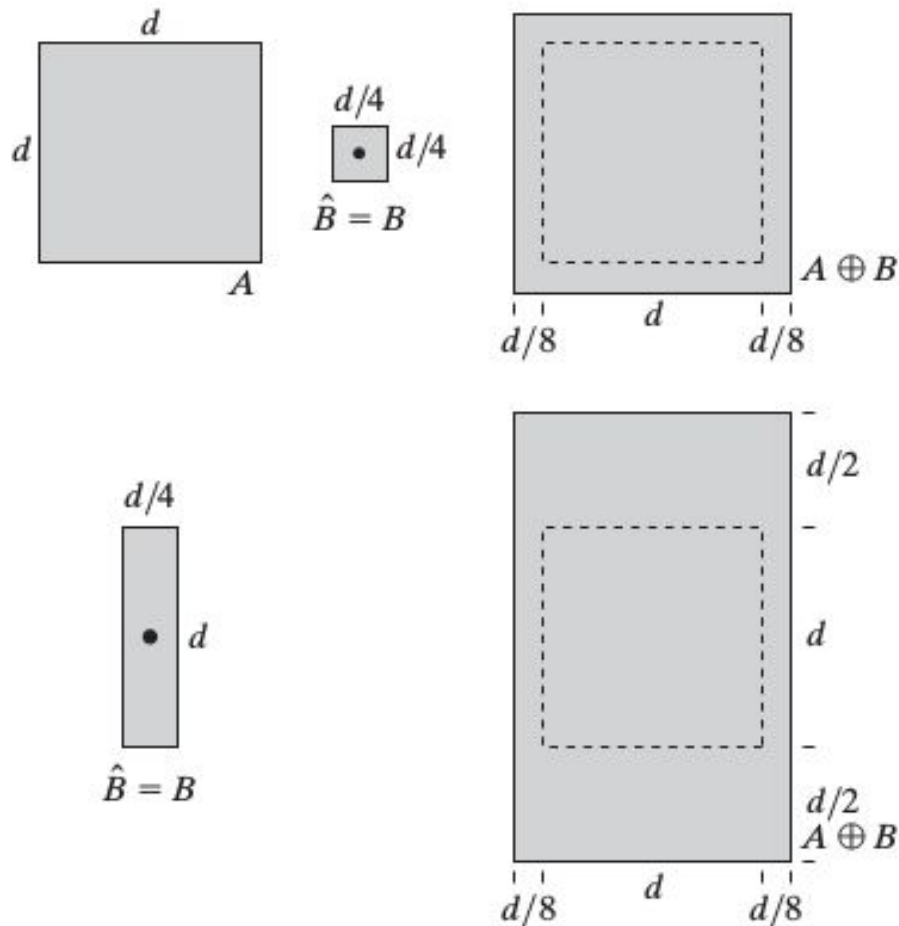
The basic process of flipping B about its origin and then successively displacing it so that it slides over set (image) A is analogous to spatial convolution

Dilation

a b c
d e

FIGURE 9.6

(a) Set A .
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B , shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A , shown only for reference





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Dilation

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



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0	1	0
1	1	1
0	1	0

One of the simplest applications of dilation is for bridging gaps



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Opening and Closing

Given an image A and a SE B :

Opening:

$$A \circ B = (A \ominus B) \oplus B$$

Erosion followed by a dilation

Closing:

$$A \bullet B = (A \oplus B) \ominus B$$

Dilation followed by an erosion



Opening and Closing

Opening: $A \circ B = (A \ominus B) \oplus B$

- smooths the contour of an object
- breaks narrow isthmuses
- eliminates thin protrusions

Closing: $A \bullet B = (A \oplus B) \ominus B$

- smooth sections of contours
- fuses narrow breaks and long thin gulfs
- eliminates small holes
- Fills gaps in the contour



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Opening and Closing

Original noisy image



Erosion



Opening



→
Noise completely removed
but some holes are created

→
Dilation partially closes the
inner holes



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Opening and Closing

Original image



Dilation



Closing



→
Most of the breaks are
restored but ridges are
thickened

→
Erosion reduces the
thickness of the ridges

Opening and Closing



Original image

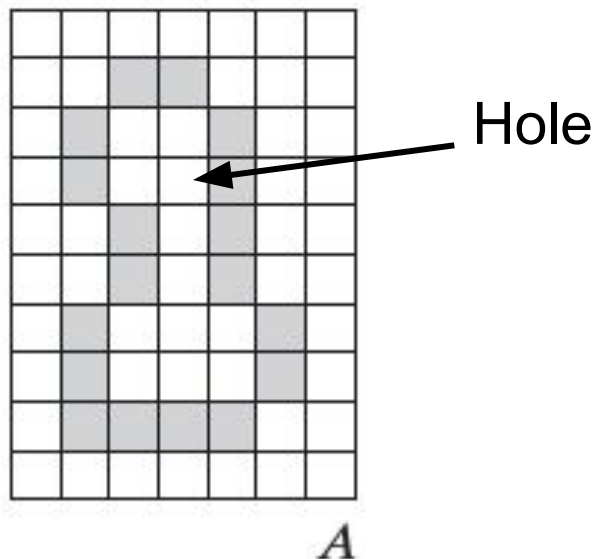


$$(A \circ B) \bullet B$$

A sequence of opening and closing both eliminate noise and close all the erroneous breaks due to an imperfect thresholding

Hole filling

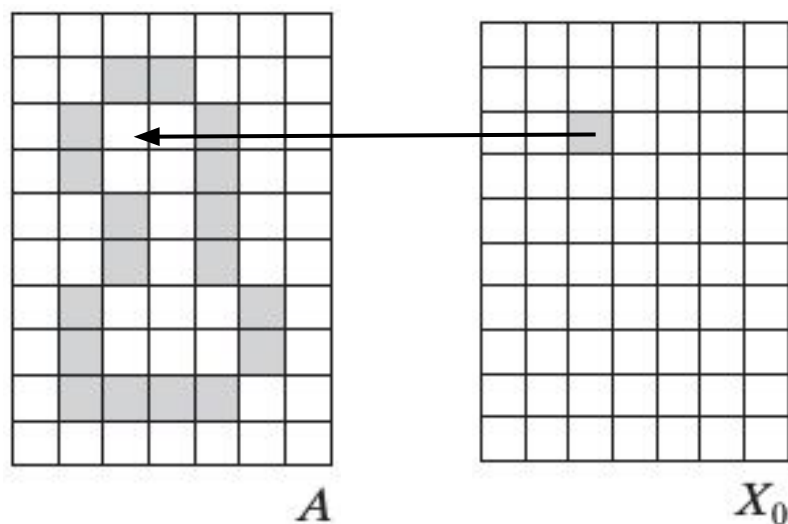
A hole may be defined as a background region surrounded by a connected border of foreground pixels.



Given an initial point inside an hole, the goal is to fill the hole with 1s

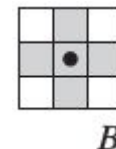
Hole filling

Let X_0 be the initial array with the same size of A , filled with 0s except at the location to a given point in each hole.

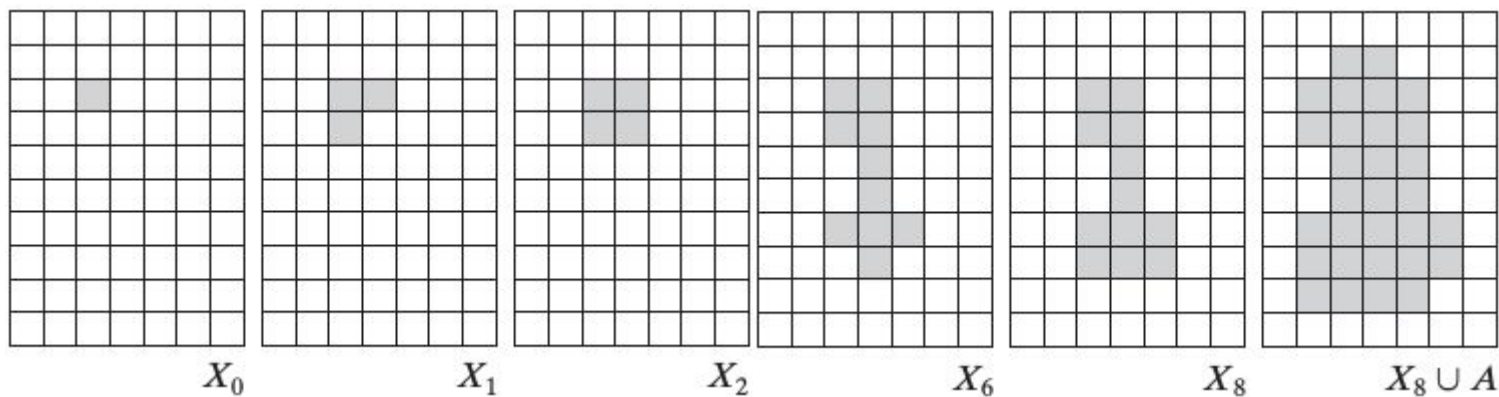
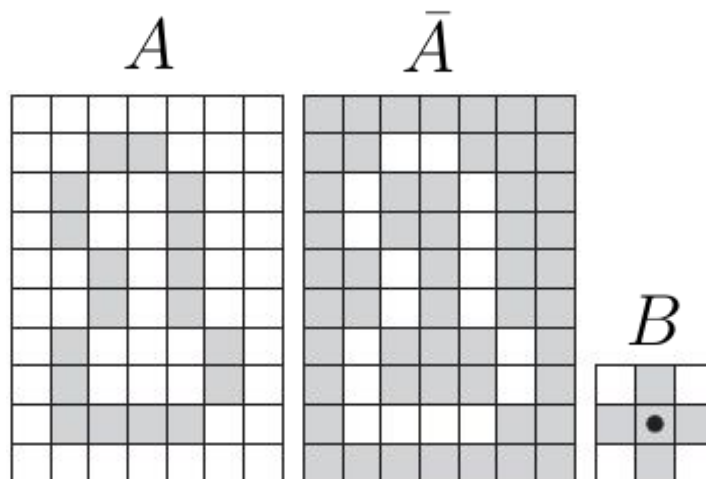


The following iterative procedure fills all holes with 1s:

$$X_k = (X_{k-1} \oplus B) \cap \bar{A}$$



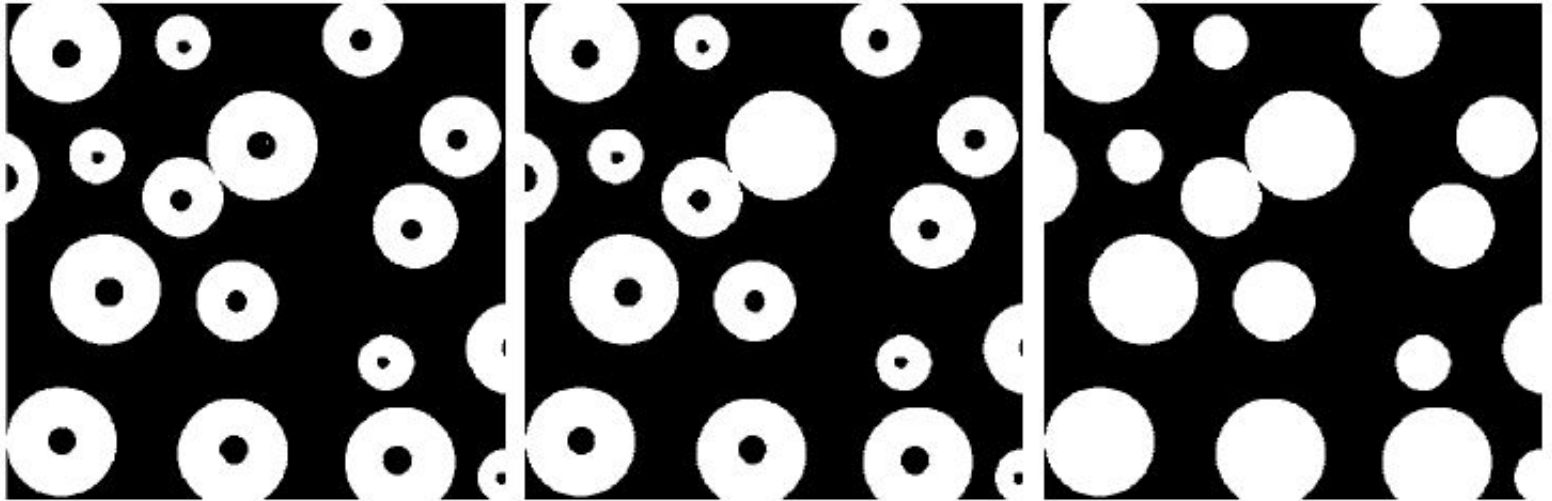
Hole filling





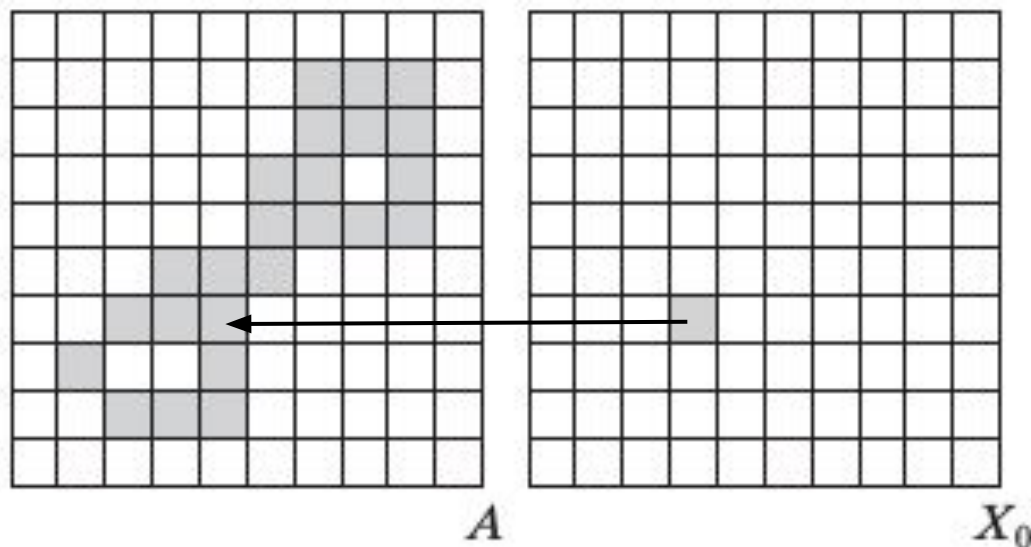
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Hole Filling



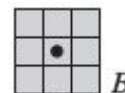
Connected components

Let X_0 be the initial array with the same size of A , filled with 0s except at the location to a known point in each region.



The following iterative procedure fills the connected component with 1s:

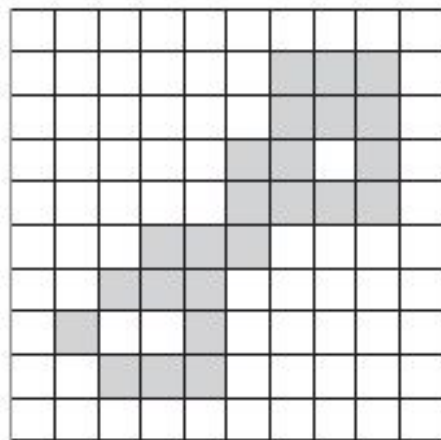
$$X_k = (X_{k-1} \oplus B) \cap A$$



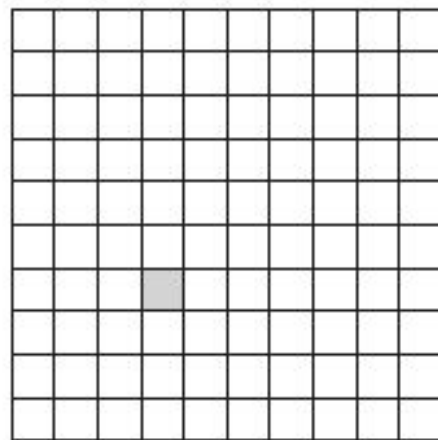


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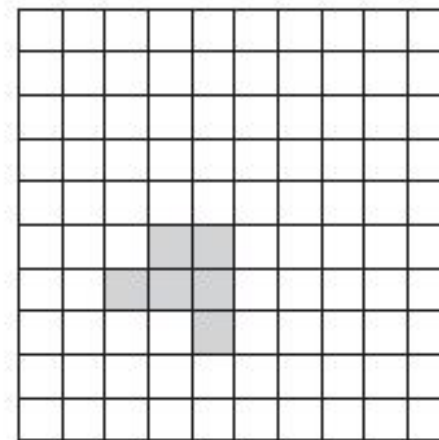
Connected components



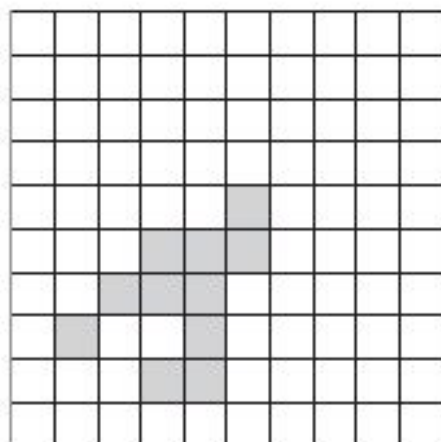
A



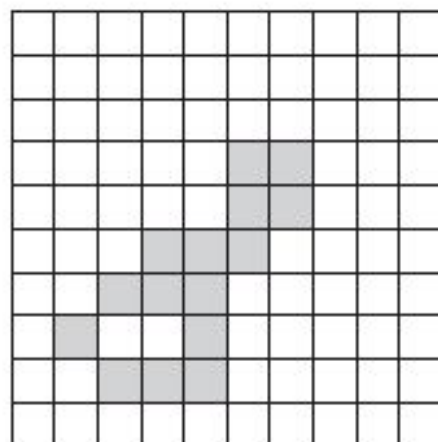
X_0



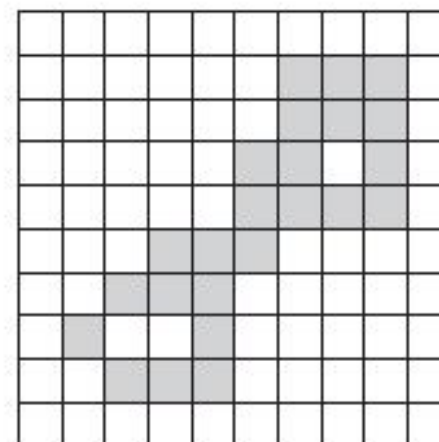
X_1



X_2



X_3

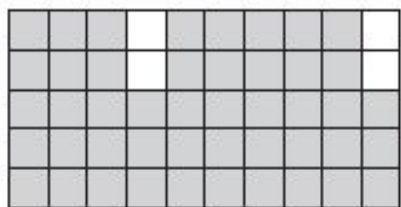


X_6

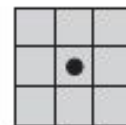
Boundary extraction

The boundary of a set A can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

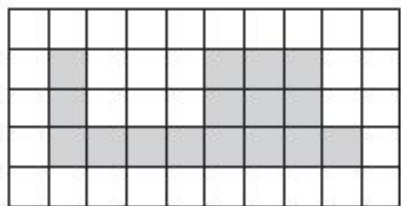
$$\beta(A) = A - (A \ominus B)$$



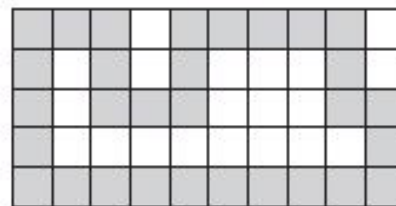
A



B



$A \ominus B$



$\beta(A)$



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Moore boundary tracking

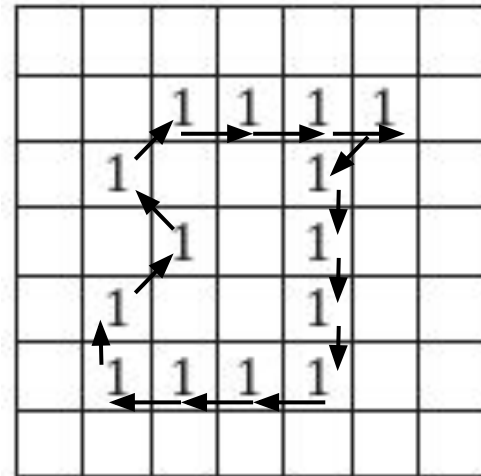
Several algorithms require to extract an ordered sequence of foreground boundary points from a region

Assumptions:

1. We are working with binary thresholded images:
0:background 1:foreground
2. Images are padded with a border of 0s so that no foreground region touches the image border



		1	1	1	1	
	1			1		
		1		1		
	1			1		
	1	1	1	1		





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Moore boundary tracking

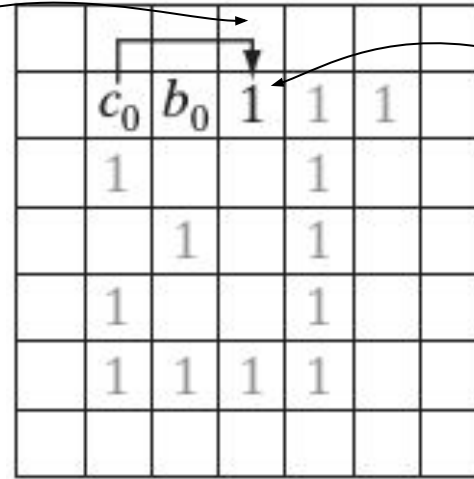
	c_0	b_0	1	1	1	
	1			1		
		1		1		
	1			1		
	1	1	1	1		

1. Let the starting point, b_0 be the uppermost, leftmost point in the image that is labeled 1
 - a. Let c_0 be the west neighbor of b_0



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Moore boundary tracking

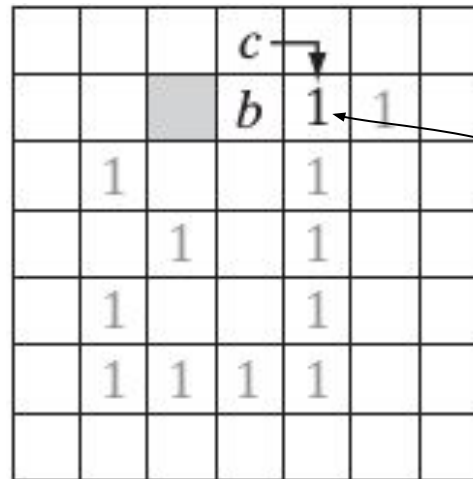


1. (initialization) Examine the 8-neighbors of b_0 , starting at c_0 and proceeding in a clockwise direction.
 - a. Let b_1 denote the first neighbor encountered whose value is 1
 - b. let c_1 be the (background) point immediately preceding b_1 in the sequence.



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Moore boundary tracking



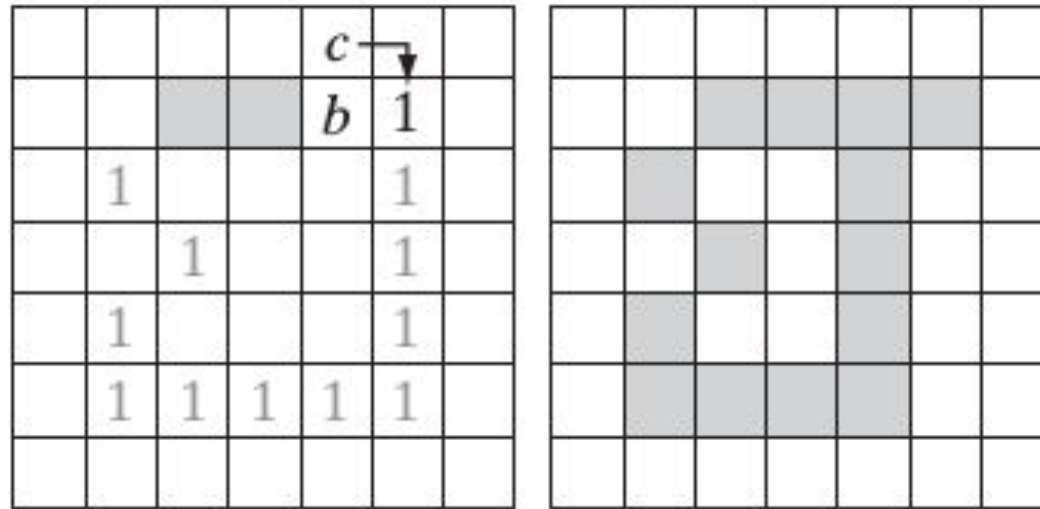
2. Let $b=b_1$ $c=c_1$

3. Let the 8-neighbors of b , starting at c and proceeding in a clockwise direction, be denoted by n_1, n_2, \dots, n_8 . Find the first n_k labeled 1.



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Moore boundary tracking



4. Let $b=n_k$, $c=n_{k-1}$

Repeat Steps 3 and 4 until $b = b_0$ and the next boundary point found is b_1 . The sequence of b points found when the algorithm stops constitutes the set of ordered boundary points.



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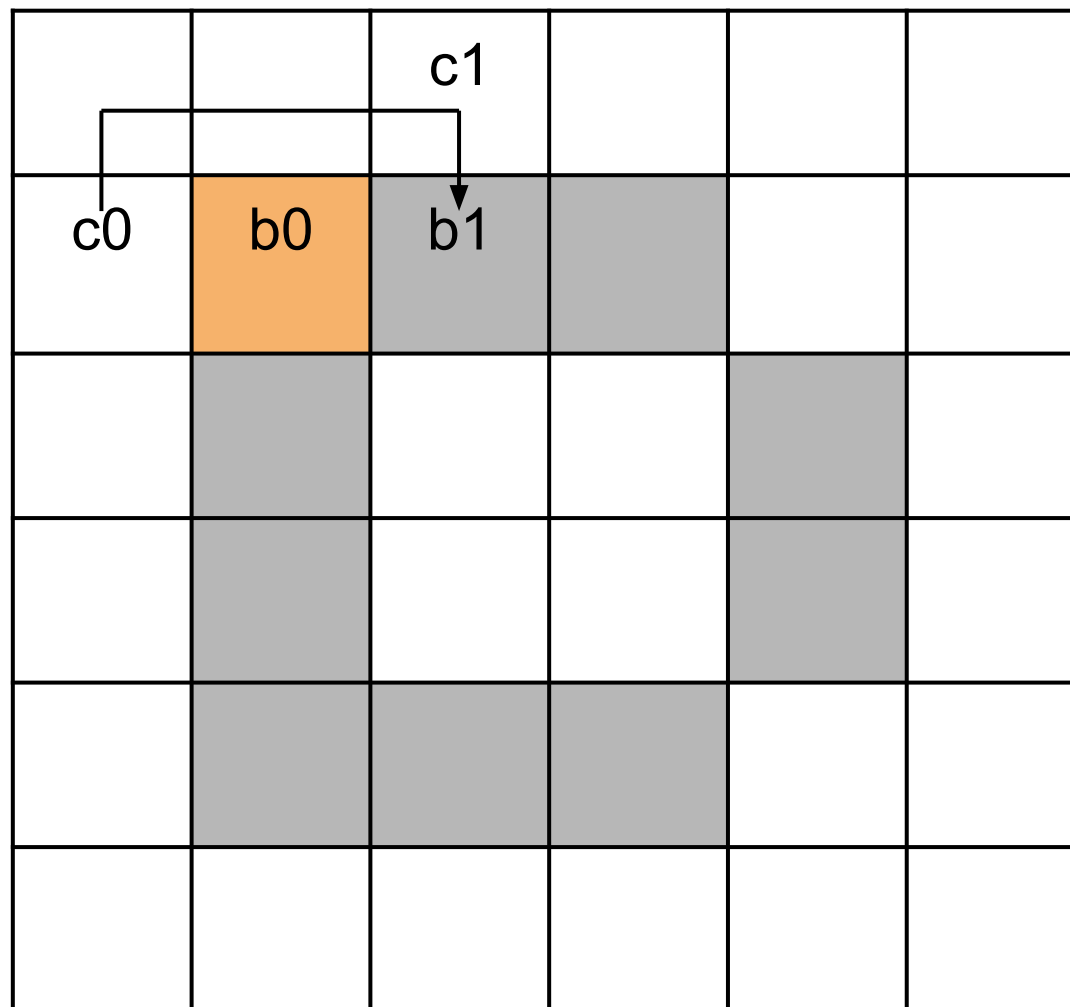
Moore boundary tracking

c0	b0				



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Moore boundary tracking





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Moore boundary tracking

		c			
c0	b0	b			



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Moore boundary tracking

			c		
c0	b0		b		



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Moore boundary tracking

c0	b0			c	
				b	



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Moore boundary tracking

c0	b0				



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Moore boundary tracking

c0	b0				



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Moore boundary tracking

c0	b0				



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Moore boundary tracking

c0	b0				



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Moore boundary tracking

c0	b0				
c	b				



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Moore boundary tracking

c0	b0				
c	b				



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Moore boundary tracking

	1	2	3		
	10			4	
	9			5	
	8	7	6		

Gray-scale morphology

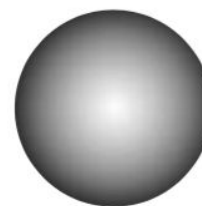
So far, we considered thresholded images to derive a set of morphological operations using set theory.

Erosion, dilation, opening and closing can also be defined for grayscale images.

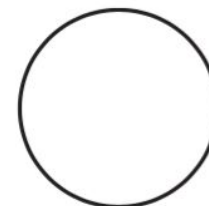
$$f(x, y) : \mathbb{Z}^2 \rightarrow \mathbb{R}$$

$$b(x, y) : \mathbb{Z}^2 \rightarrow \mathbb{R}$$

The structuring element b is used as “probe” to examine a given image for specific properties. Can be non-flat or flat (more common)



Nonflat SE



Flat SE



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Erosion

The erosion of f by a flat structuring element b at any location (x,y) is defined as the *minimum value* of the image in the region defined by b when the origin of b is at (x,y) :

$$[f \ominus b](x, y) = \min_{(s, t) \in b} \{f(x + s, y + t)\}$$

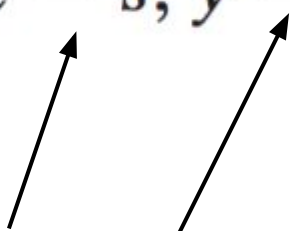
The operation is conceptually similar to convolution. The structuring element b is shifted at every pixel location of the image and the minimum operation is performed among all the pixels in the region coincident with b



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Dilation

The dilation of f by a flat structuring element b at any location (x,y) is defined as the *maximum value* of the image in the region defined by the reflection of b when the origin of it is at (x,y) :

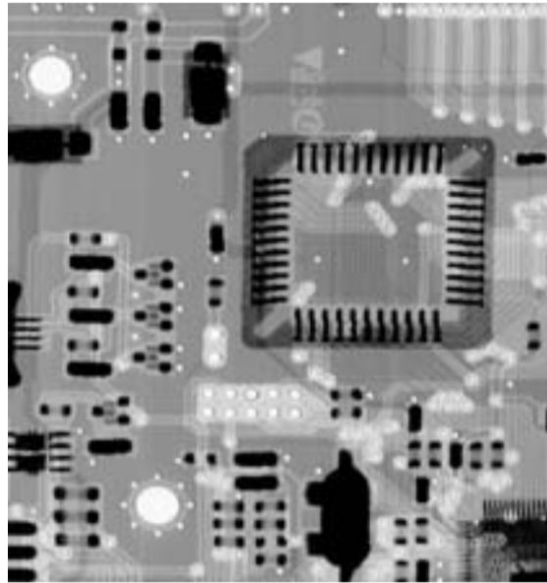
$$[f \oplus b](x, y) = \max_{(s, t) \in b} \{f(x - s, y - t)\}$$


Note the negative sign of the displacement resulting from the relation

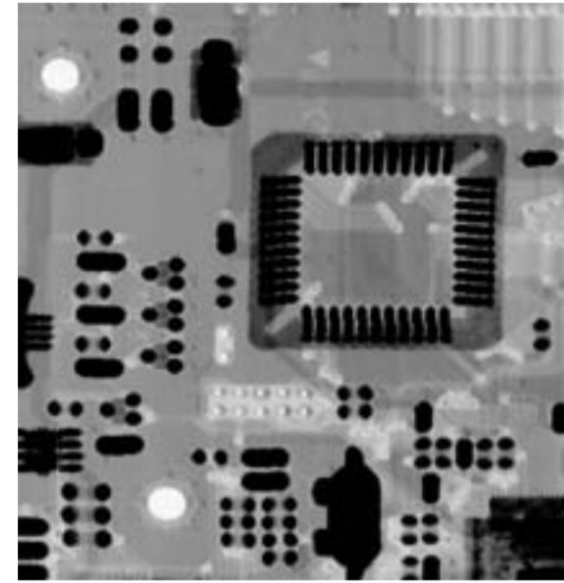
$$\hat{b} = b(-x, -y)$$

Effect of erosion

Original



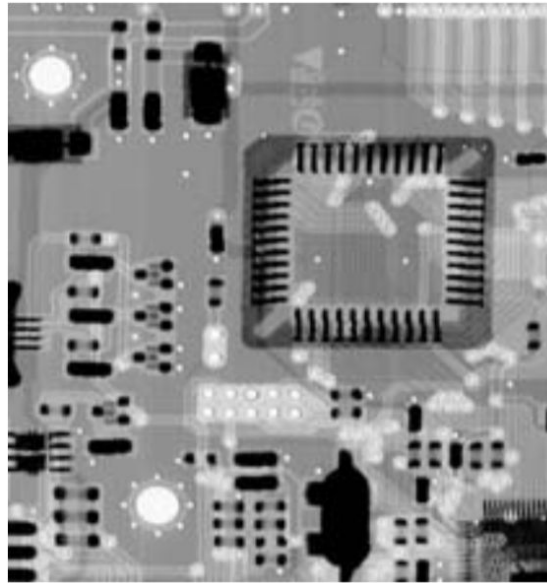
Eroded



Since erosion computes minimum intensity values over every neighborhood of (x,y) , the resulting image is darker and the size of bright features is in general reduced.

Effect of dilation

Original



Dilated



Dilation give opposite results with respect to erosion. The resulting image is brighter, bright features are thickened and the intensities of the dark features is reduced



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Opening and Closing

The expressions for opening and closing gray-scale images have the same form as their binary counterparts

$$A \circ B = (A \ominus B) \oplus B \quad A \bullet B = (A \oplus B) \ominus B$$

Geometric interpretation:

Suppose that f and b are 3D surfaces,

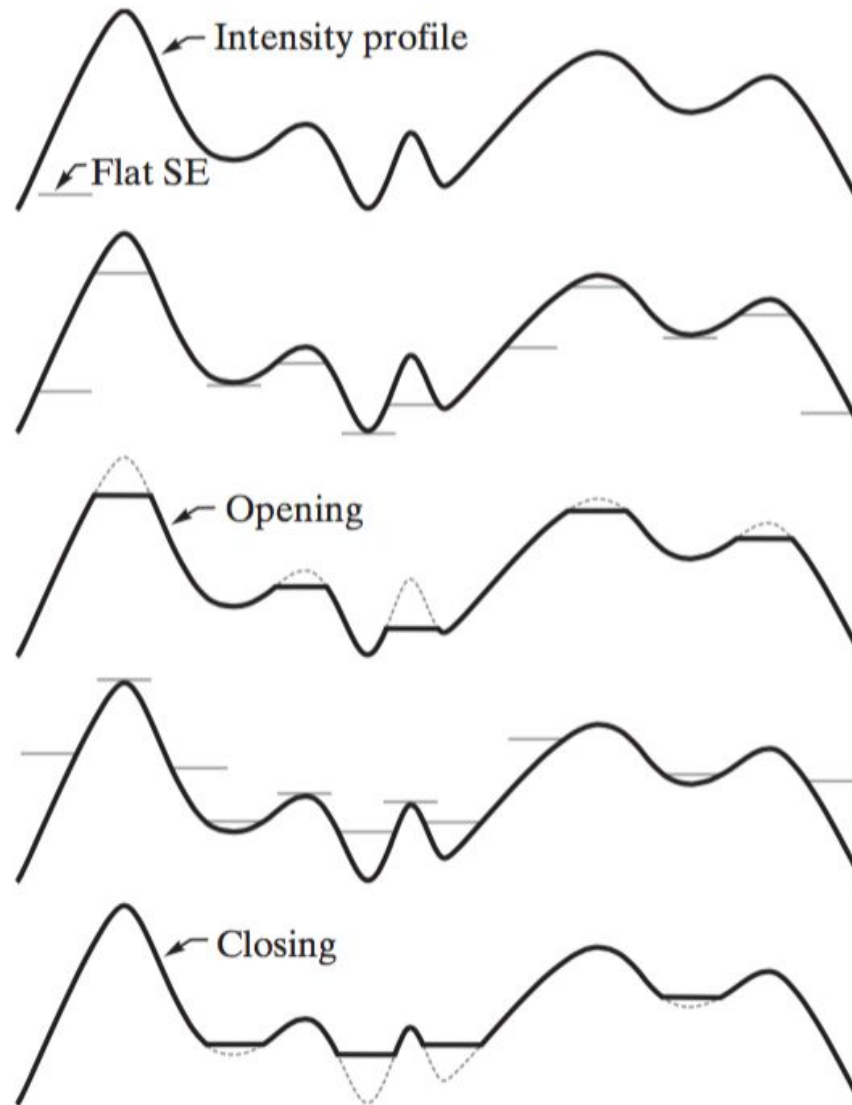
The opening of f by b is like pushing the structuring element up from below against the undersurface of f .

The closing operation is like pushing down the structuring element on top of the curve while being translated to all locations



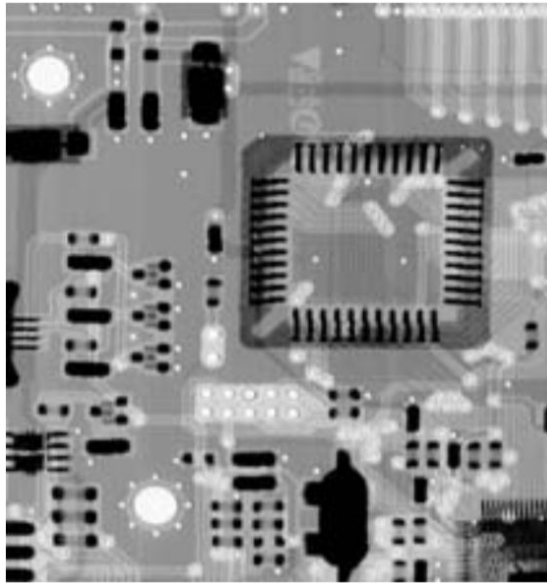
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Opening and Closing

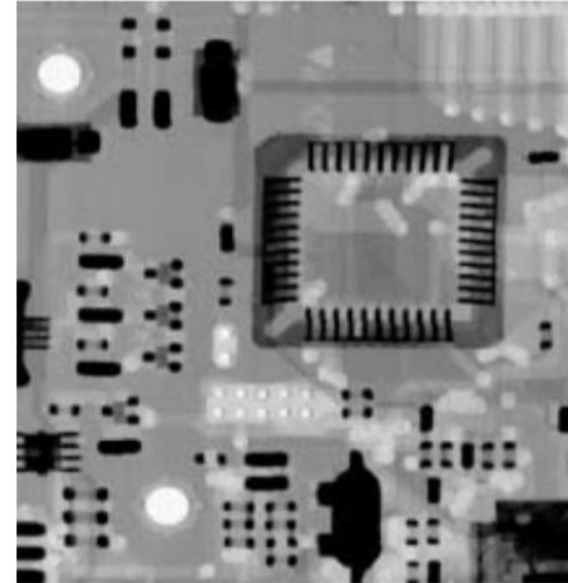


Effect of opening

Original



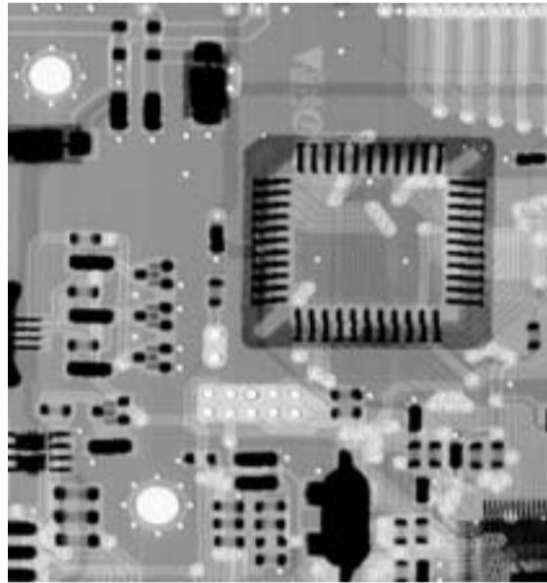
Opening



Opening attenuate bright features and has negligible effect on the dark features and the background of the image

Effect of closing

Original



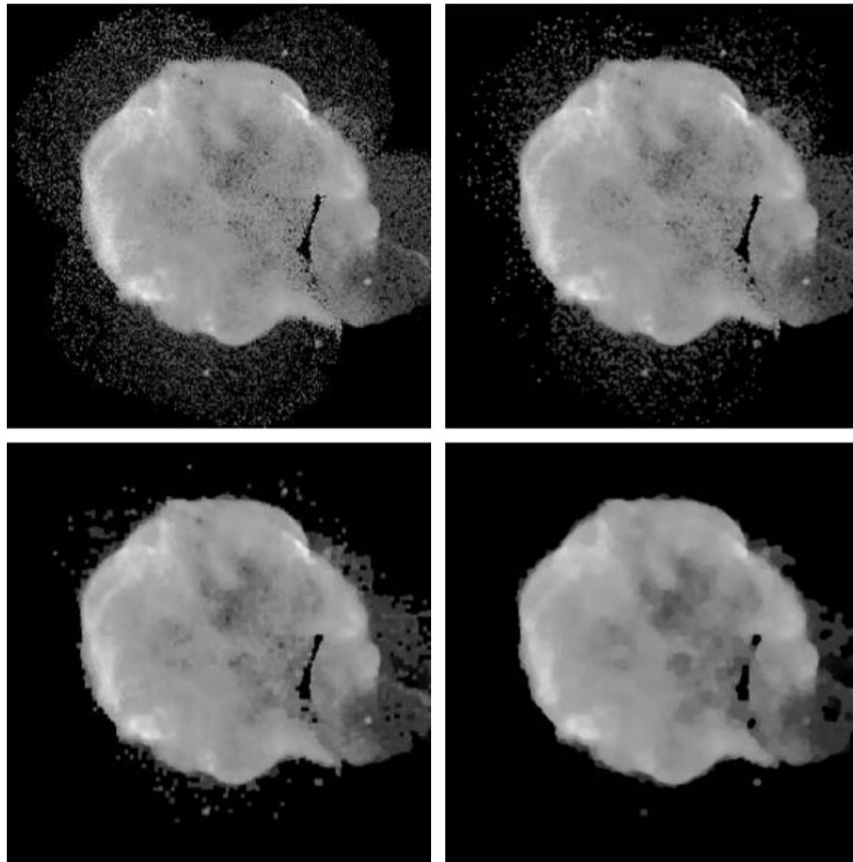
Closing



Closing attenuate dark features and has negligible effect on the bright features and the background of the image

Morphological smoothing

Because opening suppresses bright details, and closing suppresses dark details, they are used often in combination for image smoothing and noise removal



a	b
c	d

FIGURE 9.38

(a) 566×566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)



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Morphological gradient

The dilation thickens regions in an image and the erosion shrinks them. Their difference is an operation with the effect of emphasize the boundaries between regions

$$g = (f \oplus b) - (f \ominus b)$$

The net result is an image in which the edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect



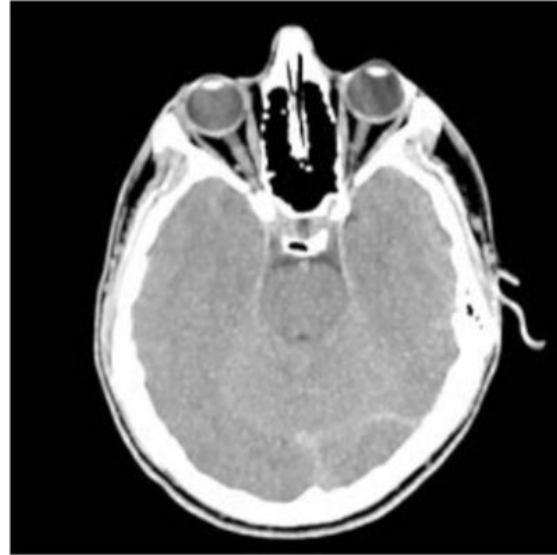
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Morphological gradient

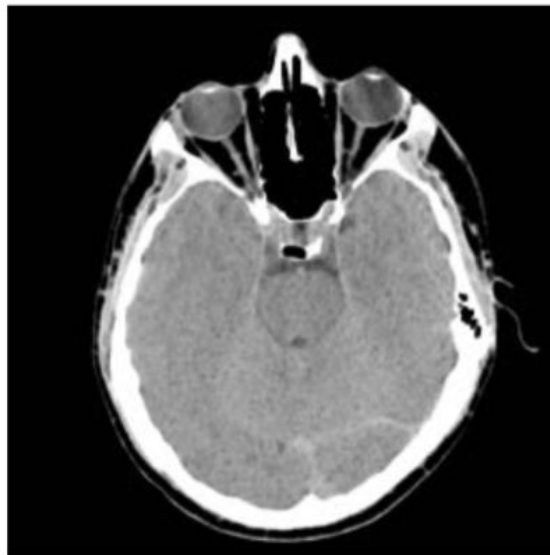
Original



Dilation



Erosion



Morphological
gradient





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Top-hat, Bottom-hat

The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{\text{hat}}(f) = f - (f \circ b)$$

the bottom-hat transformation of f is defined as the closing of f minus f :

$$B_{\text{hat}}(f) = (f \bullet b) - f$$

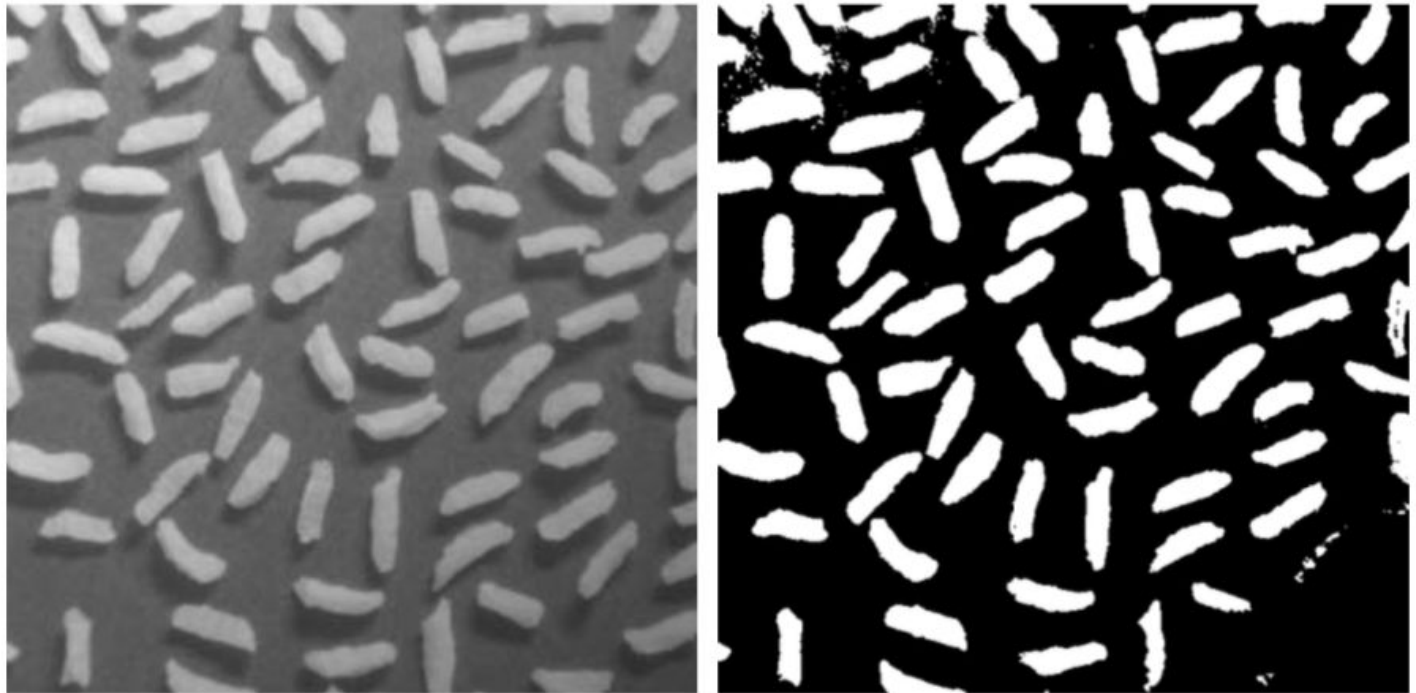
Goal: select objects from an image by using a structuring element in the opening or closing operation that does not fit the objects to be removed. The difference then removes just the selected objects



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Top-hat to correct illumination

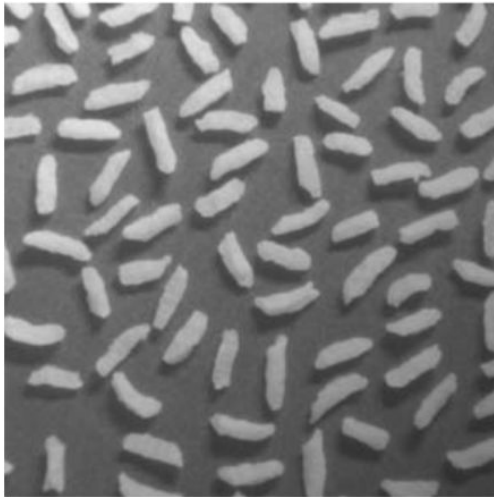
If an images exhibit uneven illumination, a global thresholding operation may fail on some areas





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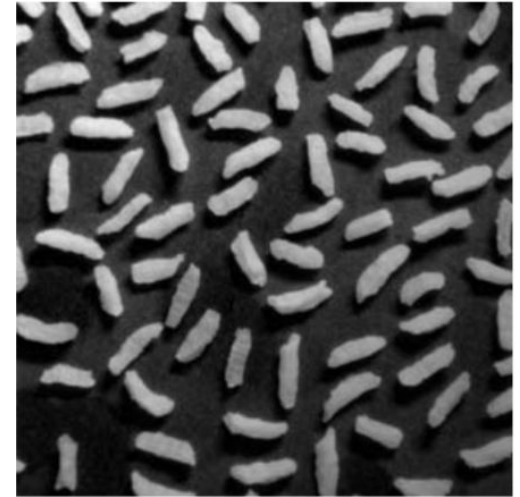
Top-hat to correct illumination



Original



Opening using a
40x40 disc

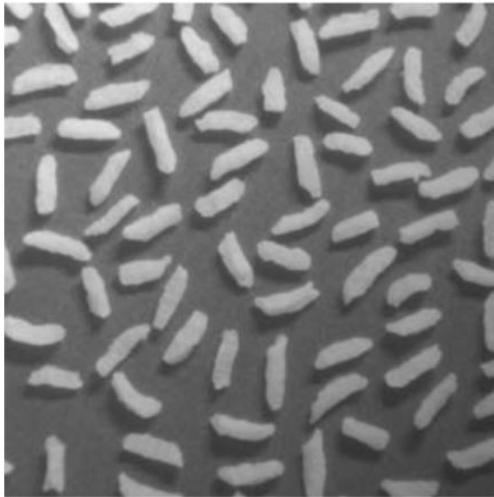


Top-hat



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Top-hat to correct illumination



Original



Otsu



Otsu after
Top-hat