Università
Ca' Foscari
Venezia

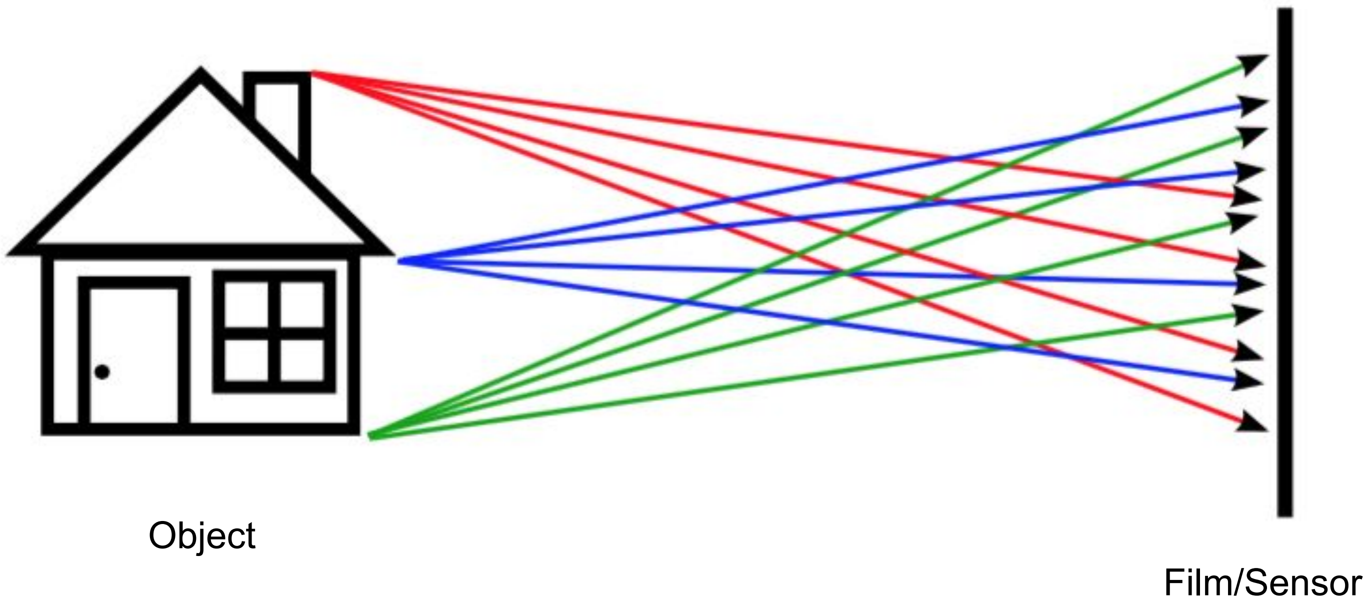
Computer Vision

The Pinhole Camera Model

Filippo Bergamasco (filippo.bergamasco@unive.it)
http://www.dais.unive.it/~bergamasco
DAIS, Ca' Foscari University of Venice
Academic year 2017/2018

Imaging device

Let's try to build a simple imaging device

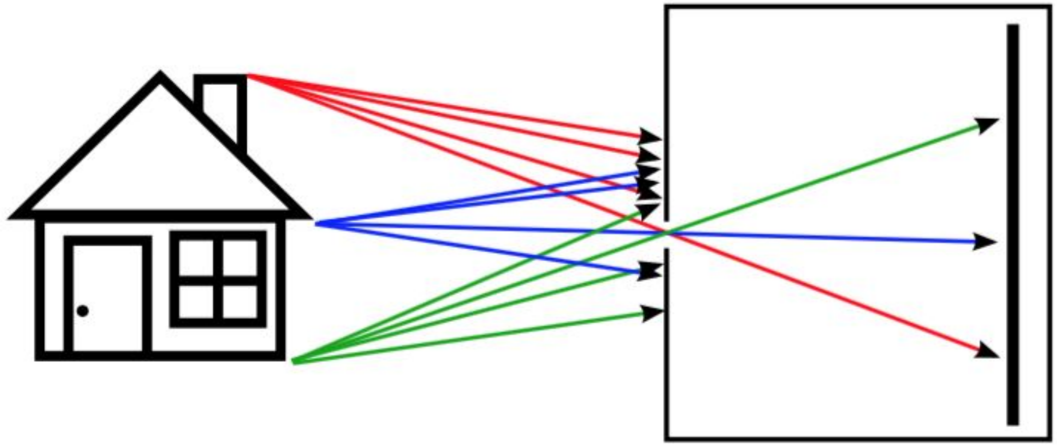


We are unable to get a reasonable image.

Can you guess why?

Camera obscura

Key idea: Put a barrier with a small hole (aperture) between the object and the sensor

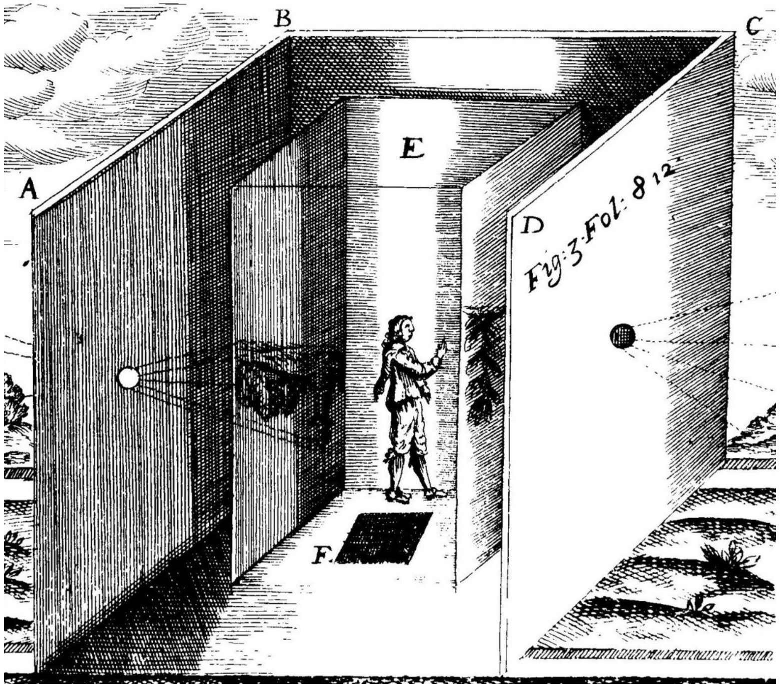


Blurring is reduced! … but the aperture should be as small as possible. This is also known as **pinhole camera**

Università
Ca' Foscari
Venezia

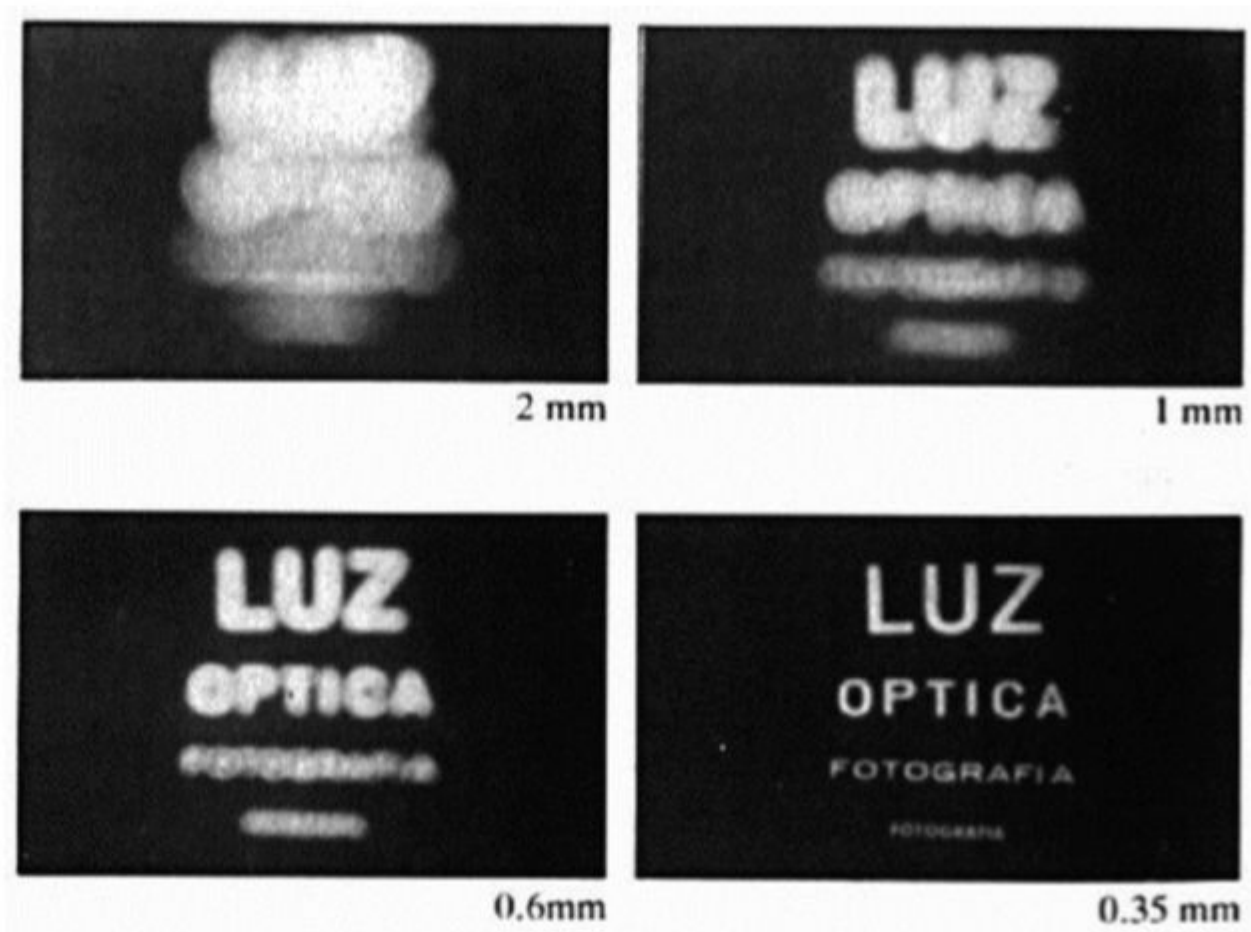
Camera obscura

Leonardo da Vinci (1452–1519), after an extensive study of optics and human vision, wrote the oldest known clear description of the camera obscura in mirror writing in a notebook in 1502



Pinhole camera

How small must the pinhole be?



Pinhole camera

How small must the pinhole be?

Large pinhole:

Rays are mixed up -> Blurring!

Small pinhole:

We gain focus, but

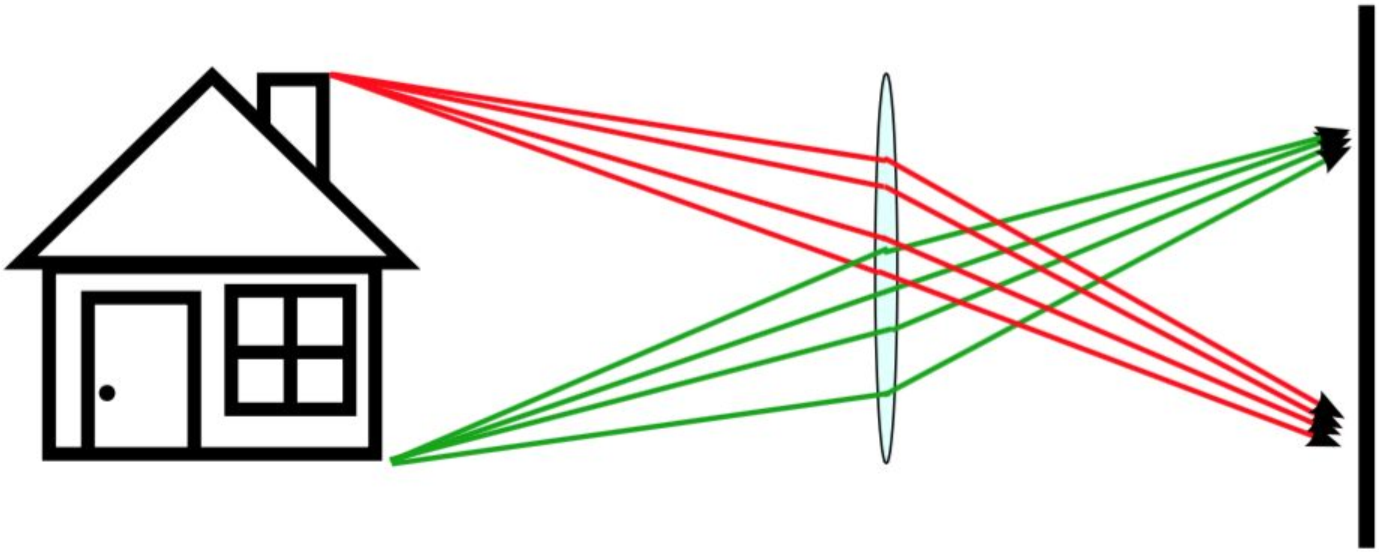
- Less light passes through (long exposure time)
- Diffraction effect (we lost focus again!)

Università
Ca' Foscari
Venezia

Pinhole camera

Cameras and Lenses

Solution: Use lenses!

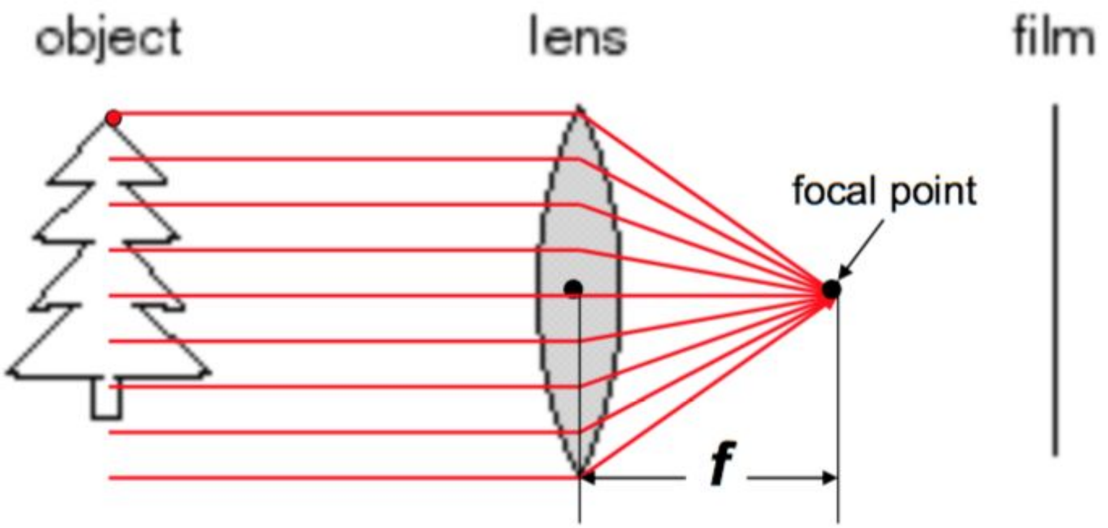


A lens focuses light onto the film/sensor

Cameras and Lenses

Università
Ca' Foscari
Venezia

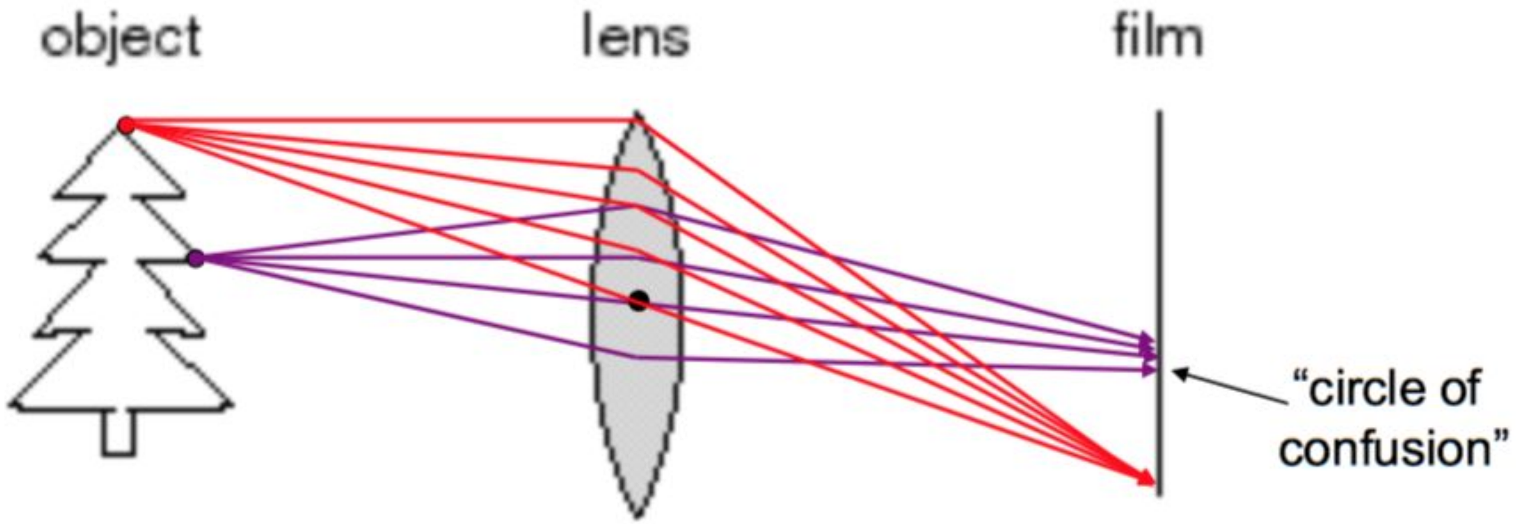
All parallel rays converge to one point on a plane located at the focal length **f**



Università Ca' Foscari Venezia

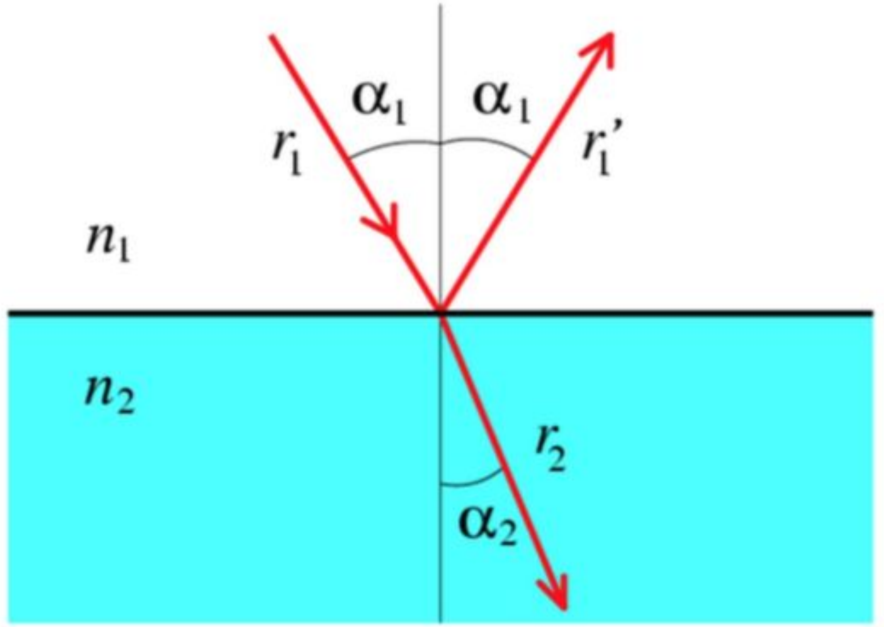
Cameras and Lenses

Unlike the ideal pinhole camera, there is a specific distance at which the objects are in focus



Cameras and Lenses

Rays are deflected when passing through the lens according to the Snell's law



$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Index of refraction

Università Ca' Foscari Venezia

Cameras and Lenses

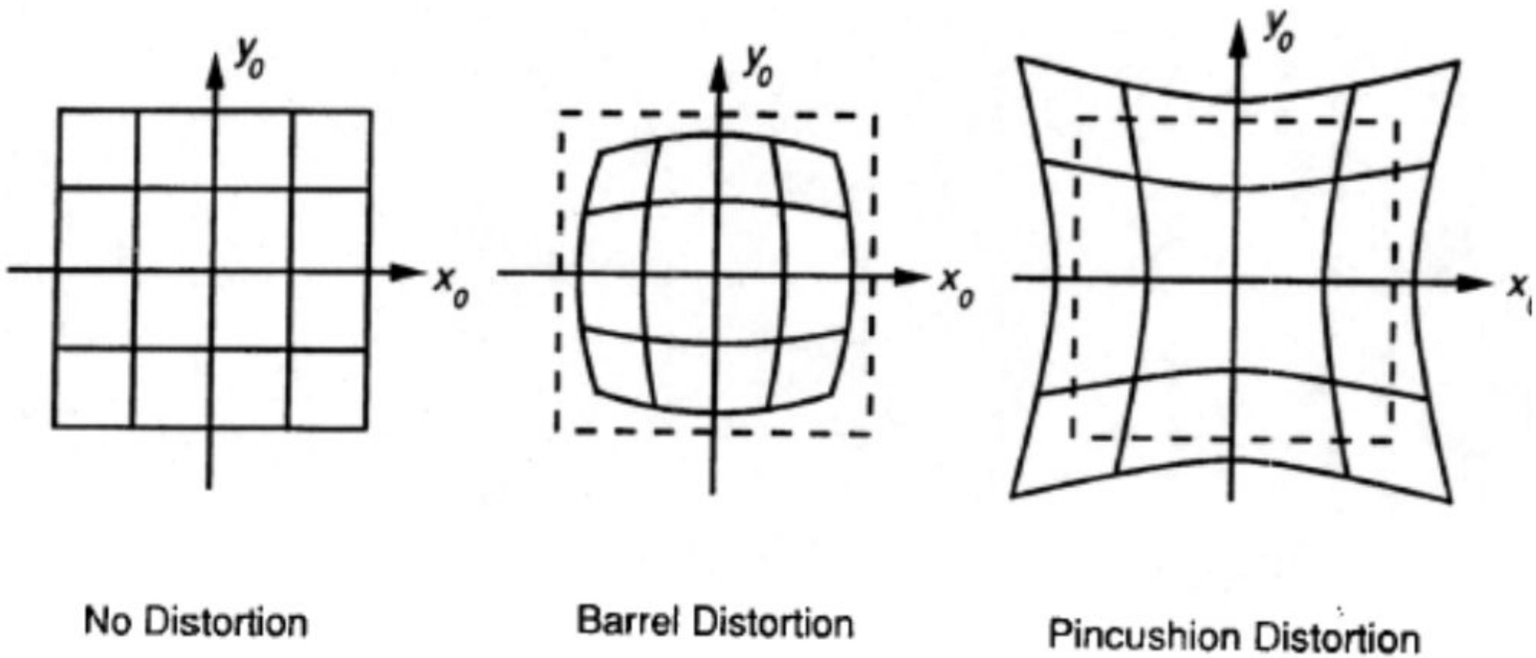
Problem: Lens has different refractive indices for different wavelengths.

Color fringing

Università
Ca' Foscari
Venezia

Cameras and Lenses

Problem: Imperfect lenses may cause radial distortion (deviations are most noticeable for rays passing through the edge of the lens)

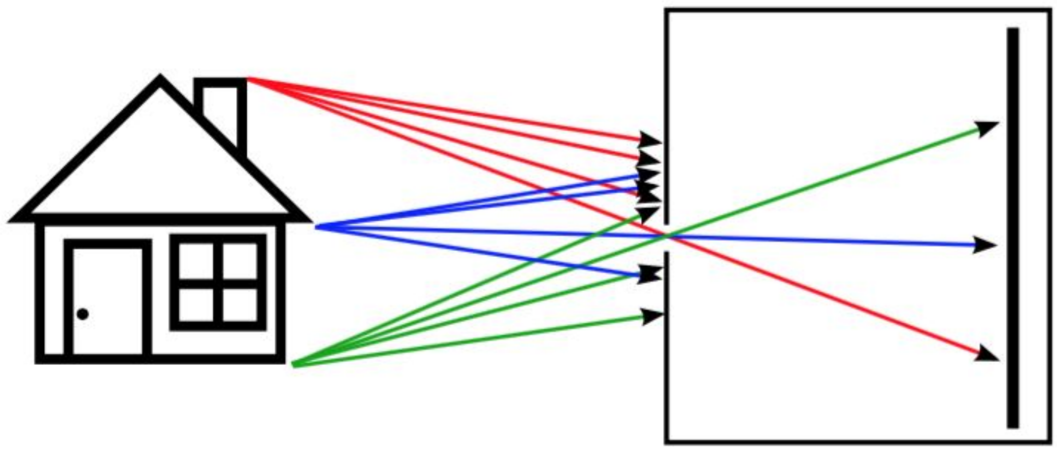


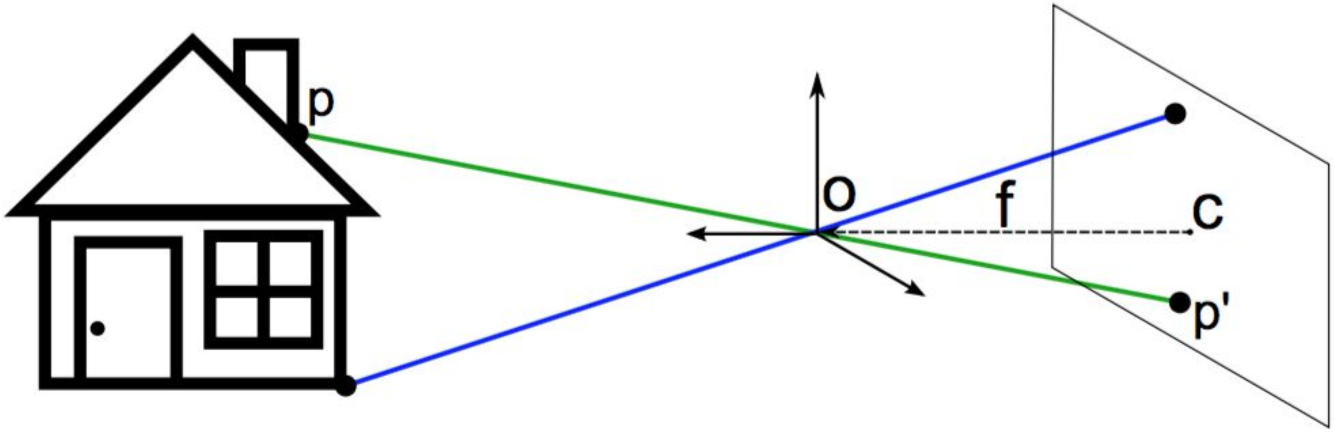
Cameras and Lenses

Problem: Imperfect lenses may cause radial distortion (deviations are most noticeable for rays passing through the edge of the lens)

Università Ca' Foscari Venezia

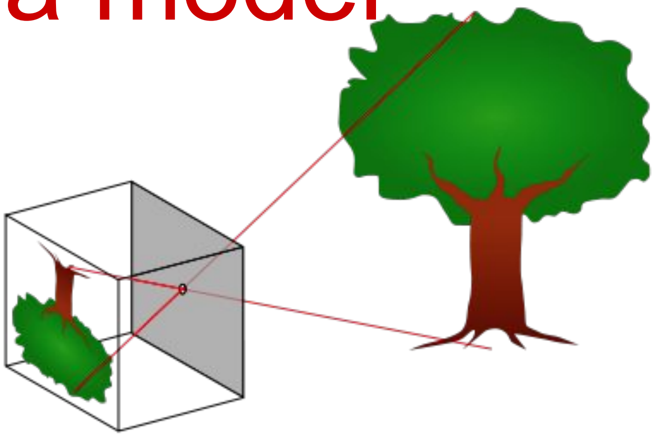
Pinhole camera model





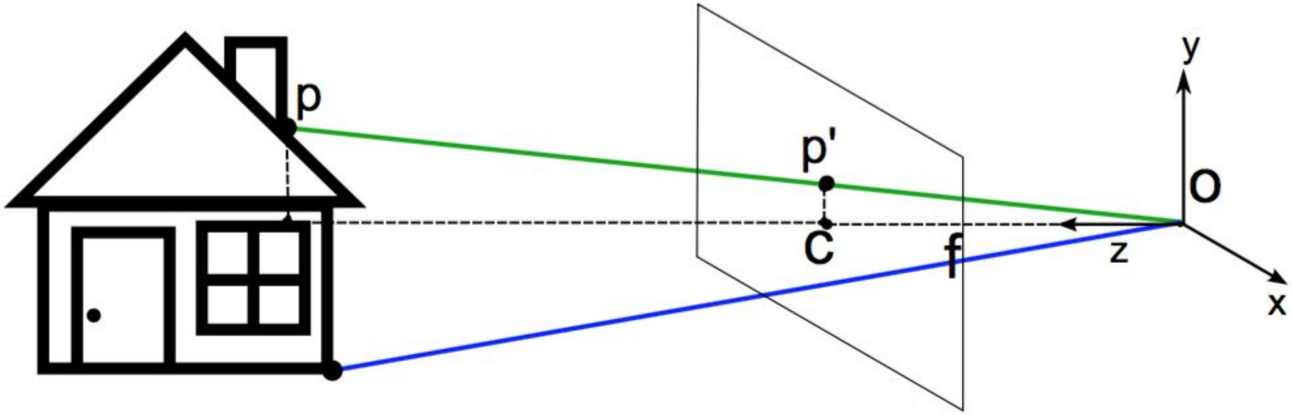
Università
Ca’ Foscari
Venezia

Pinhole camera model



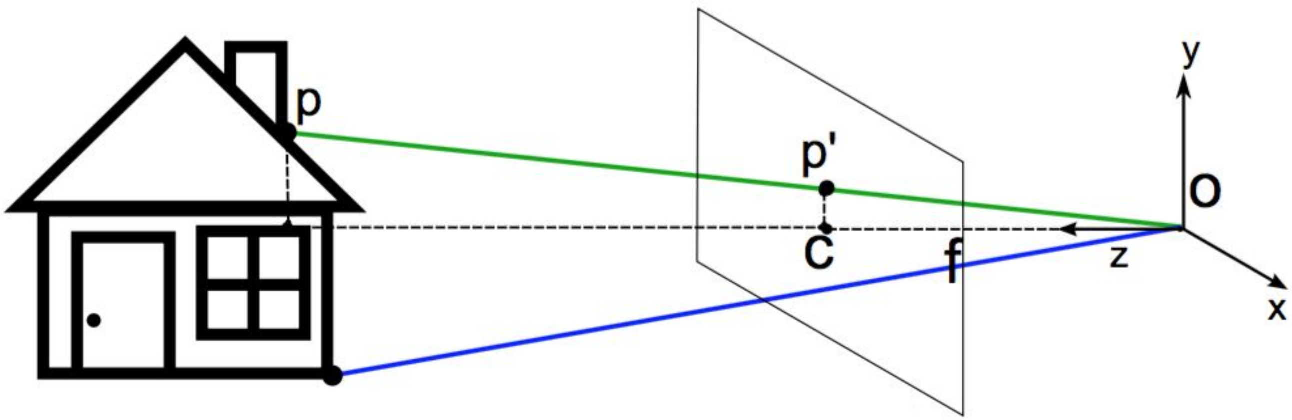
With the image plane behind the optical center (like in the real camera obscura) the image appears upside-down.

It is common to consider a virtual image plane in front of the center of projection.



Università Ca' Foscari Venezia

Pinhole camera model



Considering similar triangles, we can derive the following:

$$p = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \qquad p' = \begin{bmatrix} \frac{x_p}{z_p} f \\ \frac{y_p}{z_p} f \end{bmatrix}$$

Università Ca' Foscari Venezia

Projection

When we capture a scene with a pinhole camera, we are mapping 3D points to 2D points (onto the image plane) according to the following function:

$$E : \mathbb{R}^3 \to \mathbb{R}^2$$

$$(x,\ y,\ z) \to (\frac{x}{z}f,\ \frac{y}{z}f)$$

The function is not linear due to the division by z. How we can make it linear?

Projection

By using homogeneous coordinates, we can express the projection with a linear mapping

$$E : \mathbb{P}^3 \rightarrow \mathbb{P}^2$$

$$P' = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Projection matrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The division by z occurs only when we transform P' from homogeneous coordinates to Euclean

Projection

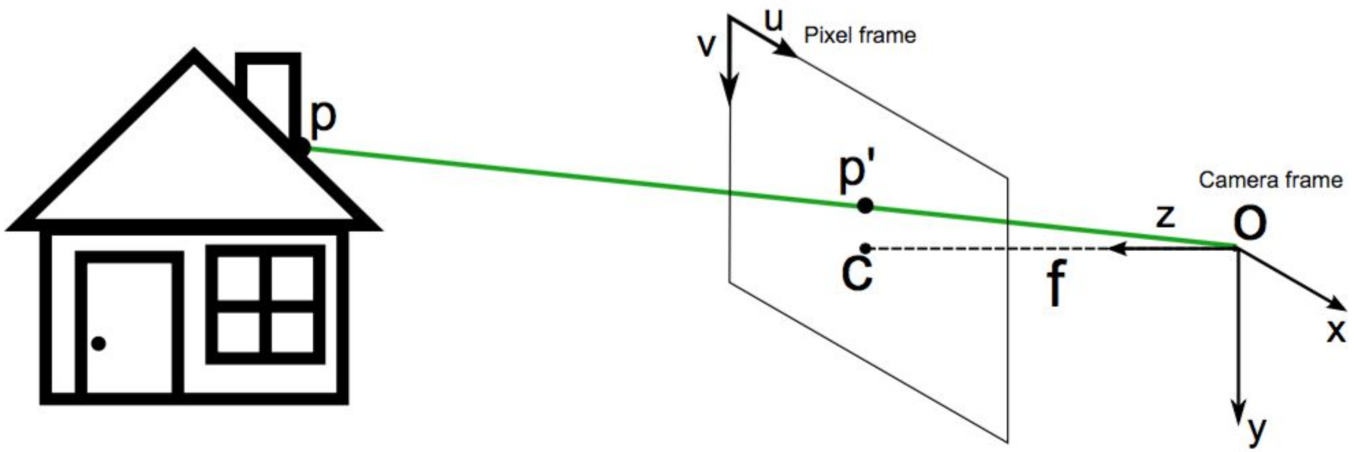
In most cameras, pixels are arranged in a grid in which the pixel (0,0) is at the top left and not at the center.

To project a 3D point in 2D pixel coordinates we need:

- The focal length be expressed in pixels (conversion from metric to pixels)
- To translate the projected point wrt. the pixel coordinates of the principal point (cx,cy)

Projection matrix

$$p' = \begin{bmatrix} fx_p + c_x z_p \\ fy_p + c_y z_p \\ z_p \end{bmatrix} = \begin{bmatrix} f & 0 & c_x & 0 \\ 0 & f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$



Università
Ca' Foscari
Venezia

Intrinsic parameters

Projection matrix

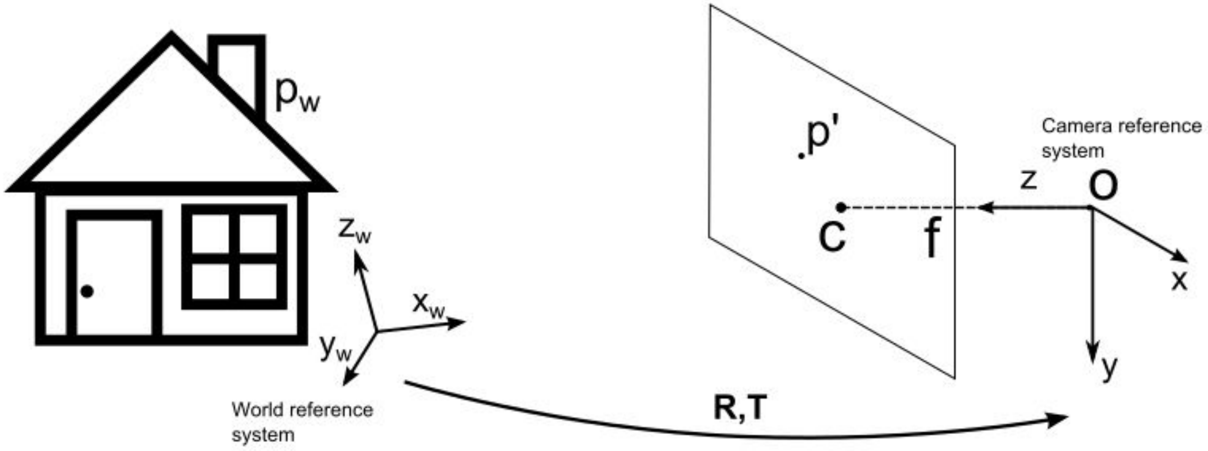
$$p' = \begin{bmatrix} fx_p + c_x z_p \\ fy_p + c_y z_p \\ z_p \end{bmatrix} = \begin{bmatrix} f & 0 & c_x & 0 \\ 0 & f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

Matrix of the intrinsic parameters

Camera pose

What we have seen so far was under the assumption that object points were expressed in the camera reference system

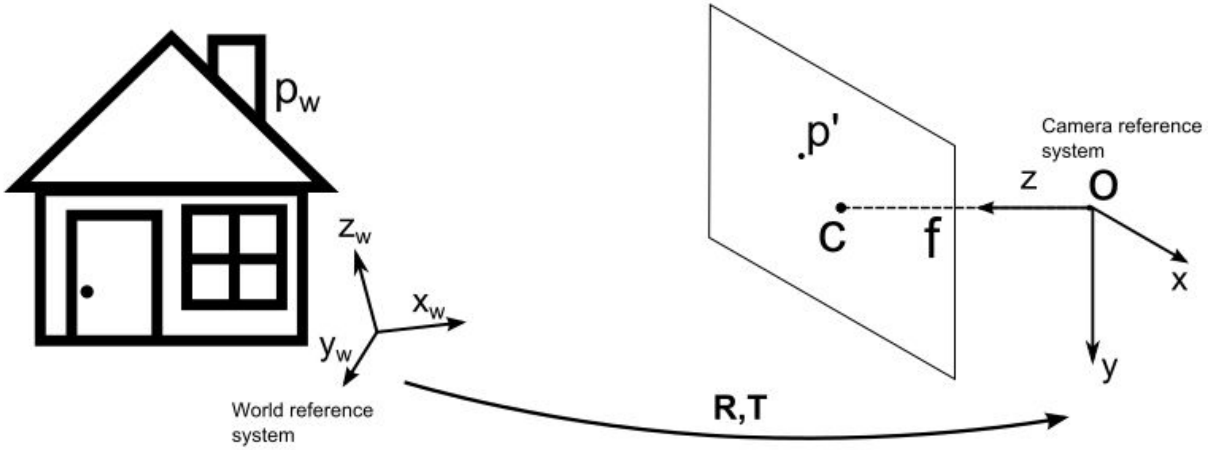
When dealing with multiple cameras it is common to represent points in a common world reference system



Camera pose

A rotation matrix **R** and a translation vector **T** express the rigid motion from a world reference system to the camera reference system (Camera pose)

- 6 degrees of freedom: **R** and **T** define the extrinsic parameters

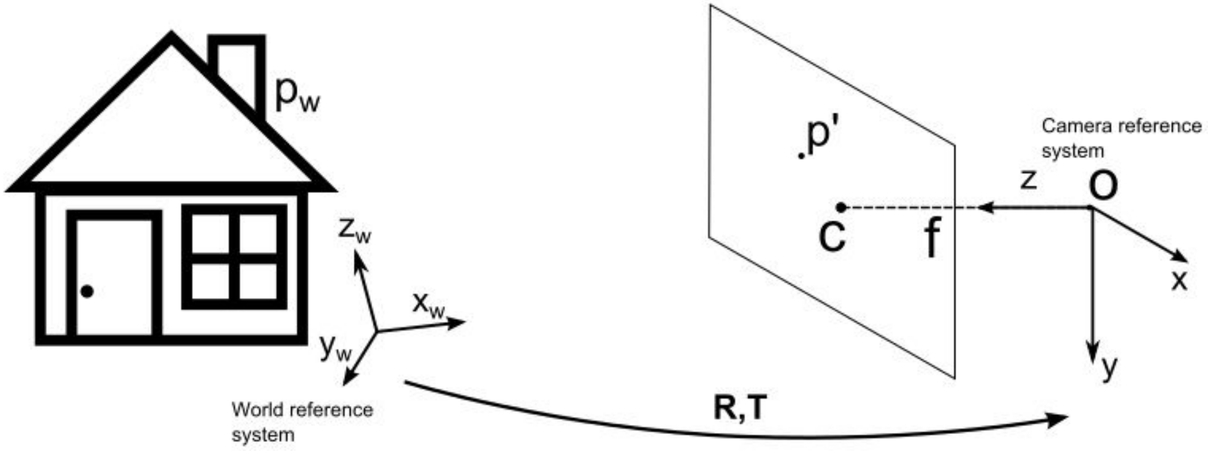


World to Camera

To be projected, a point p_w in the world reference system must first be transformed into the camera coordinate system

- In 4D homogeneous coordinates we got

  $p = [RT]p_w$



Università Ca' Foscari Venezia

Complete projection

$$p' = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} [RT] \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

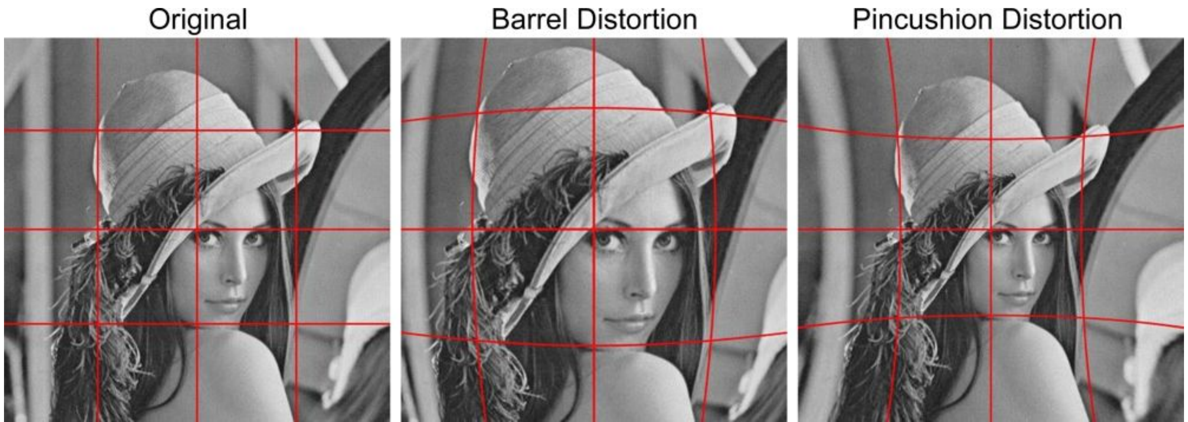
$$p' = \mathbf{K}[\mathbf{RT}]p_w$$

Lens Distortion

The pinhole camera model describes the image projection as a linear operator when working in projective spaces.

Lens distortion produces a non-linear displacement of points after their projection

> Lines does not project to lines anymore!



Radial Distortion

Università Ca' Foscari Venezia

Usually it is a good approximation to model the lens distortion with the polynomial radial distortion model:

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = (1+\mathbf{k}_1 r^2+\mathbf{k}_2 r^4+\mathbf{k}_3 r^6)\begin{bmatrix} x_d \\ y_d \end{bmatrix} + \begin{bmatrix} 2\mathbf{p}_1 x_d y_d + \mathbf{p}_2(r^2+2x_d^2) \\ \mathbf{p}_1(r^2+2y_d^2)+2p_2 x_d y_d \end{bmatrix}$$

point location in retina plane (unitary f) if the pinhole camera were perfect

Distorted location in retinal plane

$$r^2 = x_d^2 + y_d^2$$

$(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{p}_1, \mathbf{p}_2)$ are distortion parameters

Università
Ca' Foscari
Venezia

Radial Distortion