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Computer Vision

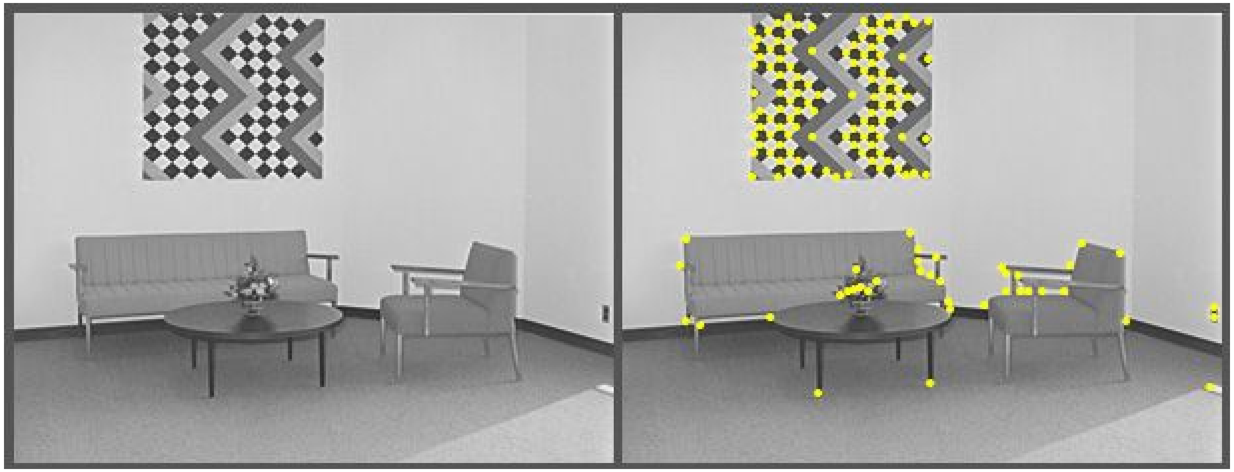
Point features

Filippo Bergamasco (filippo.bergamasco@unive.it)
http://www.dais.unive.it/~bergamasco
DAIS, Ca' Foscari University of Venice
Academic year 2016/2017

Corners

Edge detectors perform poorly at corners.

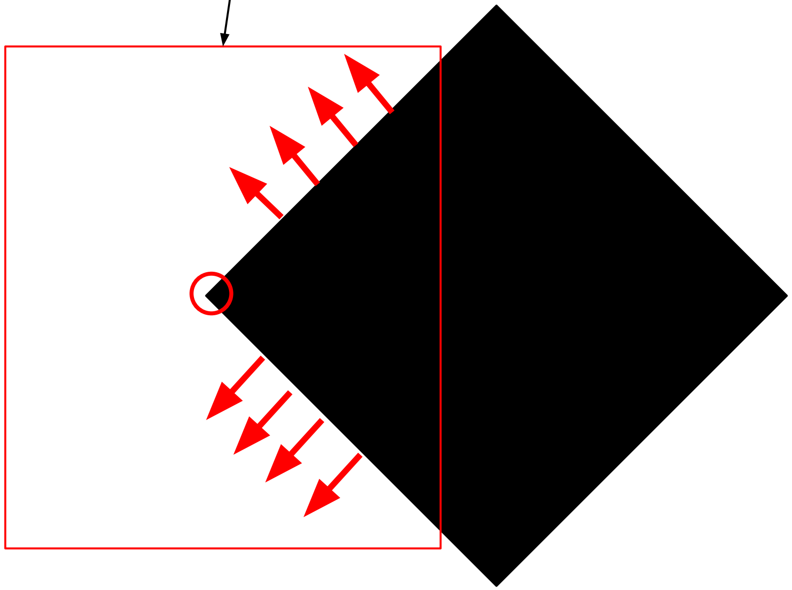
Corners provide repeatable points for matching, so are worth detecting!



Corners

How to find a corner? General idea:

- Exactly at a corner, gradient is ill defined.
- However, in the region around a corner, gradient has two or more different well-defined vectors.



Corners and gradient

Similarly to edges, a corner point exhibit strong rapid changes in the image intensities.

For a small region around a point x_0, we can consider the Taylor expansion of the image function I(x,y) and express the change of intensity as function of the image gradient and a displacement vector h:

$$I(\mathbf{x_0}+\mathbf{h}) \approx I(x_0) + \nabla I(\mathbf{x}_0)^T h$$

$$I(\mathbf{x_0}+\mathbf{h}) - I(x_0) \approx \nabla I(\mathbf{x}_0)^T h$$

Corners and gradient

$$I(\mathbf{x_0} + \mathbf{h}) - I(x_0) \approx \nabla I(\mathbf{x}_0)^T h$$

We are not interested to the sign of this variation (gradient can have any orientation) but only to its magnitude. So we can compute the square of it:

$$\left(I(\mathbf{x_0} + \mathbf{h}) - I(x_0)\right)^2 \approx \mathbf{h}^T \nabla I(\mathbf{x}_0) \nabla I(\mathbf{x}_0)^T \mathbf{h}$$

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Corners and gradient

To be more resilient to noise, we can compute this intensity difference by averaging over a region Ω_{x_0} centered at x_0:

$$\big(I(x_0+h)-I(x_0)\big)^2 \approx \sum_{x\in\Omega_{x_0}} w(x-x_0)\big(I(x+h)-I(x)\big)^2$$

this is usually a Gaussian windowing function

Corners and gradient

Considering the Taylor expansion we have seen before, we have:

$$\left(I(x_0+h)-I(x_0)\right)^2 \approx \underbrace{\sum_{x\in\Omega_{x_0}} w(x-x_0)h^T\nabla I(x)\nabla I(x)^T h}_{E(x_0)}$$

Since h does not depend to x, we can move it out from the summation

Corners and gradient

$$E(x_0) = h^T \left(\sum_{x \in \Omega_{x_0}} w(x - x_0) \nabla I(x) \nabla I(x)^T \right) h$$

$$C = \sum_{x \in \Omega_{x_0}} w(x - x_0) \begin{bmatrix} I_u^2(x) & I_u(x) I_v(x) \\ I_v(x) I_u(x) & I_v^2(x) \end{bmatrix}$$

$E(x_0)$ can then be written as: $E(x_0) = h^T C h$

And the summation can be moved inside the matrix:

$$C = \begin{bmatrix} \sum_{x \in \Omega_{x_0}} w(x - x_0) I_u^2(x) & \sum_{x \in \Omega_{x_0}} w(x - x_0) I_u(x) I_v(x) \\ \sum_{x \in \Omega_{x_0}} w(x - x_0) I_v(x) I_u(x) & \sum_{x \in \Omega_{x_0}} w(x - x_0) I_v^2(x) \end{bmatrix}$$

Second moment matrix

C form the second-moment matrix (we discard the weights for clarity)

$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix}$$

1. Depends on the first-order derivatives
2. Symmetric
3. Each element is obtained as a sum over a small region around a point x_0

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Simple case

First, consider the following ideal case:

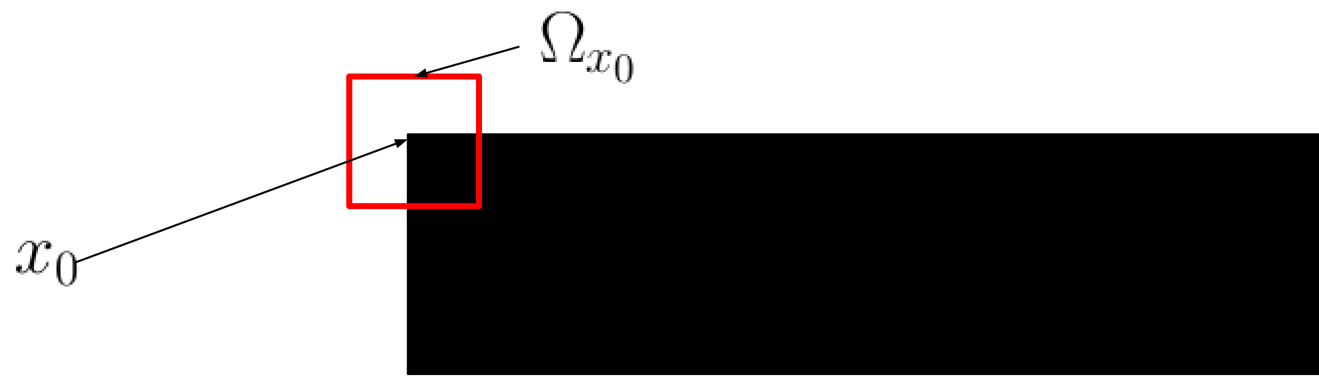


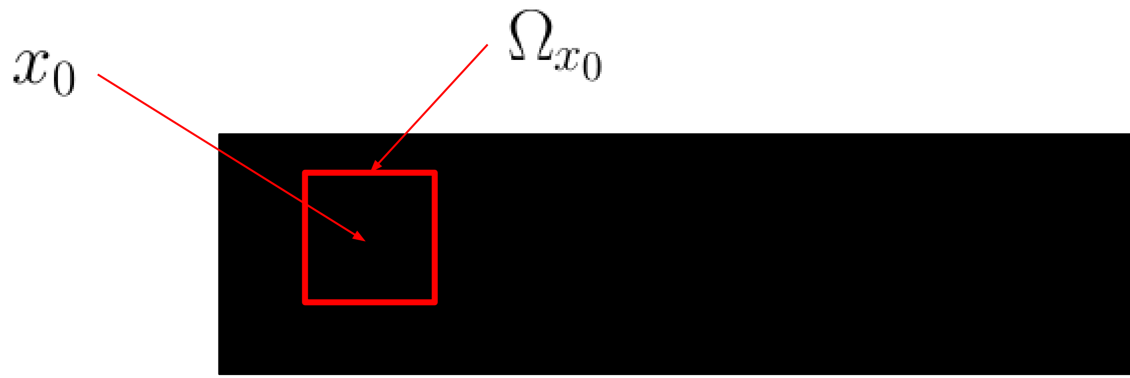
Image intensity changes either in x or y direction, but not both

$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Simple case

When x_0 is at a flat region, we expect

$$\lambda_1 = \sum I_u^2 = 0, \lambda_2 = \sum I_v^2 = 0$$

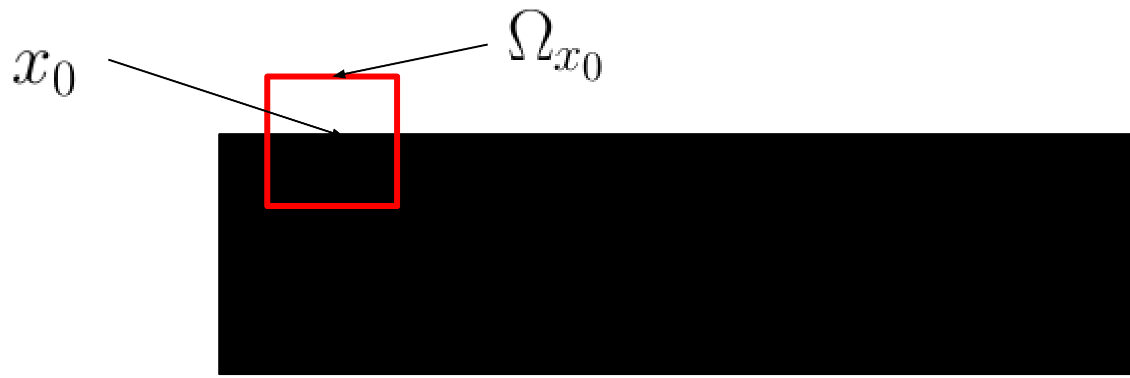


$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Simple case

When x_0 is at an horizontal edge, we expect

$$\lambda_1 = \sum I_u^2 = 0, \lambda_2 = \sum I_v^2 \gg 0$$



$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Simple case

When x_0 is at a vertical edge, we expect

$$\lambda_1 = \sum I_u^2 \gg 0, \lambda_2 = \sum I_v^2 = 0$$

$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

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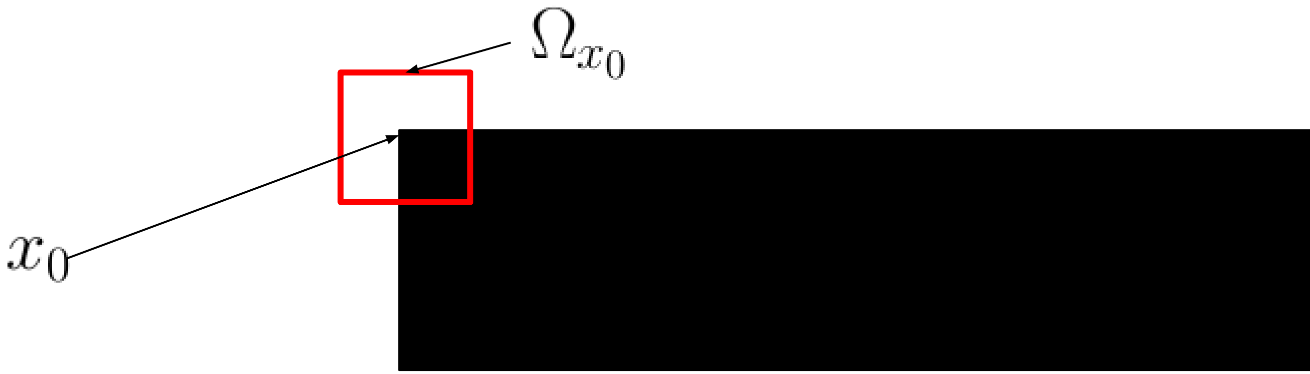
Simple case

When x_0 is at a **corner**, we expect both $\sum I_u^2$ and $\sum I_v^2$ be large (ie. λ_1, λ_2 far from zero)



$$C = \begin{bmatrix} \sum I_u^2 & \sum I_u I_v \\ \sum I_v I_u & \sum I_v^2 \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

General case

So we can detect a corner if both λ_1, λ_2 are far from zero.

What about the general case in which $\sum I_u I_v$ are not zero?

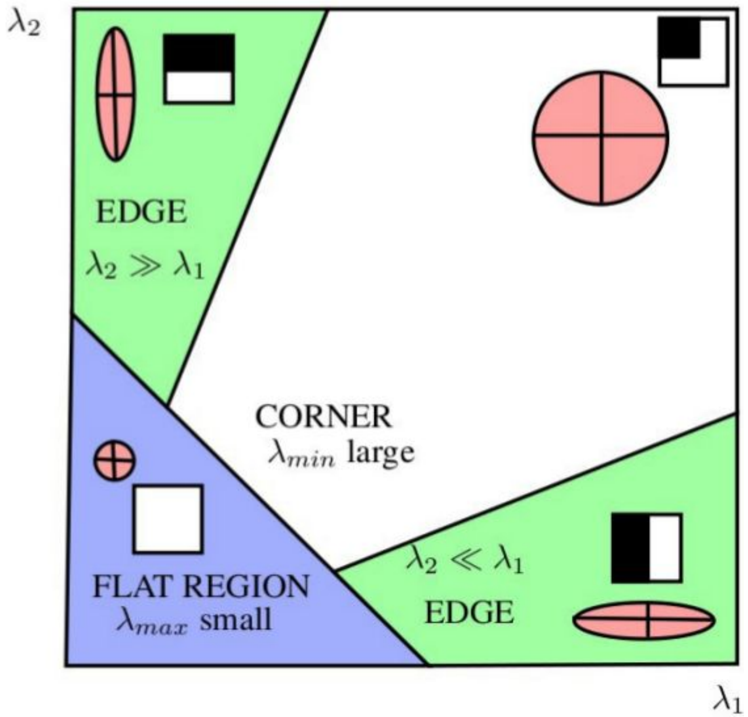
Since C is symmetric, it can be decomposed via SVD:

$$C = R \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} R^T$$

Where R is a rotation matrix and λ_1, λ_2 are the singular values of C (ie. the square-root of the eigenvalues of $C^T C$)

General case

Since the rotations do not change the magnitude of h, examining the singular values of C can tell us if x_0 is in a flat region, an edge or a corner



Harris corner detector

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Analyzing the singular values of C requires the computation of SVD at each image pixel

> This is computationally expensive in practice

Harris proposed to use the following function as a corner response:

$$R(x_0) = det(C_{x_0}) - k\ \mathrm{trace}^2(C_{x_0})$$

Where k is a constant that has to be tuned for the specific application

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Harris corner detector

$$R(x_0) = det(C_{x_0}) - k \, \mathrm{trace}^2(C_{x_0})$$

It can be shown that:

$$\mathrm{trace}(C_{x_0}) = \lambda_1 + \lambda_2$$

$$\det(C_{x_0}) = \lambda_1 \lambda_2$$

Therefore, $R(x_0) >> 0$ if we are on a corner, and $R(x_0) << 0$ if we are on an edge

Harris corner detector

Algorithm:

- Optional: Filter an image with a Gaussian to reduce noise
- Compute the image gradient $I_u(x,y), I_v(x,y)$
- Compute the matrix C for each pixel
  - 3 convolutions needed
- Compute the Harris response for each pixel
- Threshold the result and (optionally) perform non-maxima suppression

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Harris corner detector

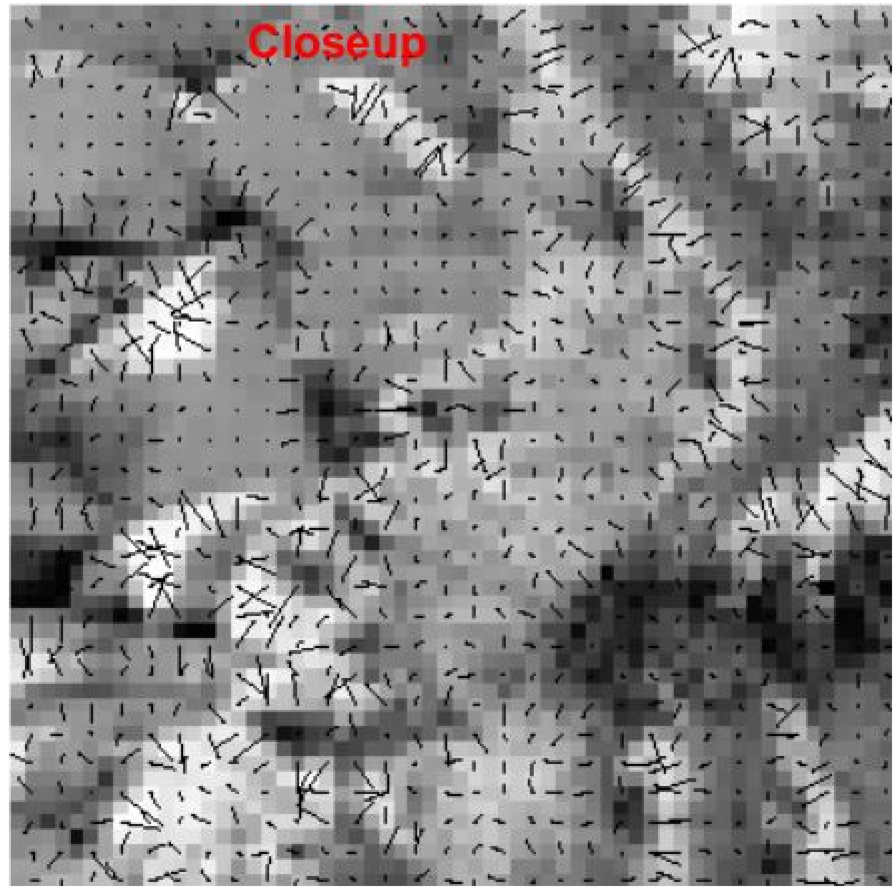

Image Gradient

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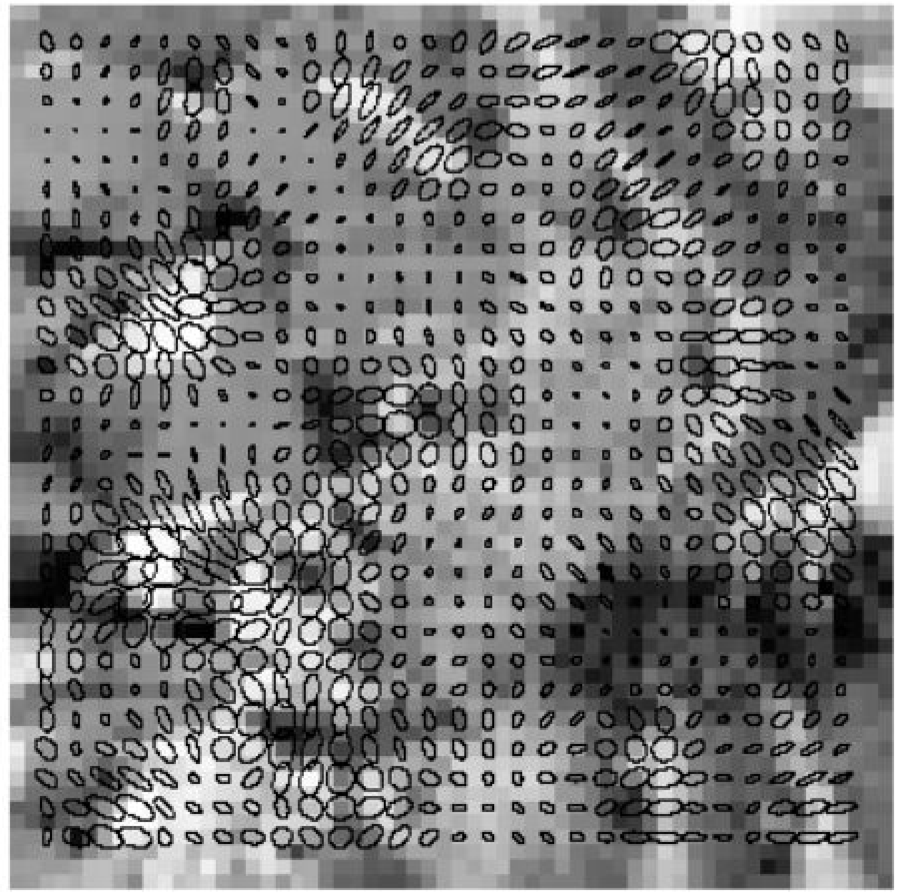
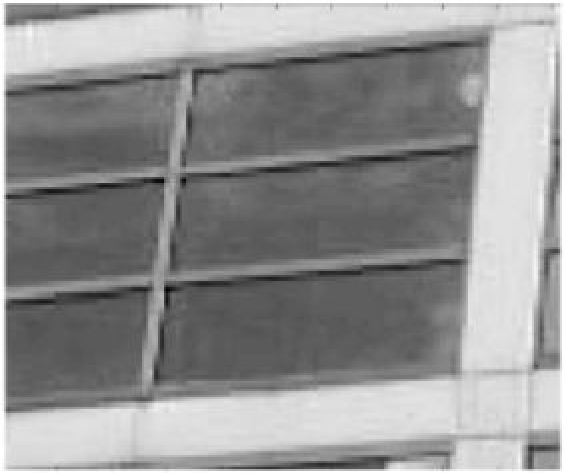
Harris corner detector

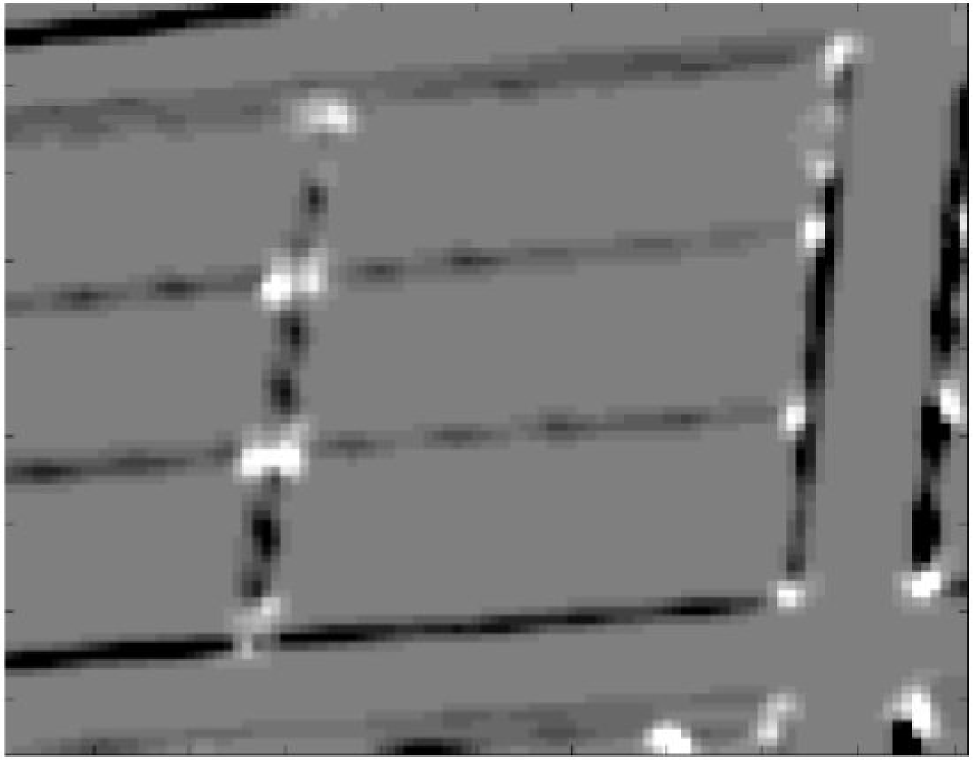


Eigenvalues plotted as ellpse axes

Harris corner detector



Input image



Harris response (R)

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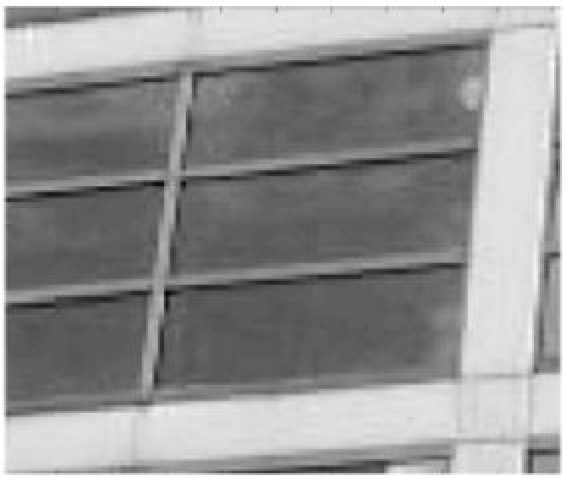
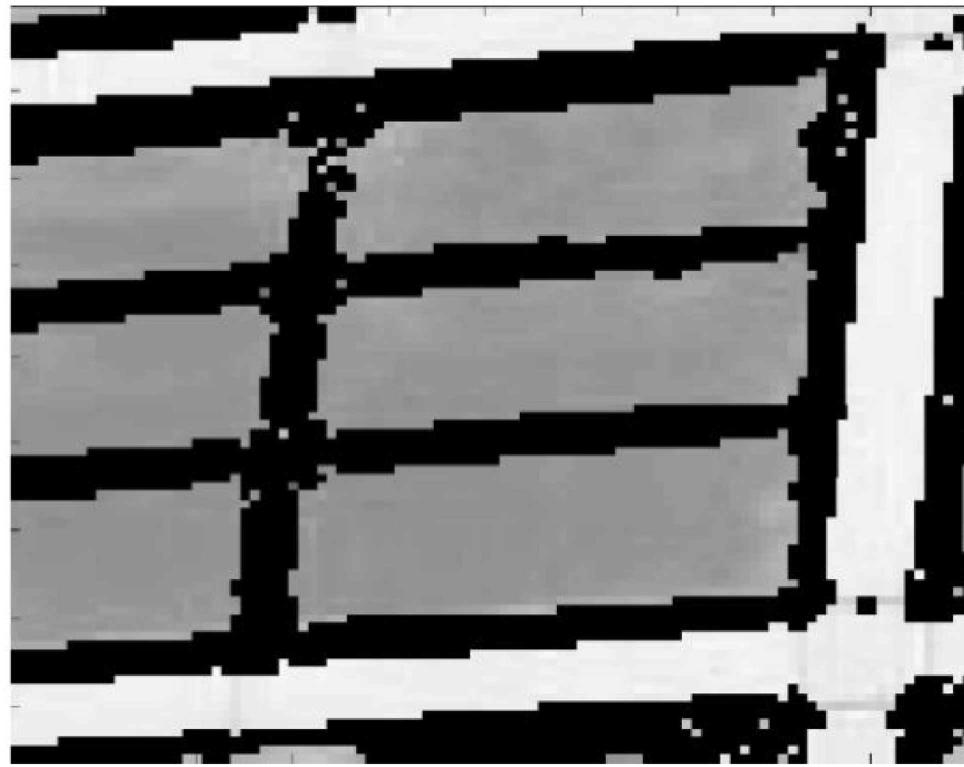
Harris corner detector



Input image



|R|<1E4
(flat regions)

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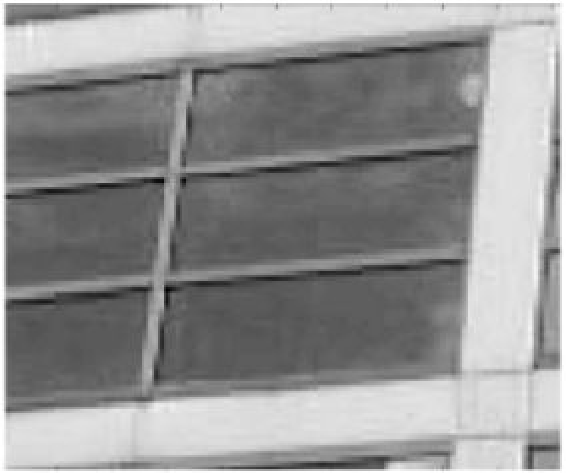
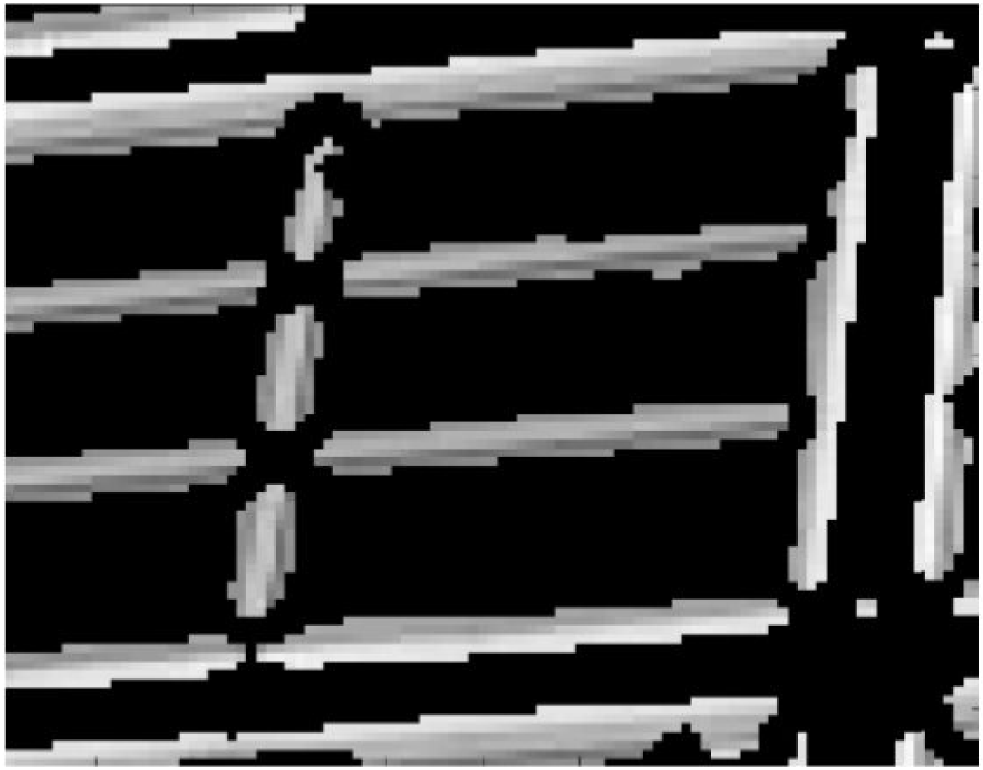
Harris corner detector



Input image



R<-1E4
(edges)

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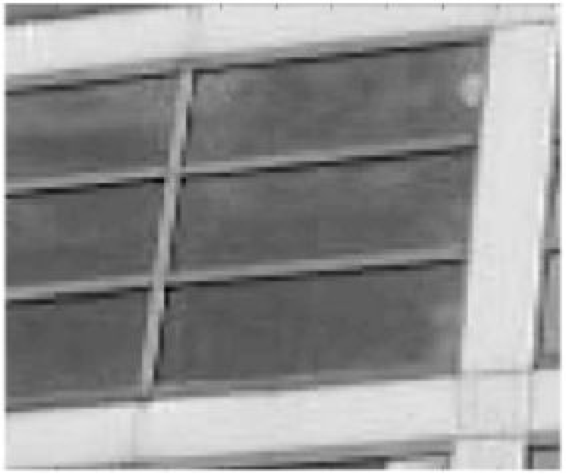
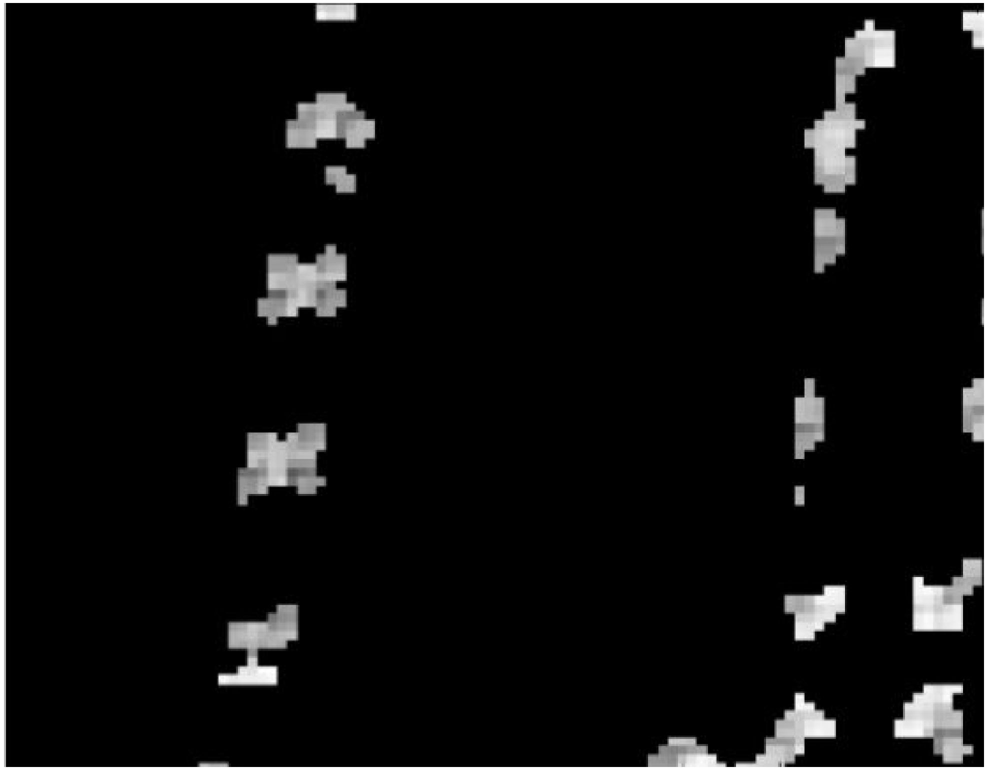
Harris corner detector



Input image



R>1E4
(corners)

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More advanced features

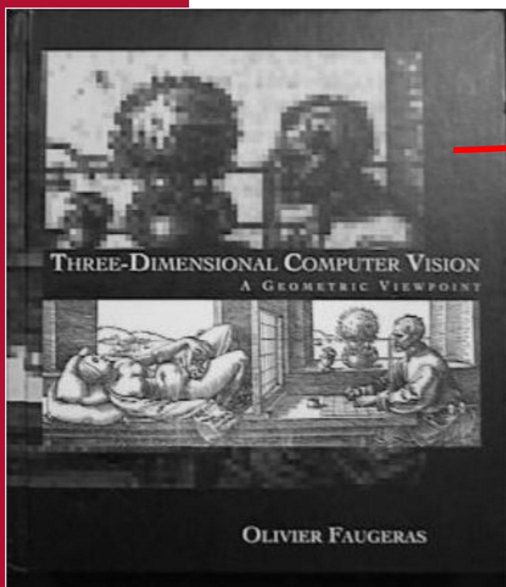
Harris corner detector works well in practice but is not invariant to scale

- The convolution window size affects the scale of the corner detected

To solve complex high-level computer vision problems we need more invariances and a way to distinguish and identify features

A typical problem

Find this:



In this picture:

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SIFT

David G. Lowe, *Distinctive image features from scale-invariant keypoints*, International Journal of Computer Vision, 60, 2 (2004), pp. 91-110.

… changed the way we approach many computer vision problems!

Invariances:

- Scaling
- Rotation
- Illumination

While still providing very good localization

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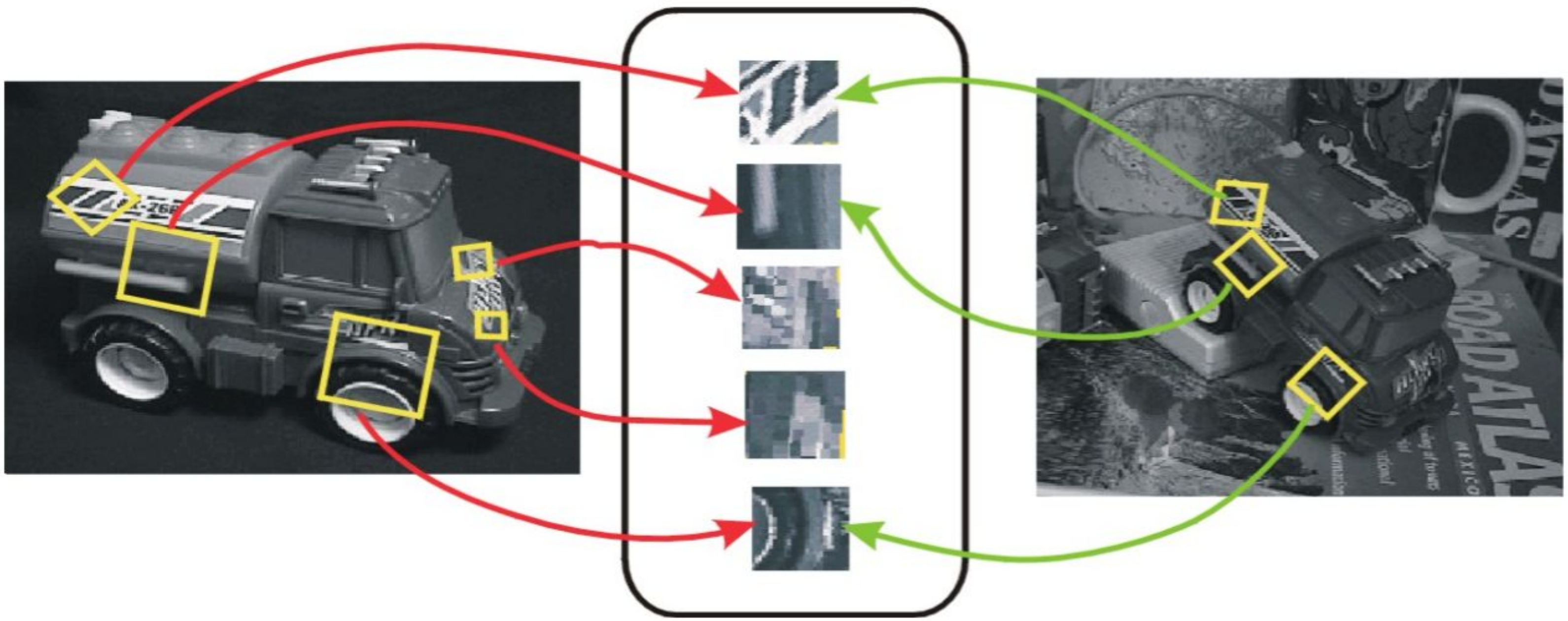
SIFT: General idea

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



SIFT: Advantages

Locality: features are local, so robust to occlusion and clutter (no prior segmentation)

Distinctiveness: individual features can be matched to a large database of objects

Quantity: many features can be generated for even small objects

Efficiency: close to real-time performance

Extensibility: can easily be extended to wide range of differing feature types, with each adding robustness

SIFT Algorithm

Keypoint Localization:

1. **Enforce invariance to scale:** Compute Gaussian difference max, for may different scales; non-maximum suppression, find local maxima: keypoint candidates
2. **Localizable corner:** For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.
3. **Eliminate edges:** Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.

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SIFT Algorithm

Signature computation:

4. **Enforce invariance to orientation:** Compute orientation by finding the strongest gradient direction in the smoothed image (possibly multiple orientations). Rotate patch so that orientation points upward.

5. **Compute feature signature (descriptor):** Compute a "gradient histogram" of the local image region in a 4x4 pixel region. Do this for 4x4 regions of that size. Orient so that largest gradient points up (possibly multiple solutions). Result: feature vector with 128 values (15 fields, 8 gradients).

SIFT Algorithm

6. **Enforce invariance to illumination change and camera saturation:**
Normalize the descriptor to unit length to increase invariance to illumination. Then, threshold all gradients, to become invariant to camera saturation.

SIFT - Step 1

Enforce invariance to scale: Compute Gaussian difference max, for may different scales;non-maximum suppression, find local maxima: keypoint candidates

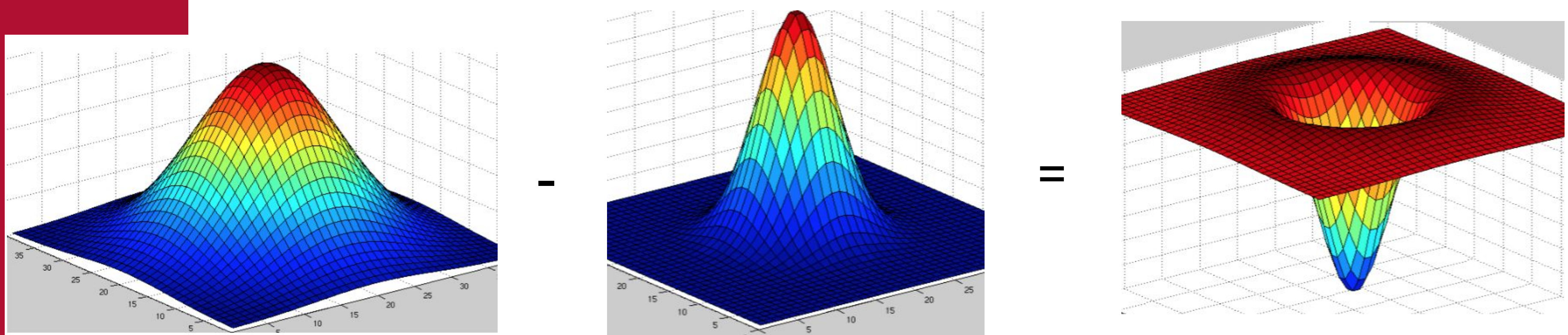
Main idea: Find corners as in Harris, but achieve scale invariance

Method:

- Convolve with Difference of Gaussians (DoG) to identify interesting image pixels
- DoG is performed at multiple resolutions and the local maximum (in space and scale is selected)

Difference of Gaussians

Essentially an High-pass filter which approximates well the LoG.



Why use that? We can efficiently compute the DoG at different scales using image pyramid

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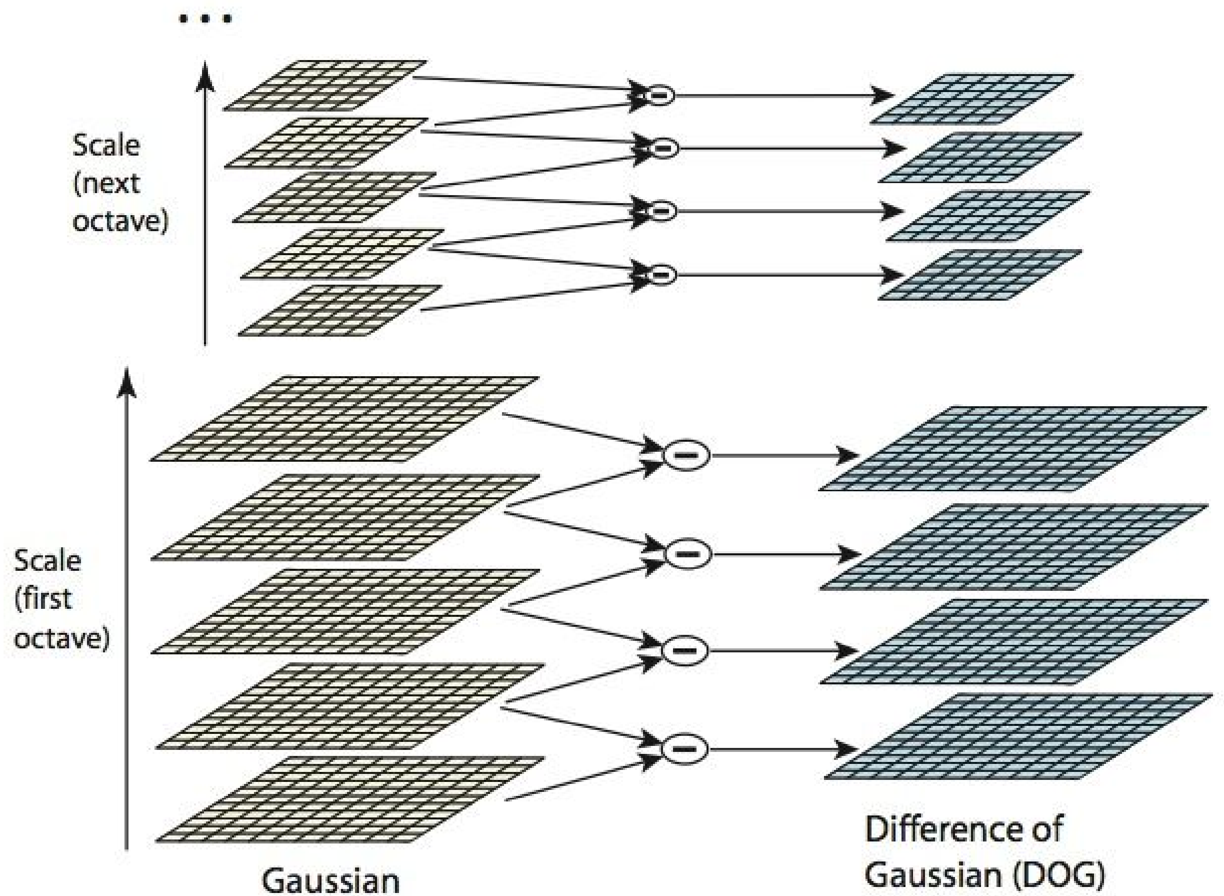
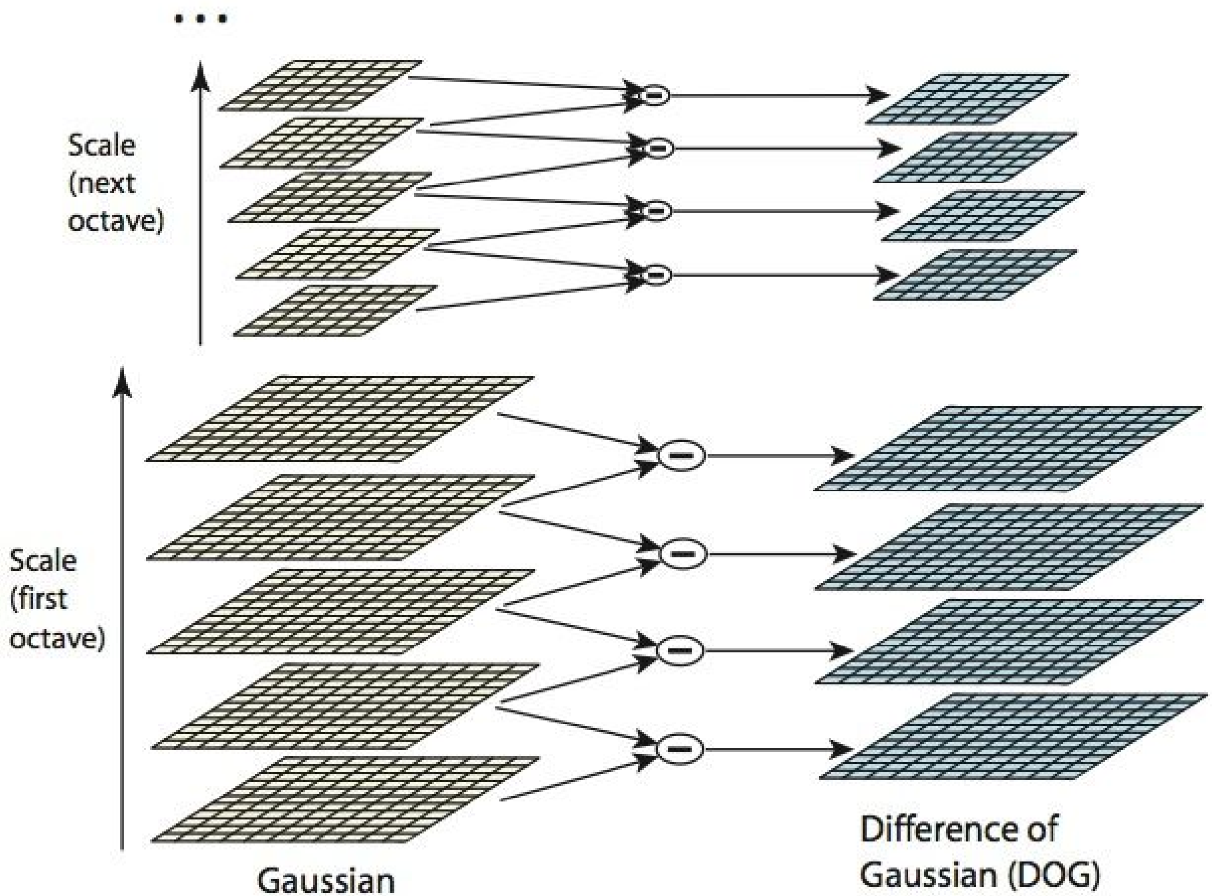
DoG & Image Pyramid

Image is scaled several times (size is halved), each scale is called an octave



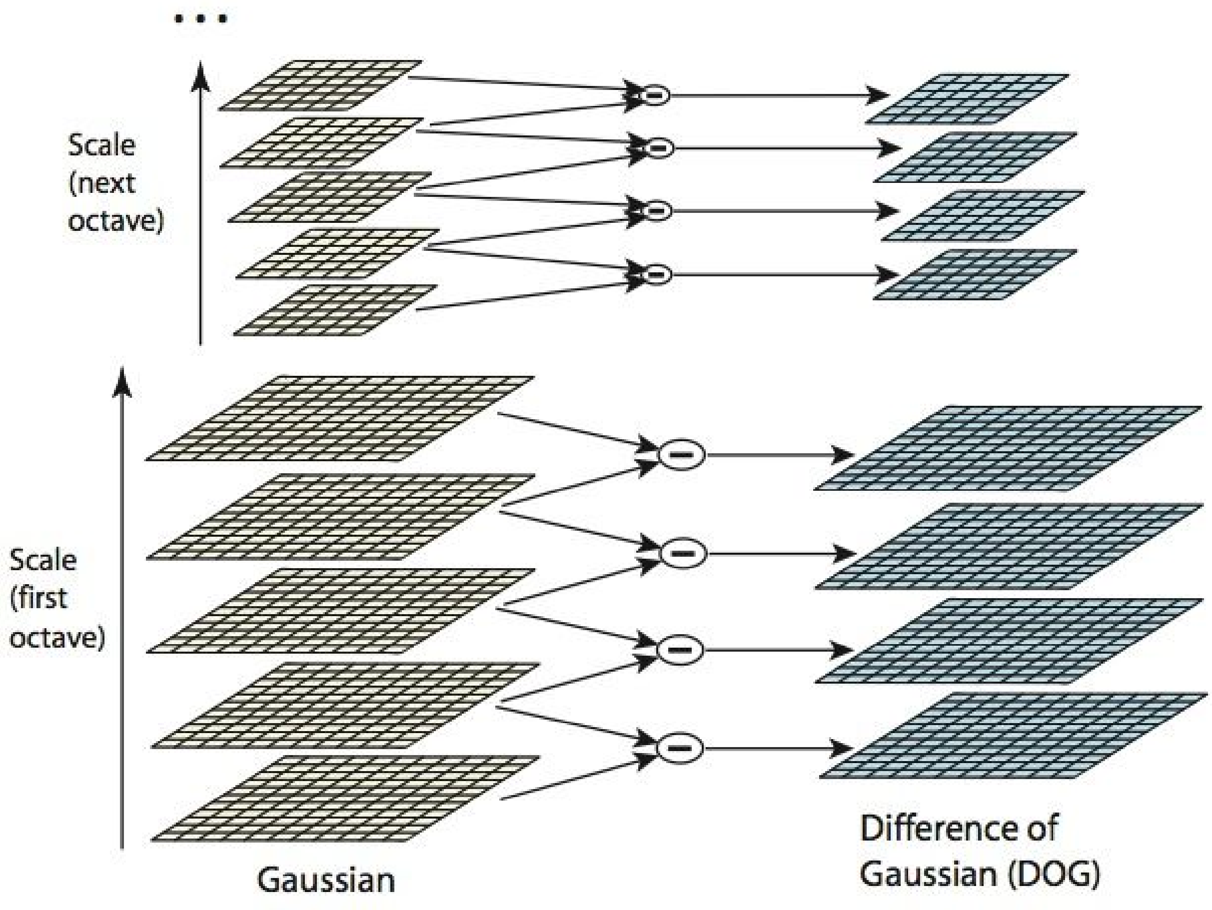
DoG & Image Pyramid

For each octave of scale space, the initial image is repeatedly convolved with Gaussians



DoG & Image Pyramid

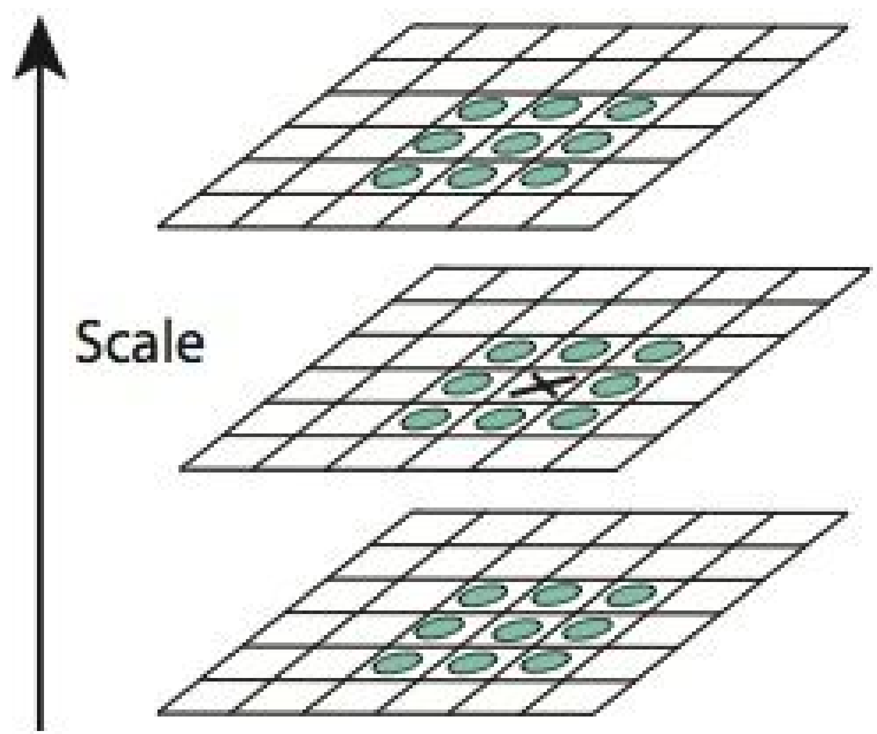
Adjacent Gaussian images are subtracted to produce the difference-of-Gaussian images



Keypoint localization

Once the DoG pyramids are built, the local-maxima are extracted considering both current-scale and adjacent scales

Local-maximum is checked against 9+8+9=26 neighbours



SIFT - Step 2 & 3

2. Localizable corner: For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.

3. Eliminate edges: Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.

Threshold on value at DoG peak and on ratio of principal curvatures (similar to Harris approach)

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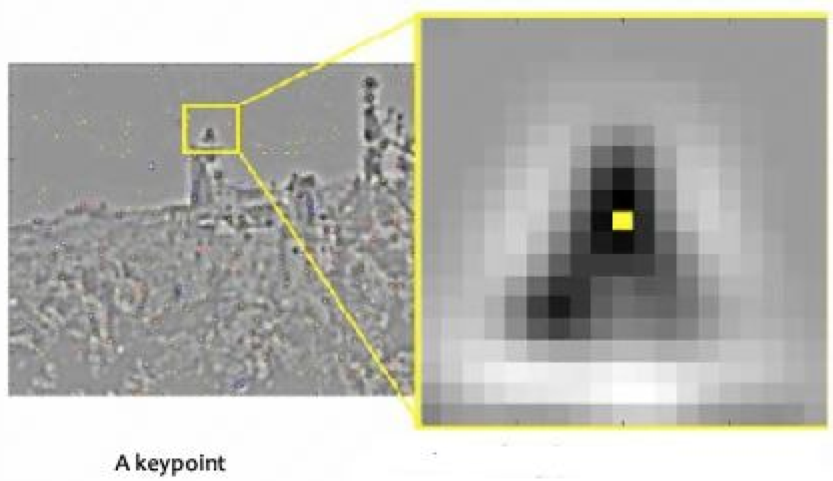
SIFT - Step 4

4. Enforce invariance to orientation

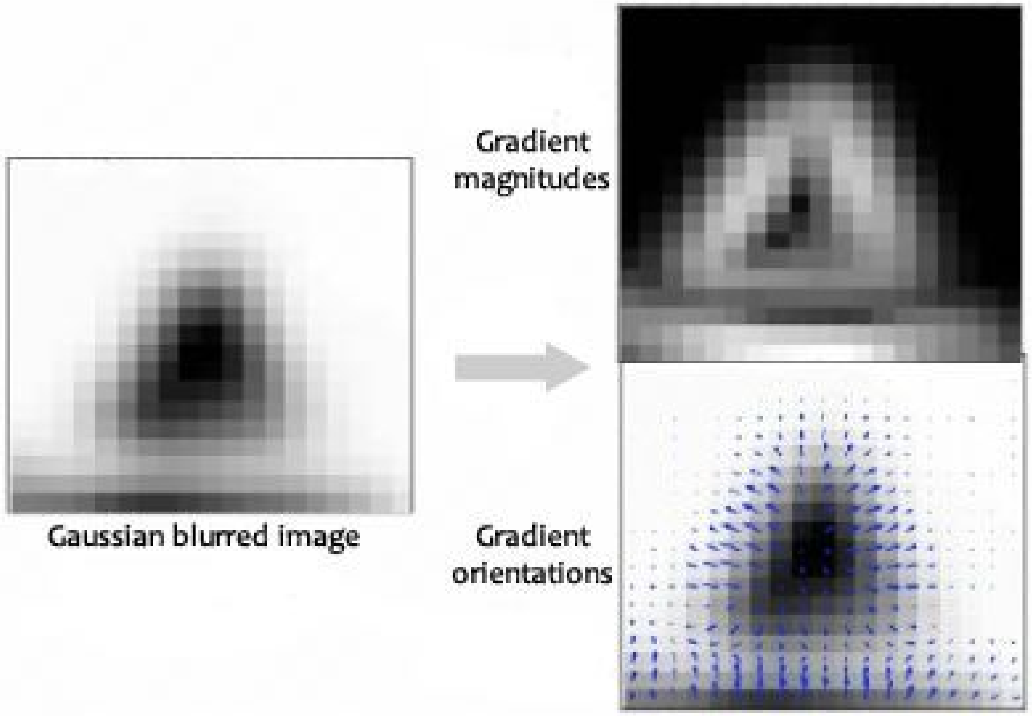
By assigning a consistent orientation to each key point based on local image properties we can achieve invariance to image rotation.

Suppose that we want to assign an orientation to this detected keypoint:



SIFT - Step 4

Gradient magnitude and orientation is calculated for each pixel in the keypoint region (region size depends on the detected scale)



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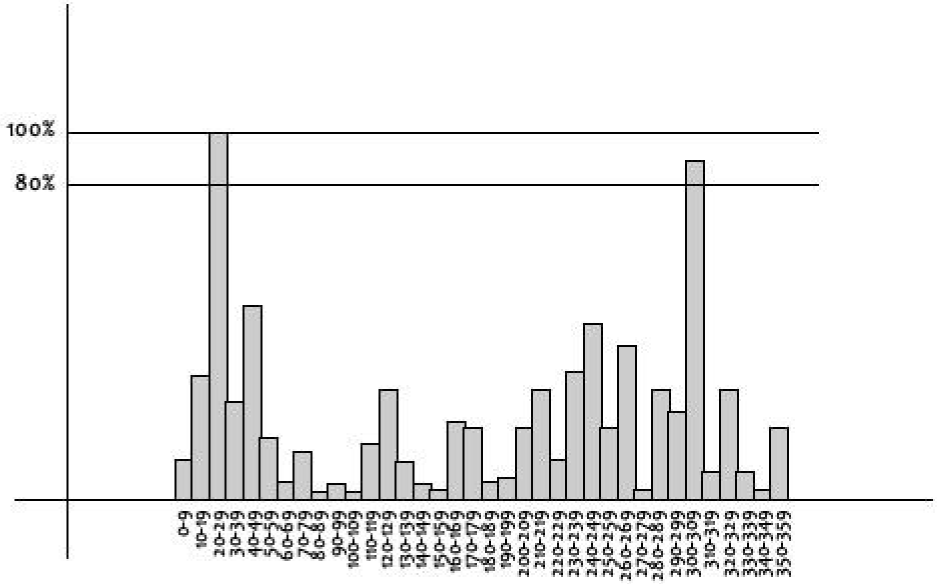
SIFT - Step 4

And an orientation histogram is formed with these orientations and magnitudes



The maximum value of the histogram gives the orientation of the detected keypoint

SIFT - Step 4

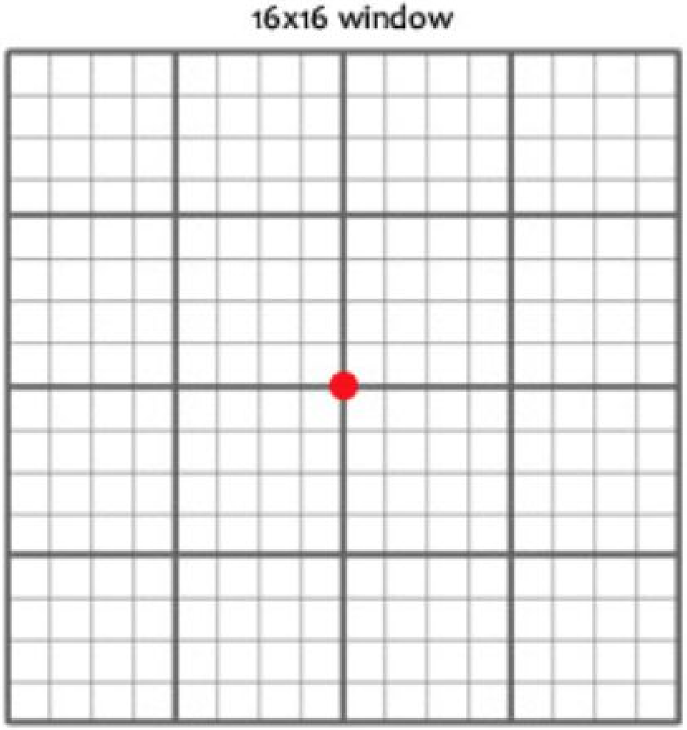
After the step 4, each detected keypoint is characterized by:

- A coordinate in the image space (x,y)
- A scale
- An orientation

Steps 5 and 6 aims to create a signature (or descriptor) that can be used to uniquely identify the features with respect to the others

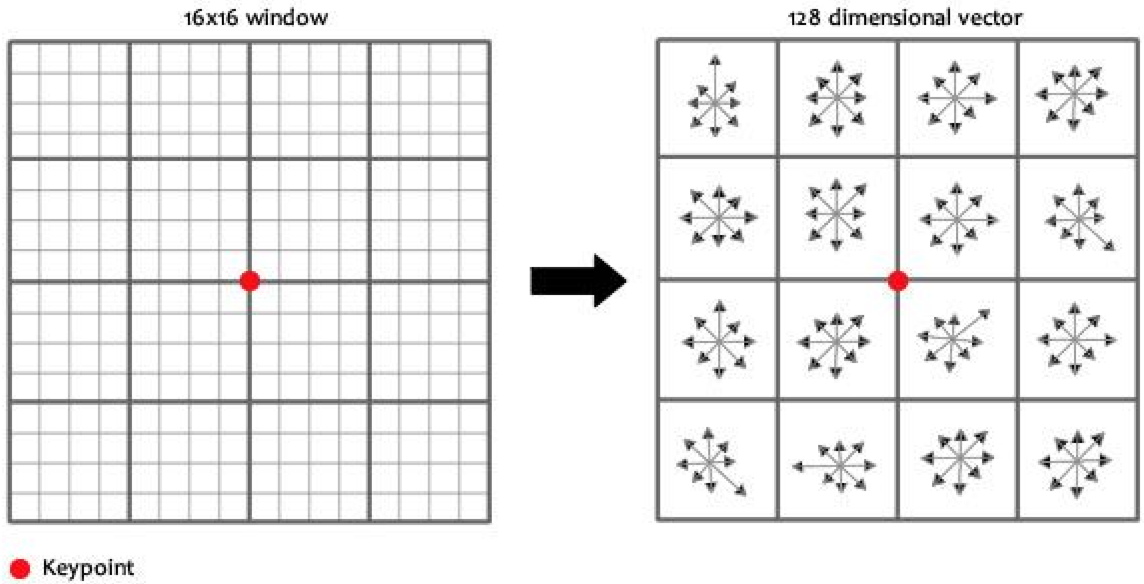
SIFT - Step 5

To create the descriptor we look at the gradient vectors in a 16x16 window around each keypoint (window is rotated with respect to the keypoint orientation)



SIFT - Step 5

- The 16x16 window is divided into 16 4x4 windows.
- For each 4x4 window, an 8-bins (45° steps) histogram of gradient orientation is formed
- Histograms are concatenated all together to produce a 16x8=128-values feature vector



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SIFT - Step 6

To reduce the effect of illumination change, feature vectors (descriptor) are normalized to have unitary length.