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Computer Vision

Morphological image processing

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Academic year 2016/2017

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Preliminaries

Morphology offers a unified and powerful approach to numerous image processing problems

> The language of mathematical morphology is **set theory**

We will consider thresholded images containing only white and black pixels.

The "set of white pixels" contains the complete morphological description of the image.

Such sets are subsets of Z^2 where each vector express coordinates of white (or black) pixels

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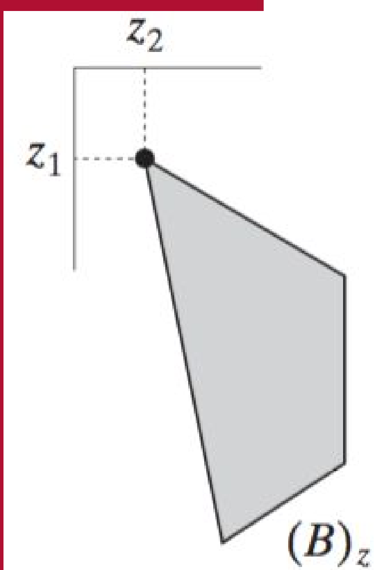
Set translation

The translation of a set B by point z = (z1, z2), denoted $(B)_z$, is defined as:

$$(B)_z = \{c \mid c = b + z \ \text{ for } \ b \in B\}$$



if B is the set of pixels representing an object in an image, then $(B)_z$ is the set of points in B whose (x,y) coordinates have been replaced by

(x + z1, y + z2)

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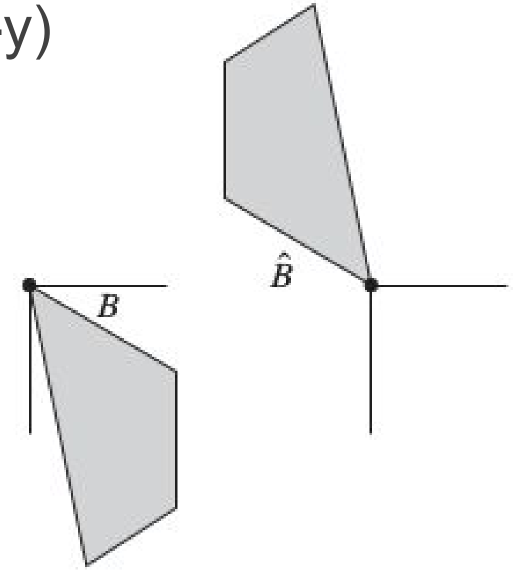
Set reflection

The reflection of a set B, denoted $\hat{B}$ is defined as:

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

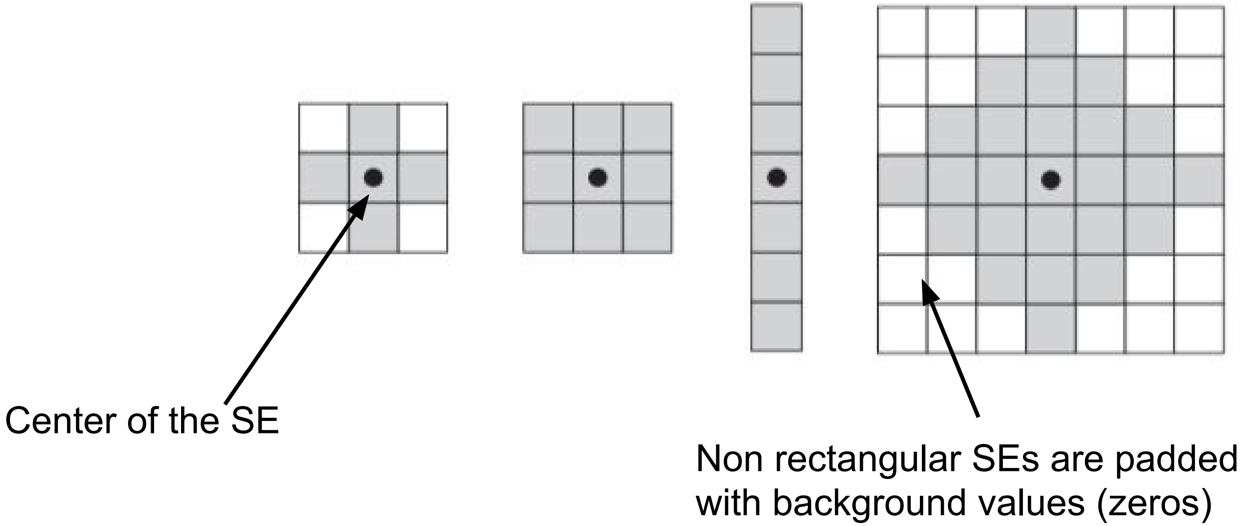
The reflection of B is composed by the points whose coordinates are replaced with (-x,-y)



Structuring elements

Morphological operations are based on so-called structuring elements (SEs):

SEs: small sets or subimages used to probe an image under study for properties of interest



Erosion

With A and B as sets in Z^2 , the erosion of A by B, denoted $A \ominus B$, is defined as:

$$A \ominus B = \{z \mid (B)_z \cap \bar{A} = \emptyset\}$$

In words, the erosion of A by B is the set of all points z such that B, translated by z, is contained in A (the intersection between $(B)_z$ and the complement of A is empty)

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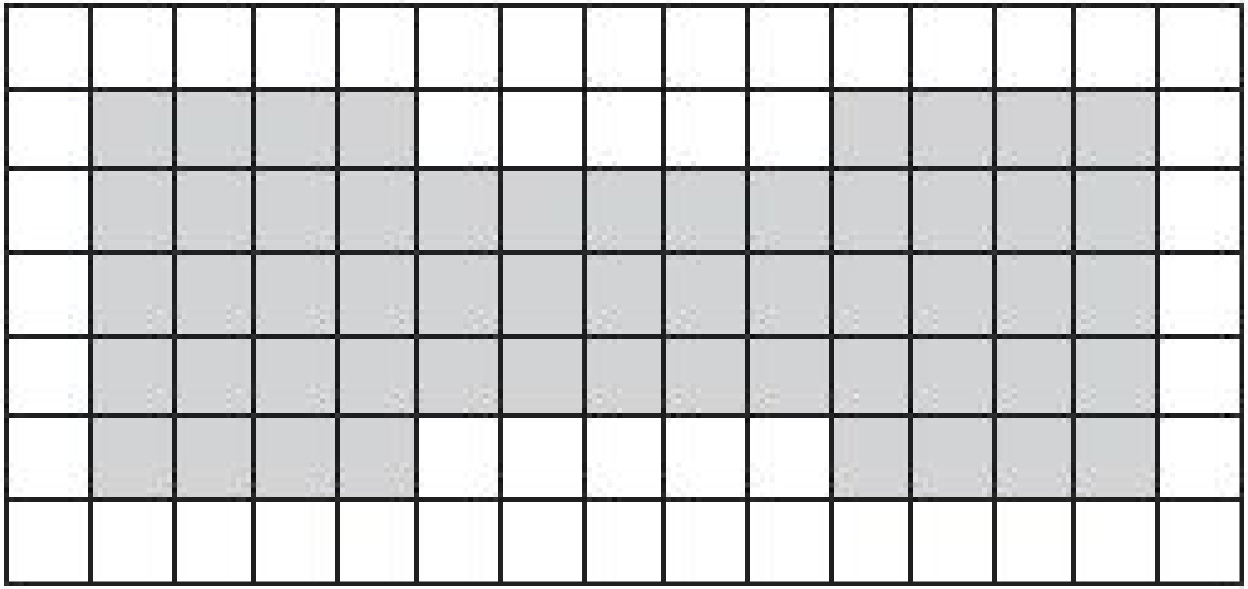
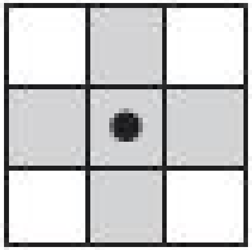
Erosion example

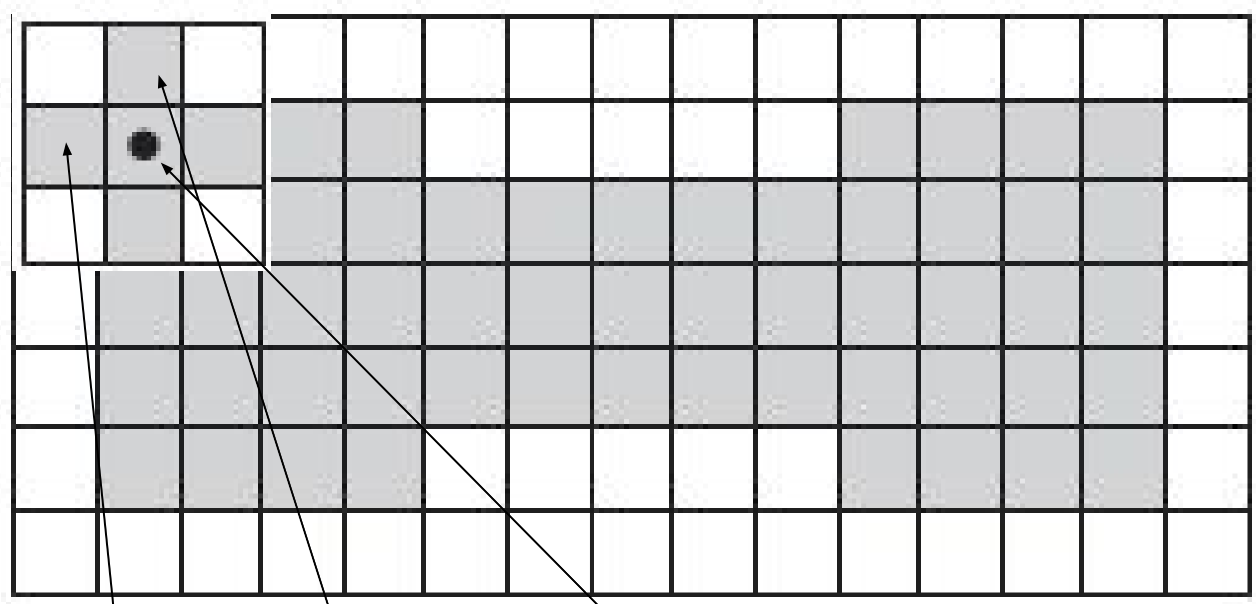
A



B



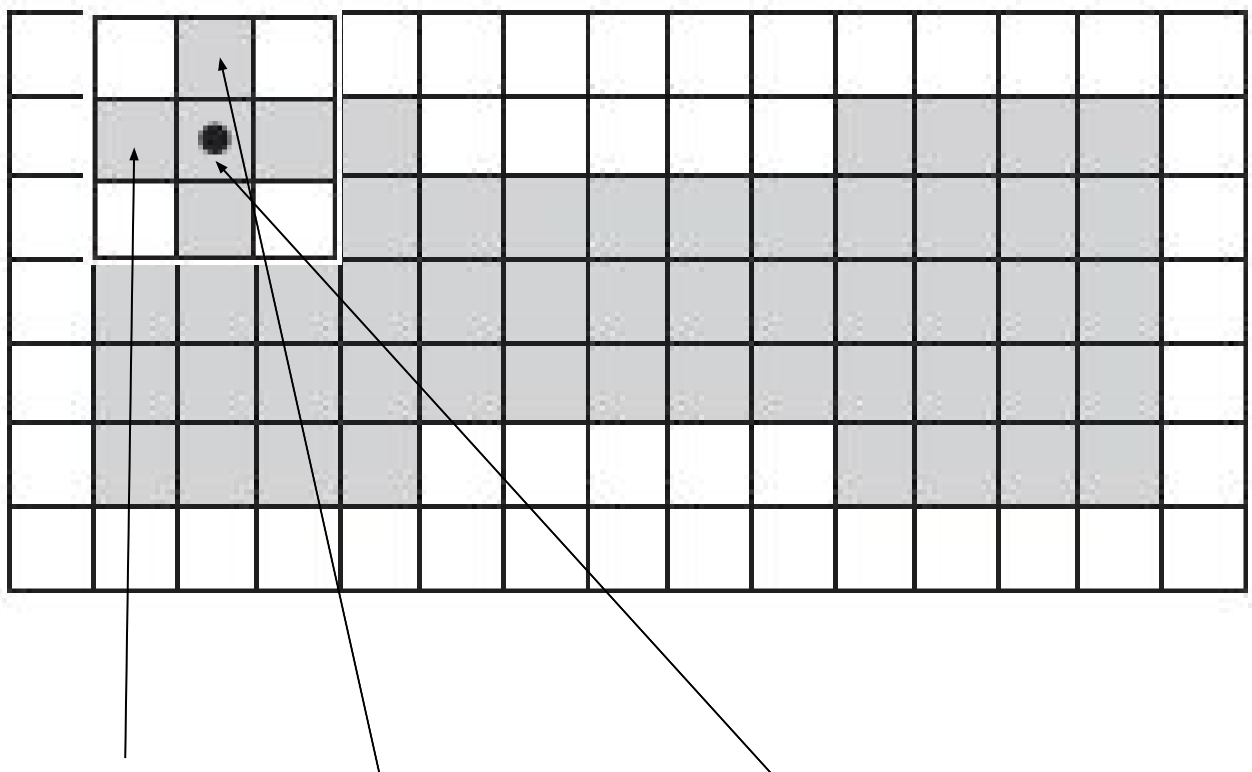
Erosion example



$(B)_z \cap \bar{A} \neq \emptyset$

This point will not be present in the output

Erosion example



$(B)_z \cap \bar{A} \neq \emptyset$

This point will not be present in the output

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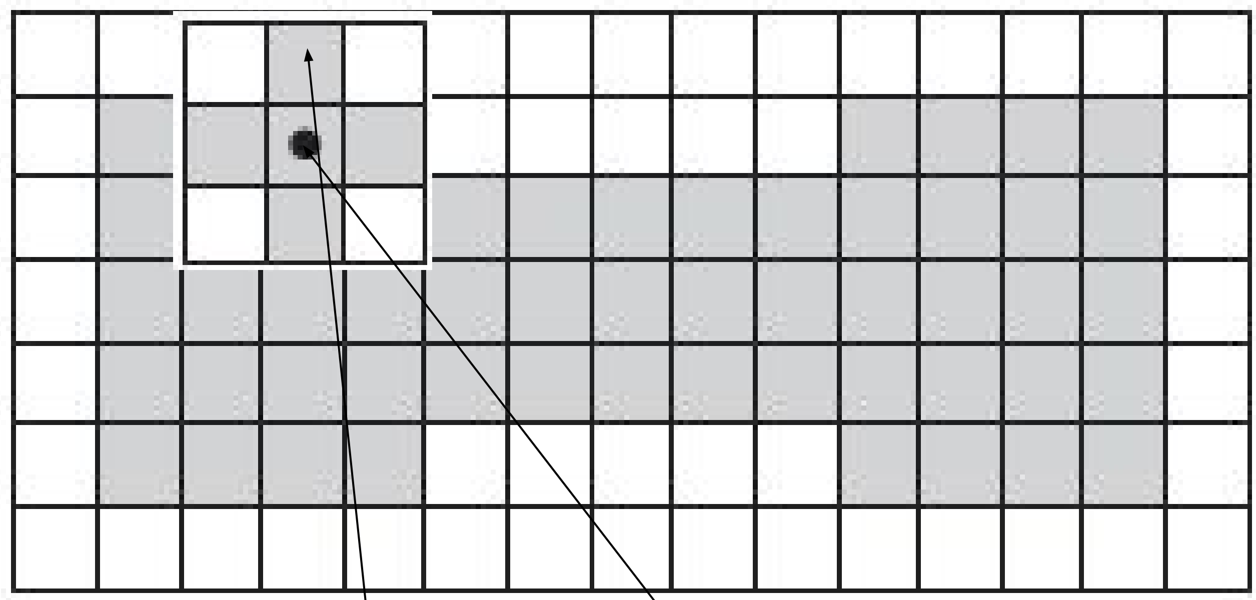
Erosion example



$$(B)_z \cap \bar{A} \neq \emptyset$$

This point will not be present in the output

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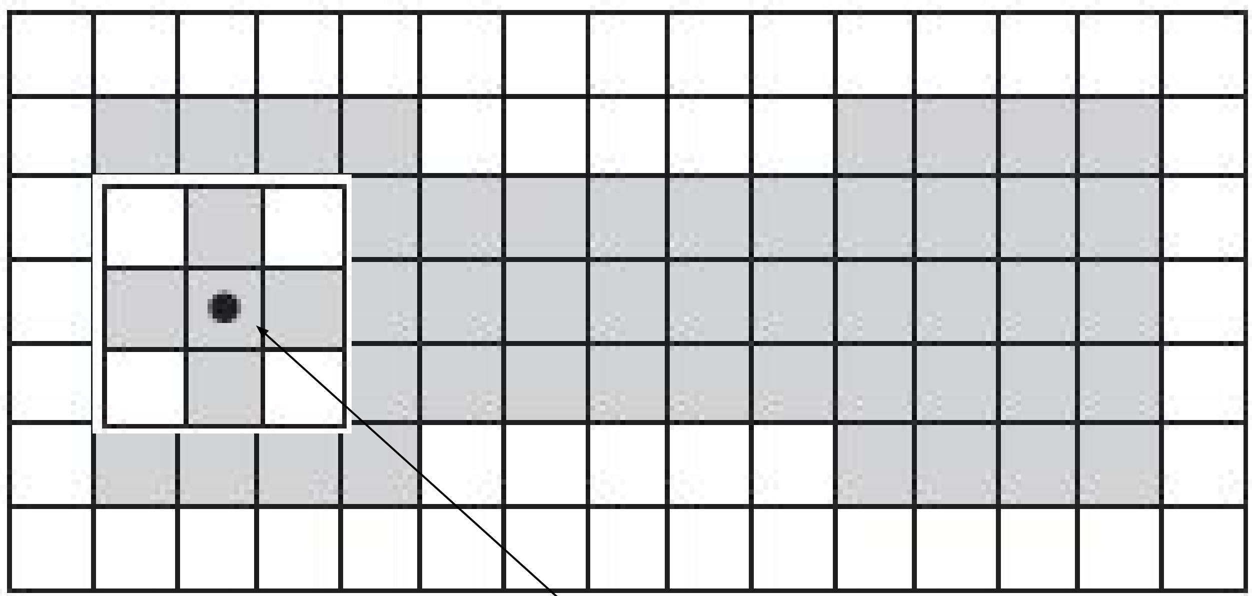
Erosion example

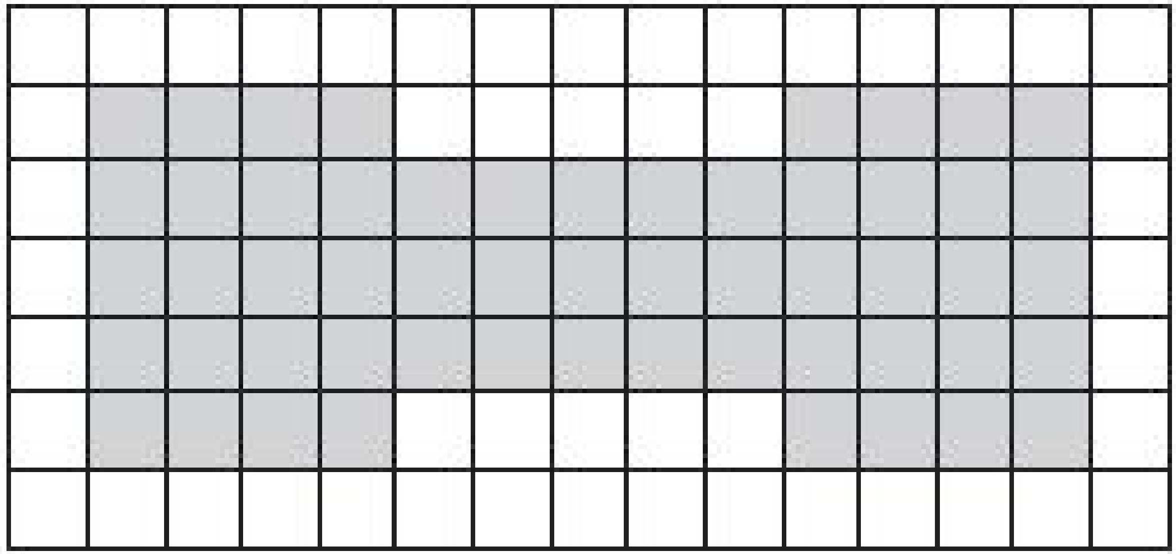


$$(B)_z \cap \bar{A} = \emptyset$$

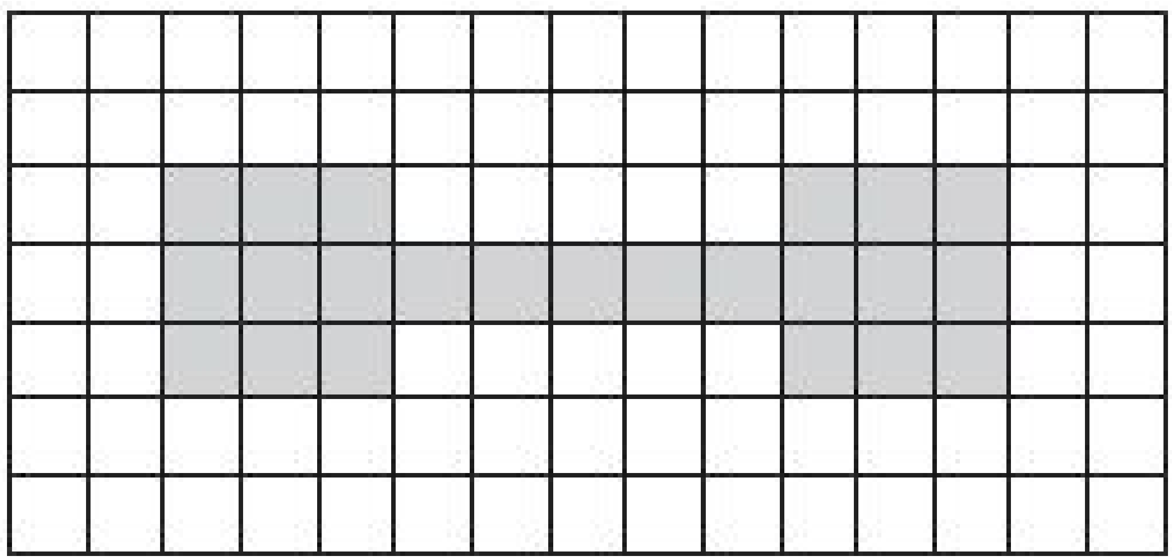
This point will be present in the output

Erosion example

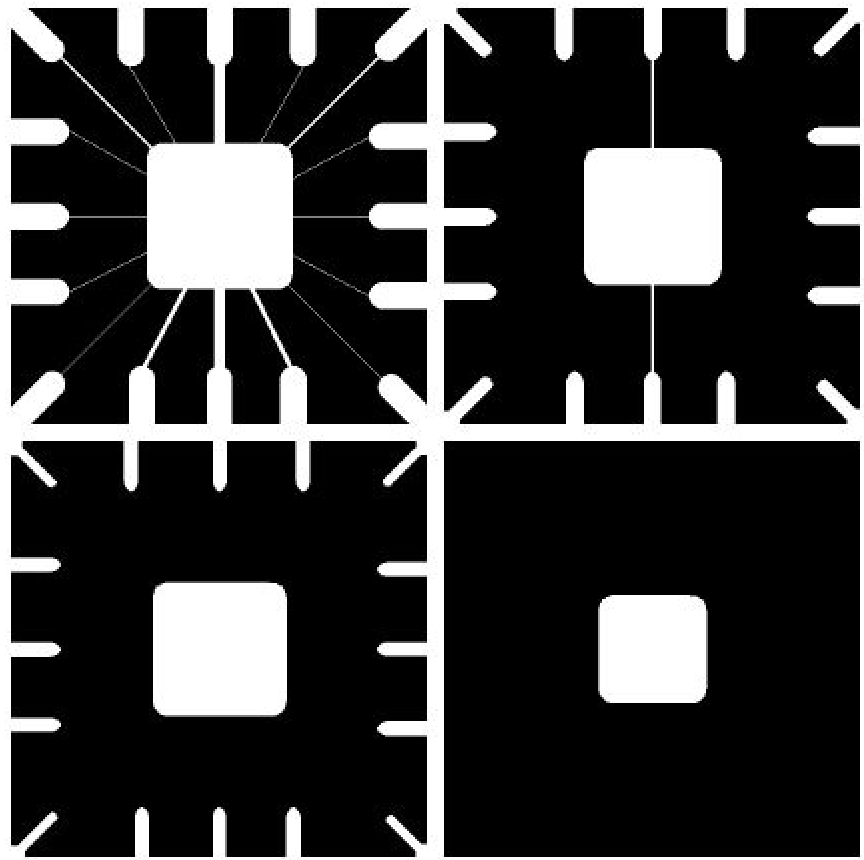
A



$A \ominus B$



Erosion example



Original image

11x11 erosion mask

15x15 erosion mask

45x45 erosion mask

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Dilation

With A and B as sets in Z^2 , the dilation of A by B, denoted A $\ominus$ B, is defined as:

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

The dilation of A by B is the set of all displacements such that the reflection of B and A overlap by at least one element.

The basic process of flipping B about its origin and then successively displacing it so that it slides over set (image) A is analogous to spatial convolution

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Dilation

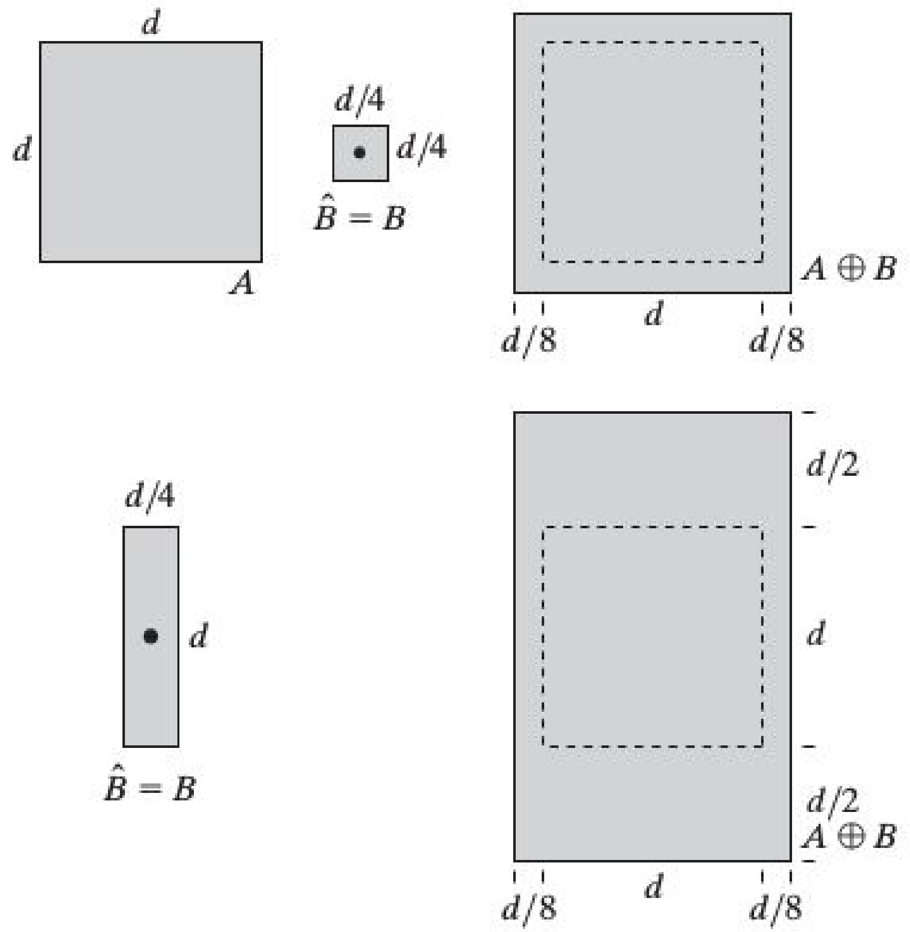

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

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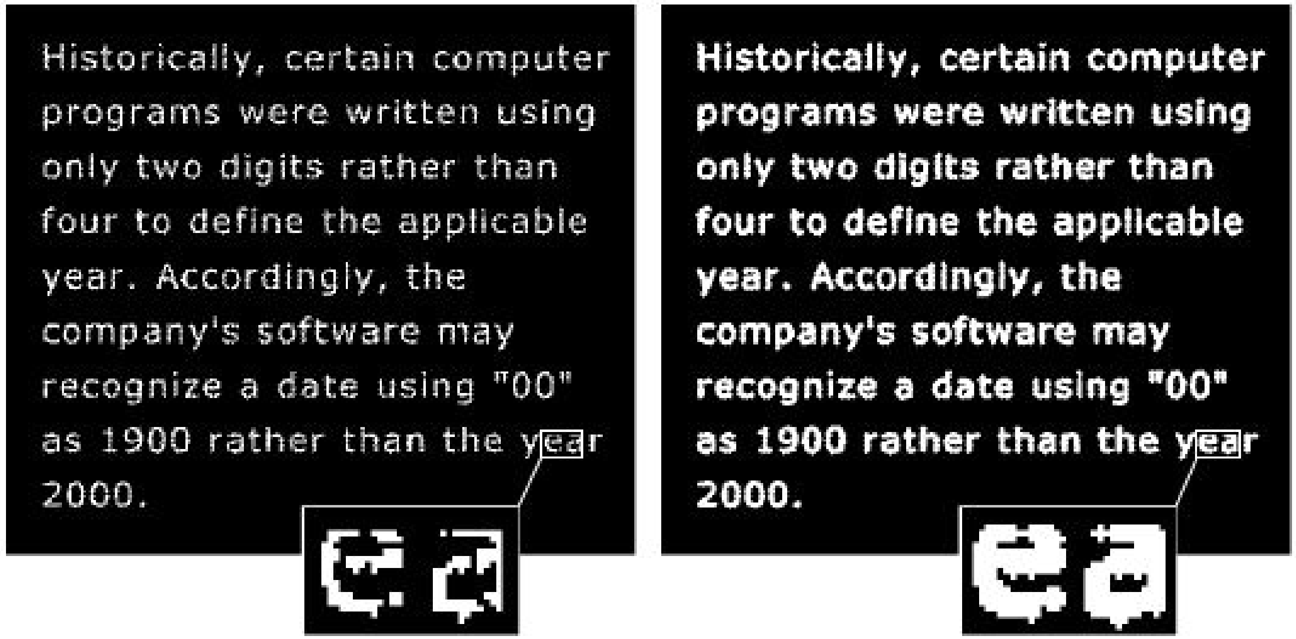
Dilation



0	1	0
1	1	1
0	1	0

One of the simplest applications of dilation is for bridging gaps

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Opening and Closing

Given an image A and a SE B:

Opening:

$$A \circ B = (A \ominus B) \oplus B$$

Erosion followed by a dilation

Closing:

$$A \bullet B = (A \oplus B) \ominus B$$

Dilation followed by an erosion

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Opening and Closing

Opening: $$A \circ B = (A \ominus B) \oplus B$$

- smoothes the contour of an object
- breaks narrow isthmuses
- eliminates thin protrusions

Closing: $$A \bullet B = (A \oplus B) \ominus B$$

- smooth sections of contours
- fuses narrow breaks and long thin gulfs
- eliminates small holes
- Fills gaps in the contour

Opening and Closing

Original noisy image

Erosion

Opening

→ Noise completely removed but some holes are created

→ Dilation partially closes the inner holes

Opening and Closing

Original image

Dilation

Closing

Most of the breaks are restored but ridges are thickened

Erosion reduces the thickness of the ridges

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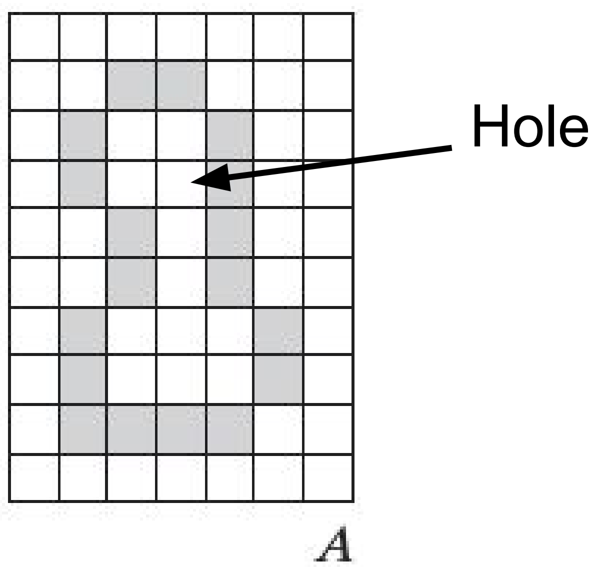
Opening and Closing

Original image

$$(A \circ B) \bullet B$$

Hole filling

A hole may be defined as a background region surrounded by a connected border of foreground pixels.



Given an initial point inside an hole, the goal is to fill the hole with 1s

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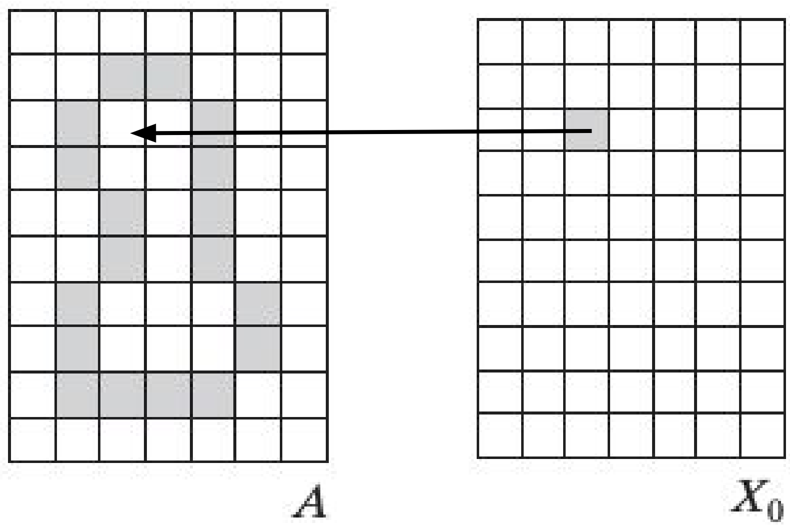
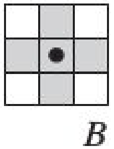
Hole filling

Let X_0 be the initial array with the same size of A, filled with 0s except at the location to a given point in each hole.



The following iterative procedure fills all holes with 1s:

$$X_k = (X_{k-1} \oplus B) \cap \bar{A}$$



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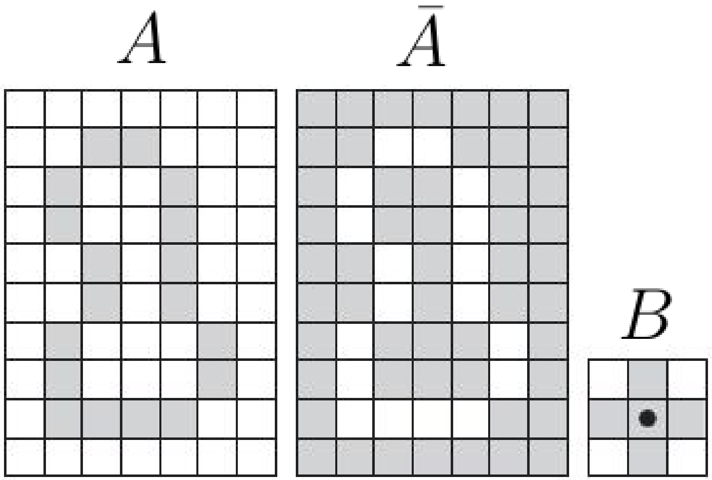
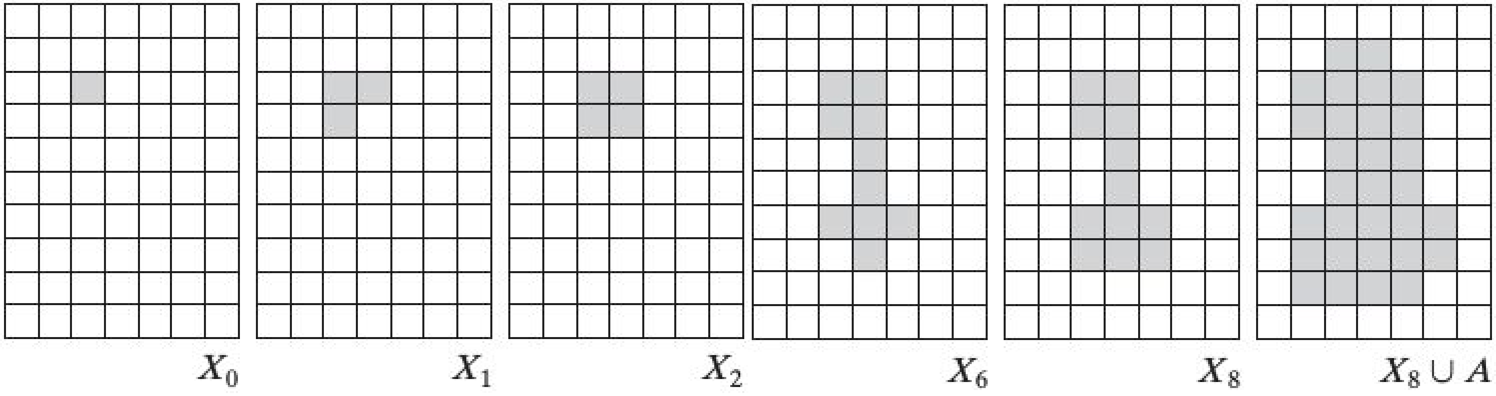
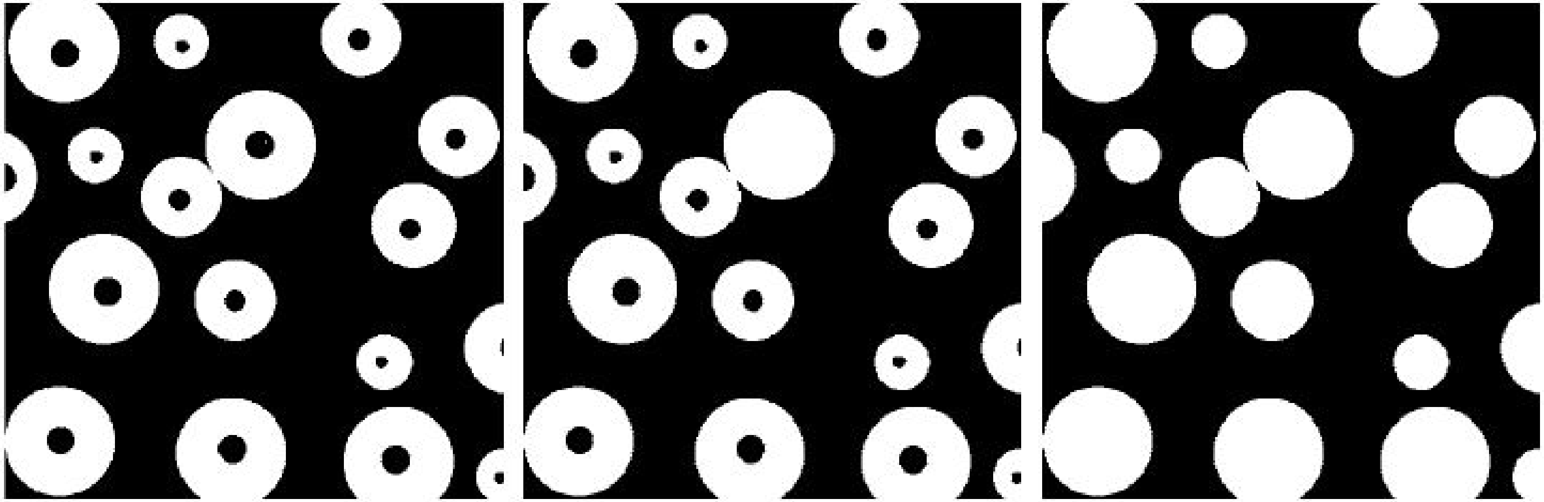
Hole filling



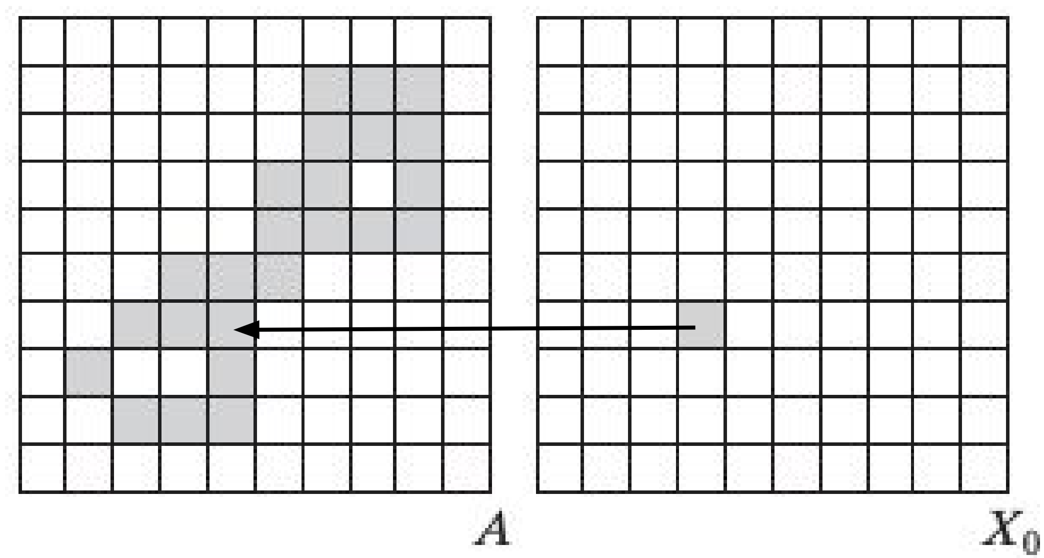


Hole Filling



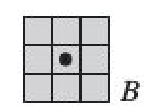
Connected components

Let X_0 be the initial array with the same size of A, filled with 0s except at the location to a known point in each region.

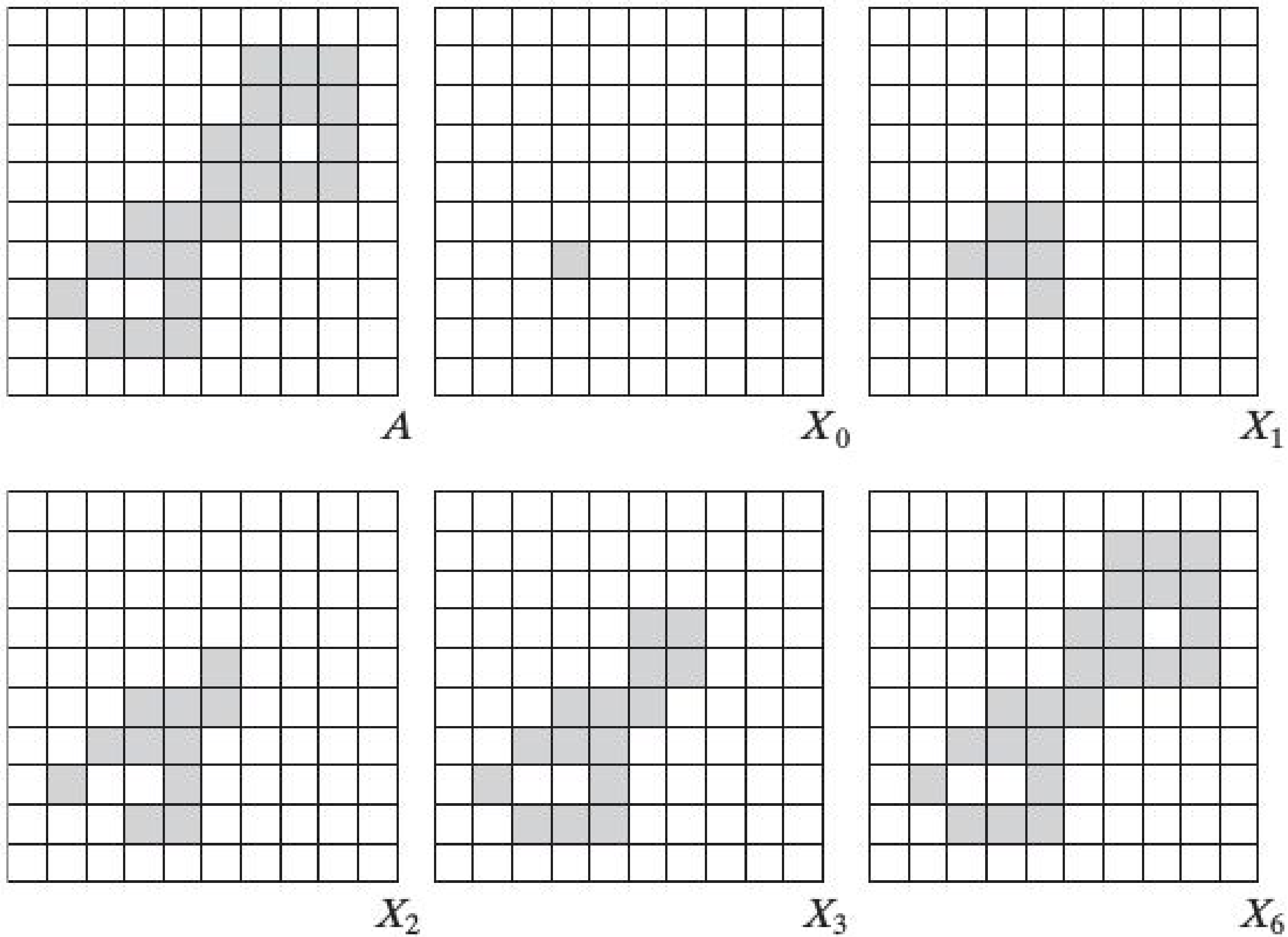


The following iterative procedure fills the connected component with 1s:

$$X_k = (X_{k-1} \oplus B) \cap A$$



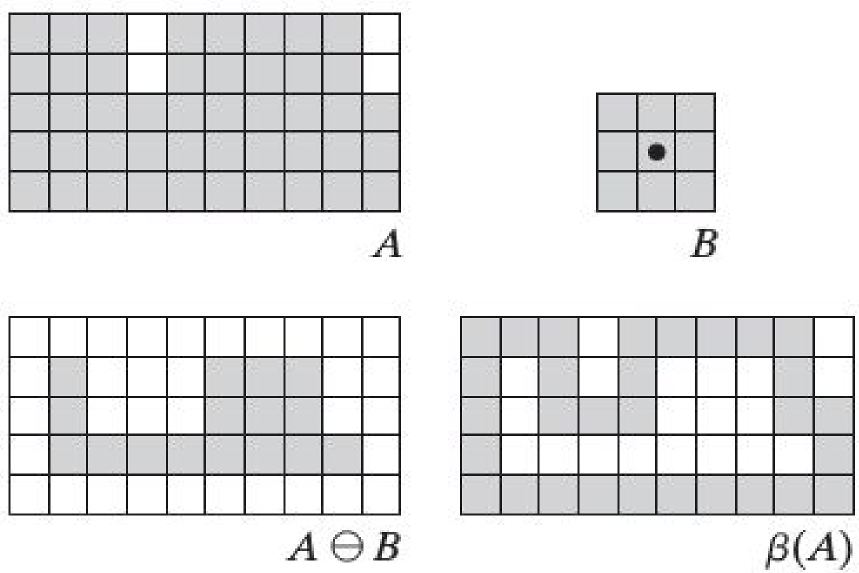
Connected components



Boundary extraction

The boundary of a set A can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

$$\beta(A) = A - (A \ominus B)$$



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Moore boundary tracking

Several algorithms require to extract an ordered sequence of foreground boundary points from a region

Assumptions:

1. We are working with binary thresholded images: 0:background 1:foreground
2. Images are padded with a border of 0s so that no foreground region touches the image border

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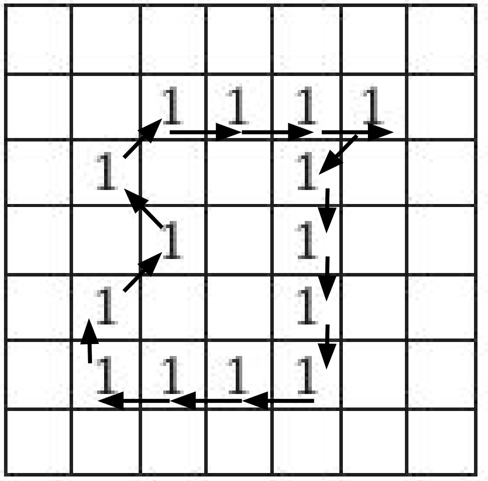
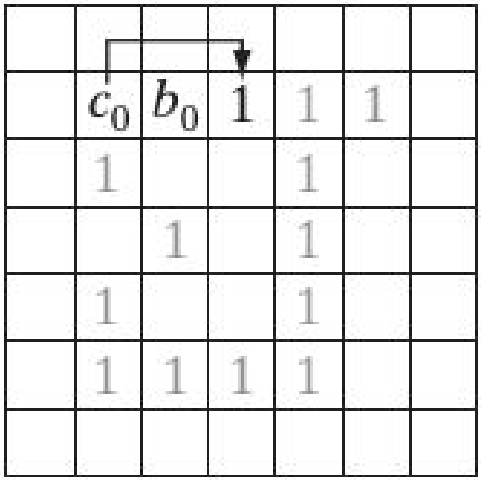
Moore boundary tracking

		1	1	1	1	
	1			1		
		1		1		
	1			1		
	1	1	1	1		



Moore boundary tracking



1. Let the starting point, b0 be the uppermost, leftmost point in the image that is labeled 1
   a. Let c0 be the west neighbor of b0

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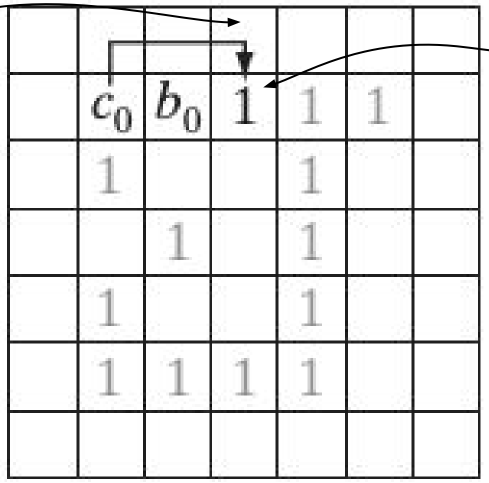
Moore boundary tracking



1. (initialization) Examine the 8-neighbors of b0, starting at c0 and proceeding in a clockwise direction.
   a. Let b1 denote the first neighbor encountered whose value is 1
   b. let c1 be the (background) point immediately preceding b1 in the sequence.

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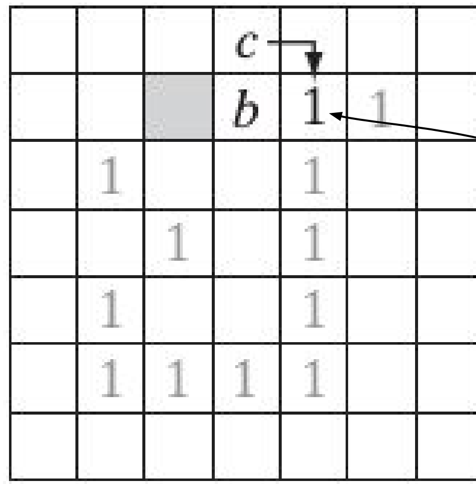
Moore boundary tracking



2. Let b=b1 c=c1

3. Let the 8-neighbors of b, starting at c and proceeding in a clockwise direction, be denoted by n_1, n_2 , ... , n_8 . Find the first n_k labeled 1.

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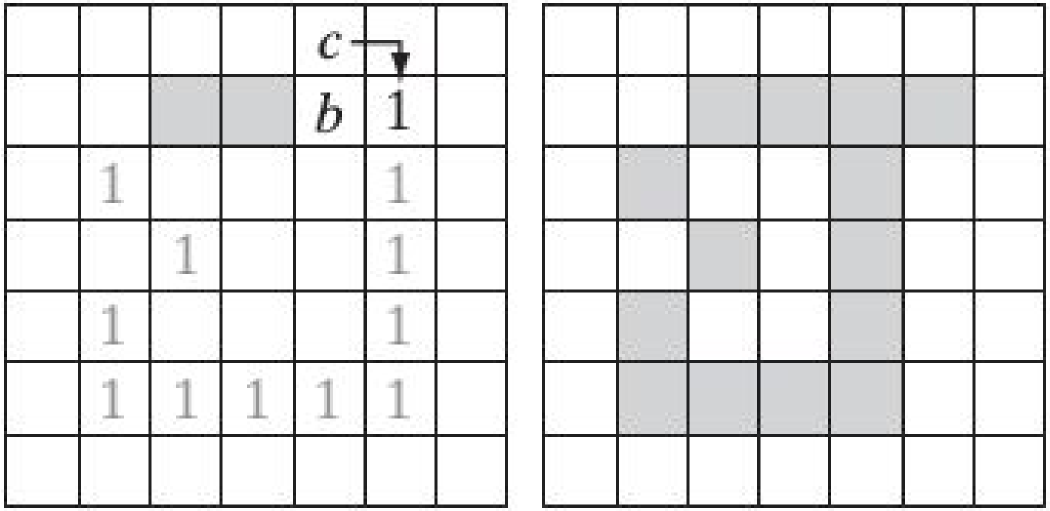
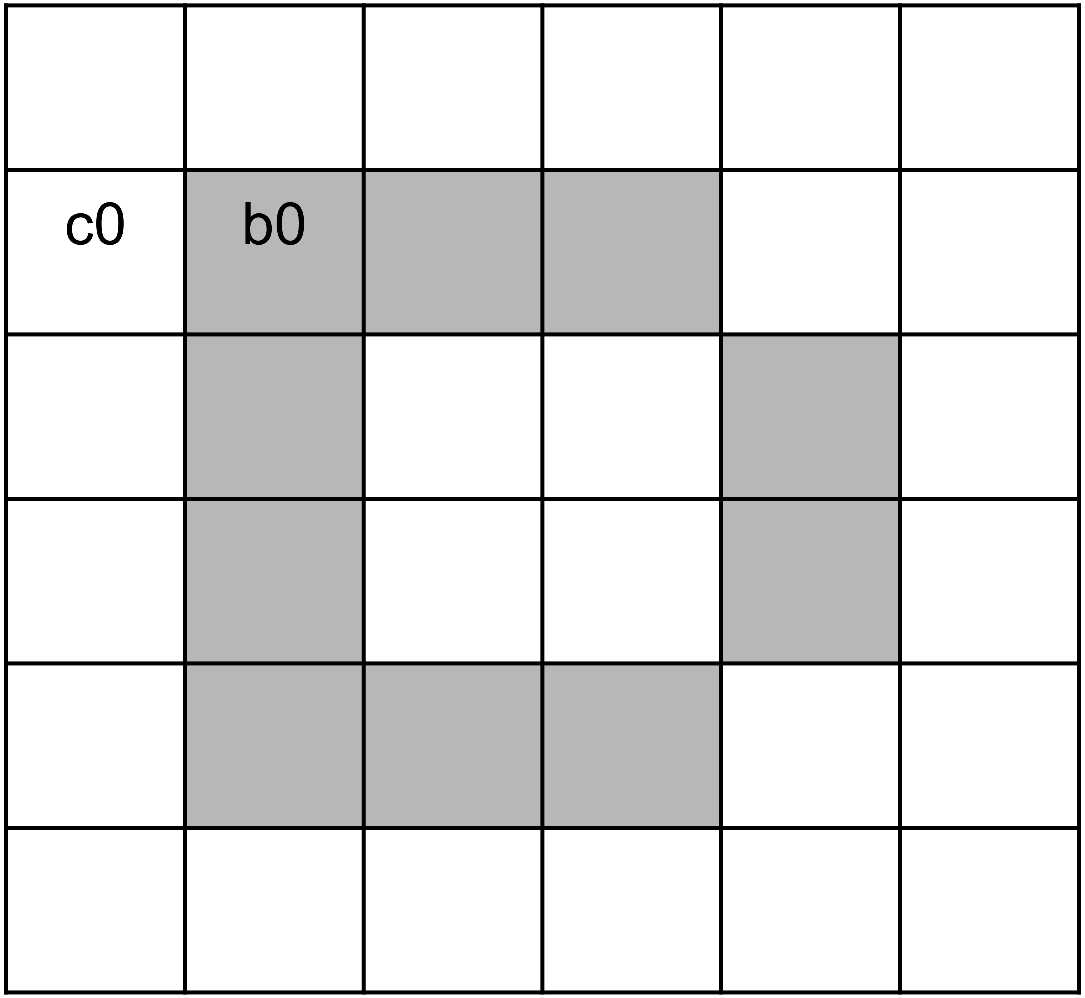
Moore boundary tracking



4. Let $b=n_k$, $c=n_{k-1}$

Repeat Steps 3 and 4 until $b = b_0$ and the next boundary point found is b_1.The sequence of b points found when the algorithm stops constitutes the set of ordered boundary points.

Moore boundary tracking



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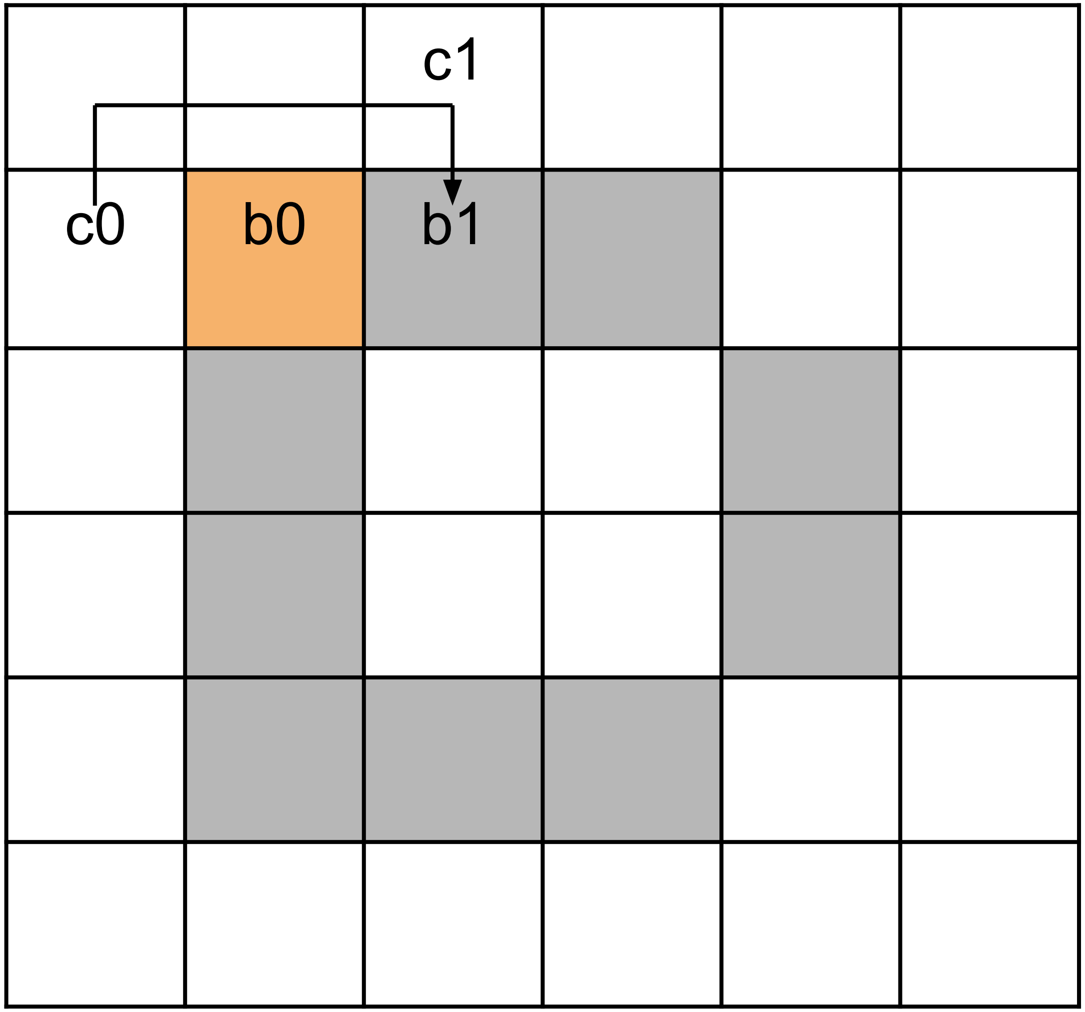
Moore boundary tracking



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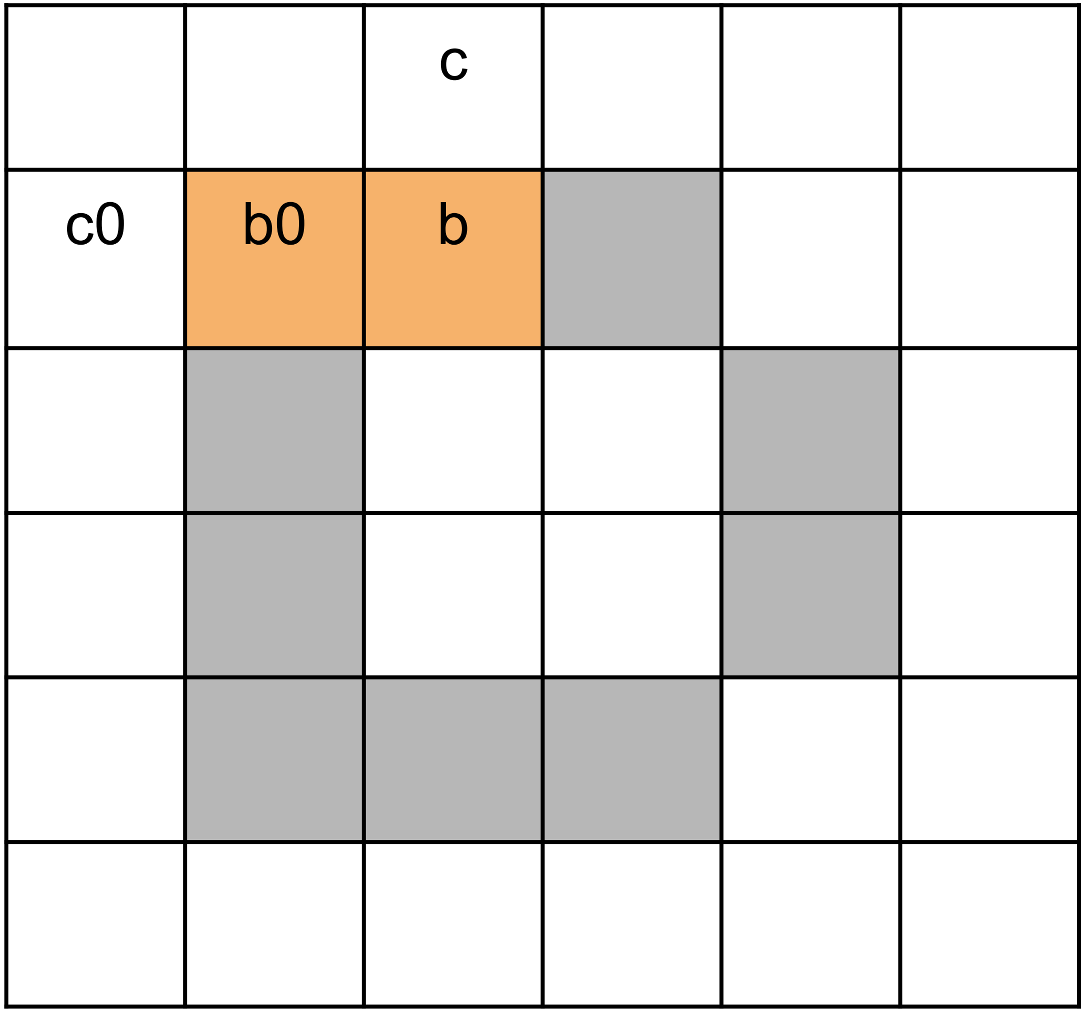
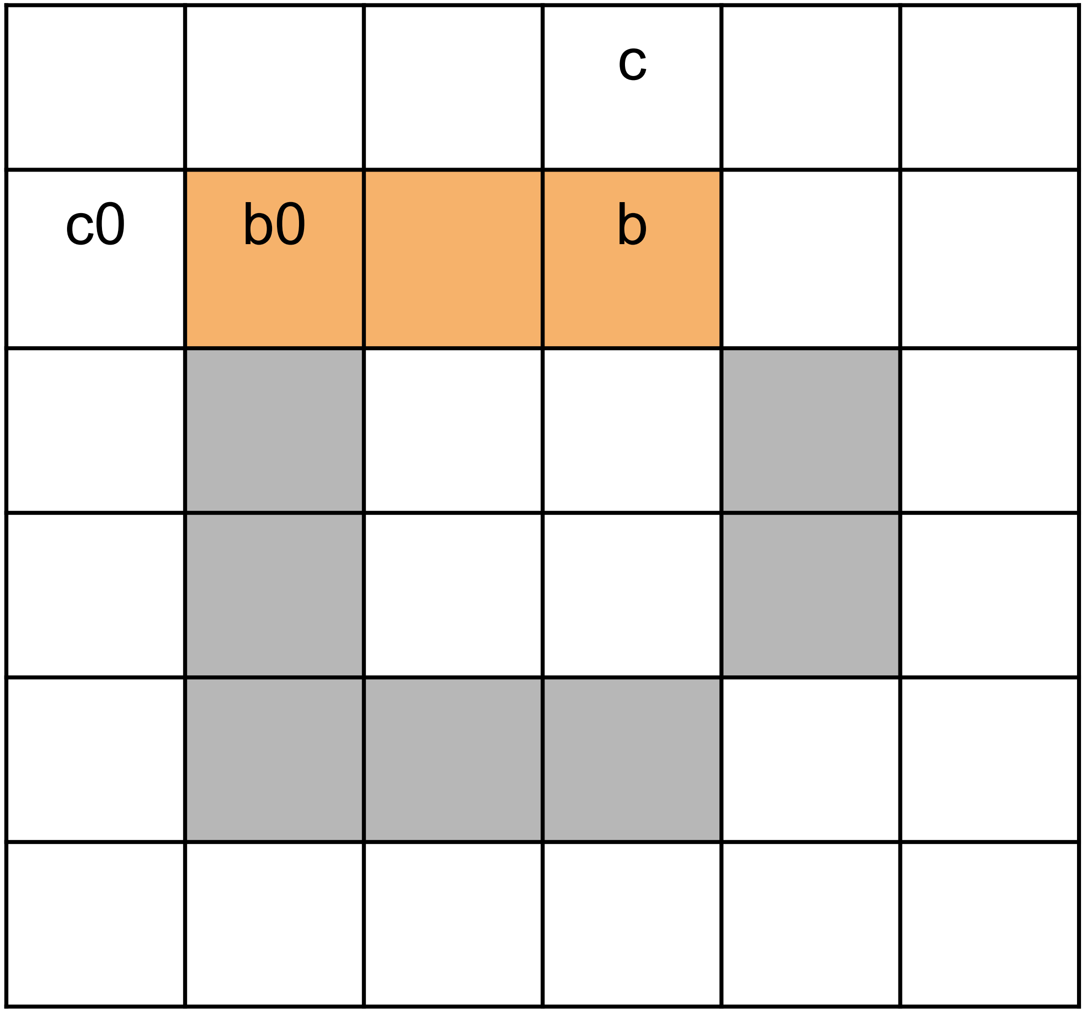
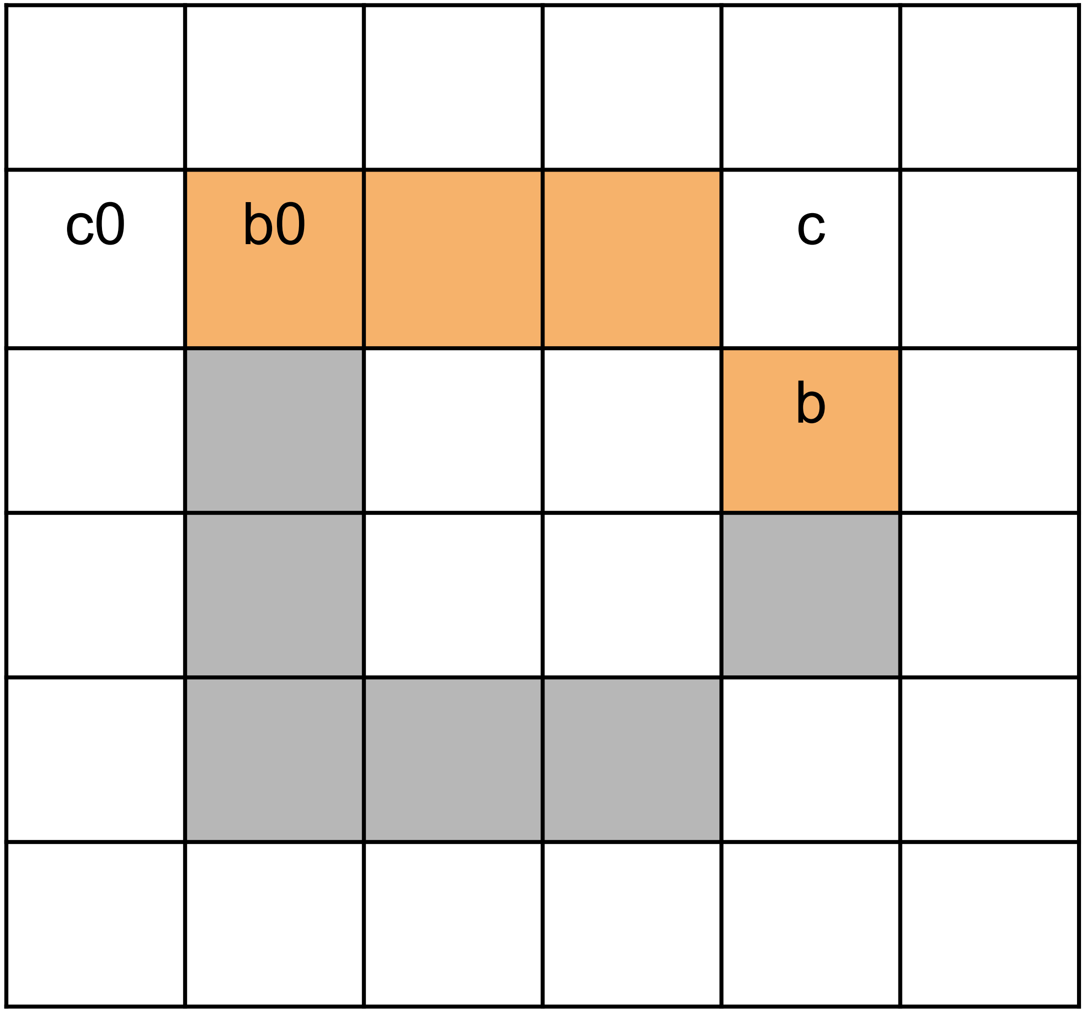


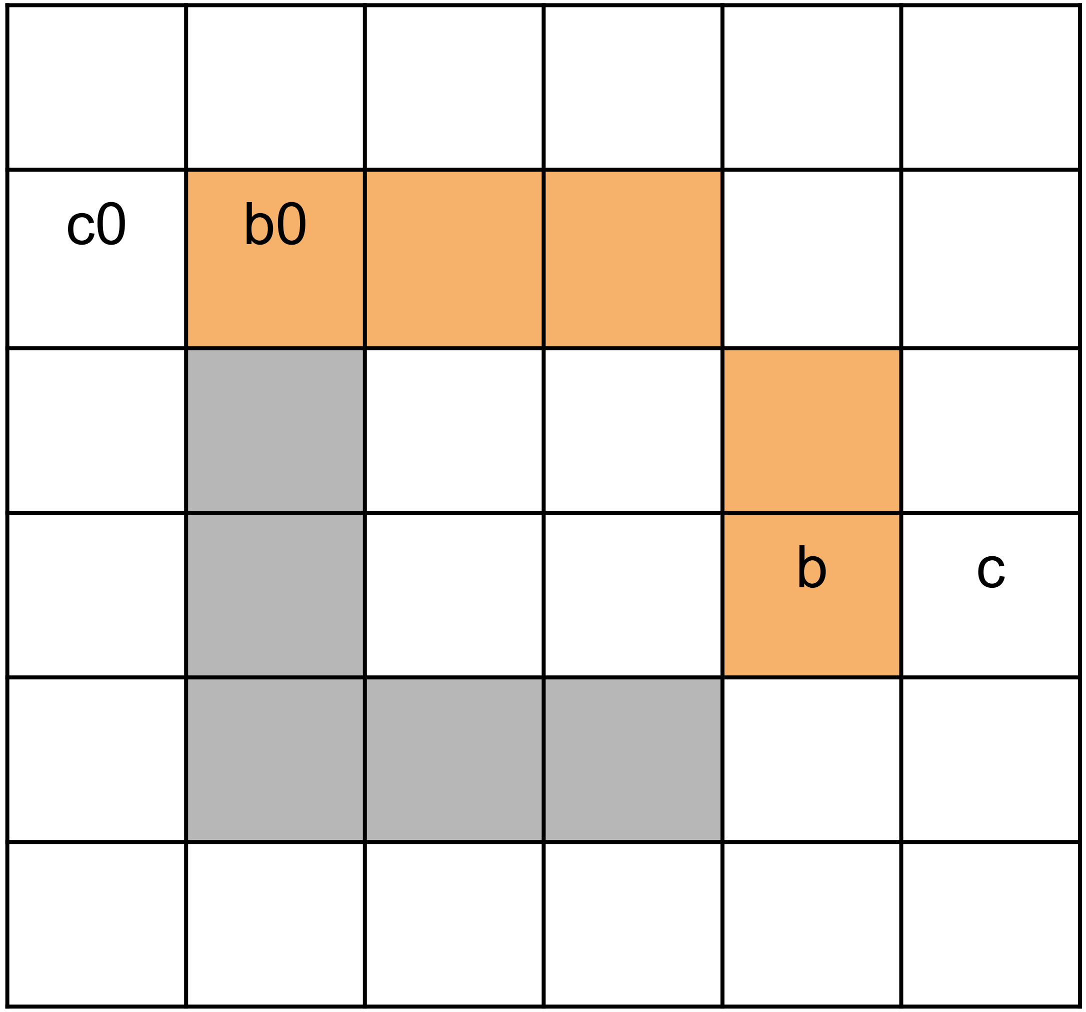
Moore boundary tracking



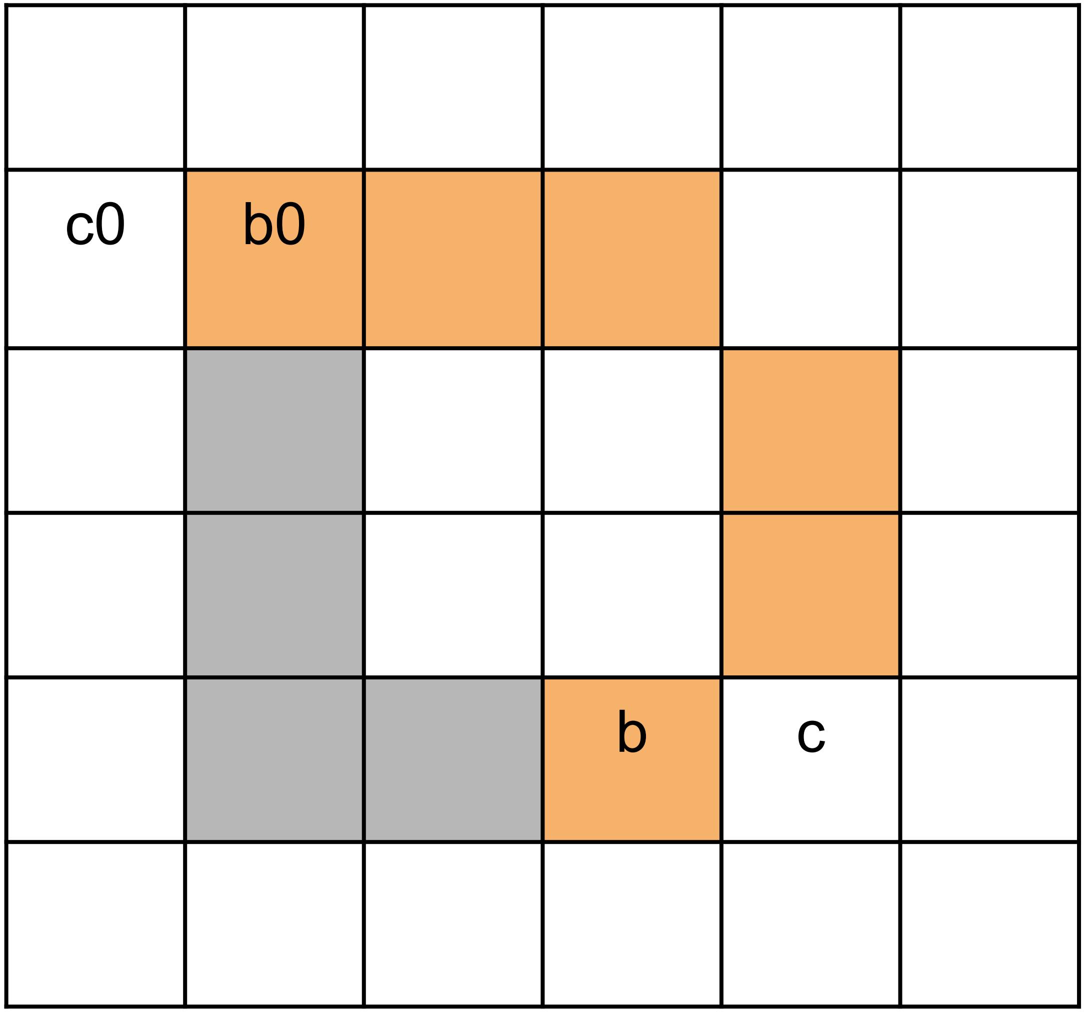
Moore boundary tracking



Moore boundary tracking



Moore boundary tracking



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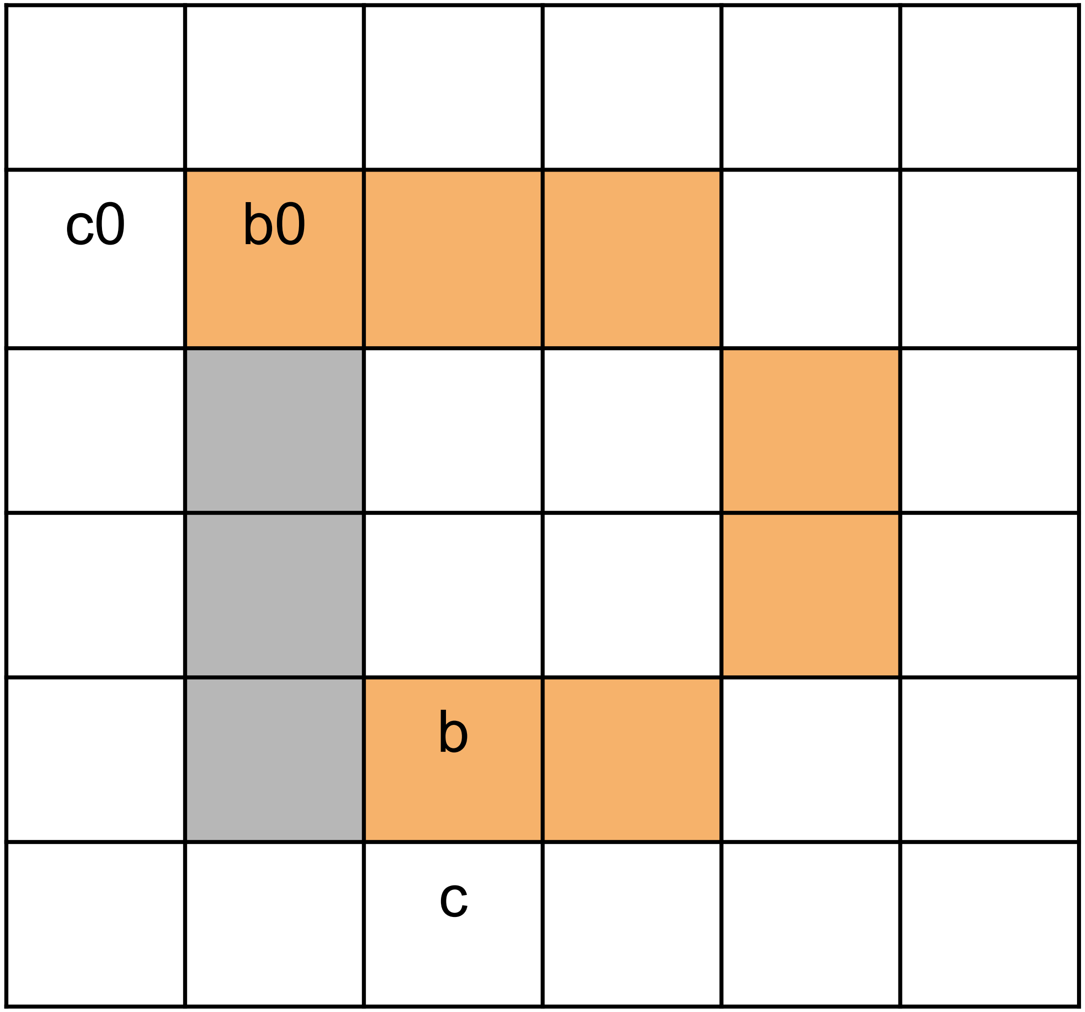
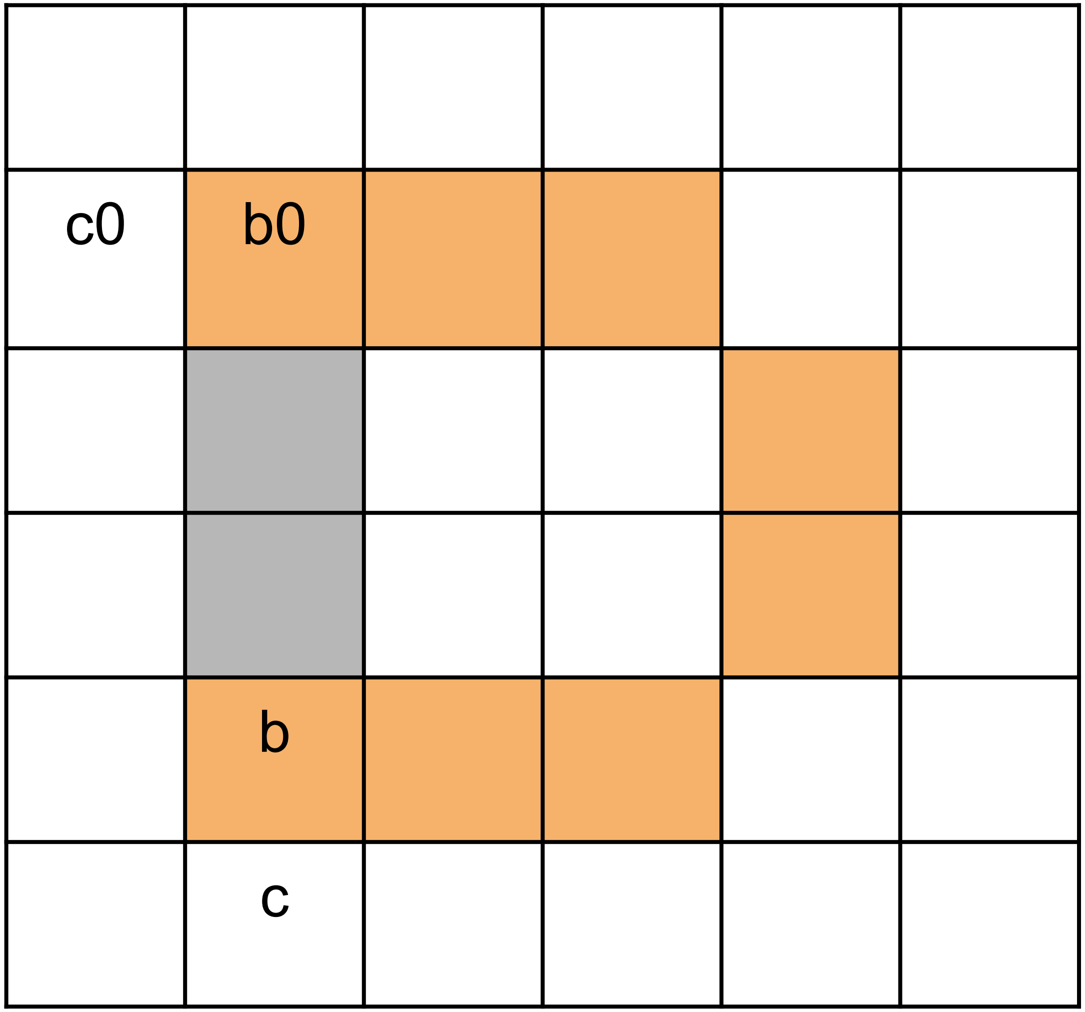
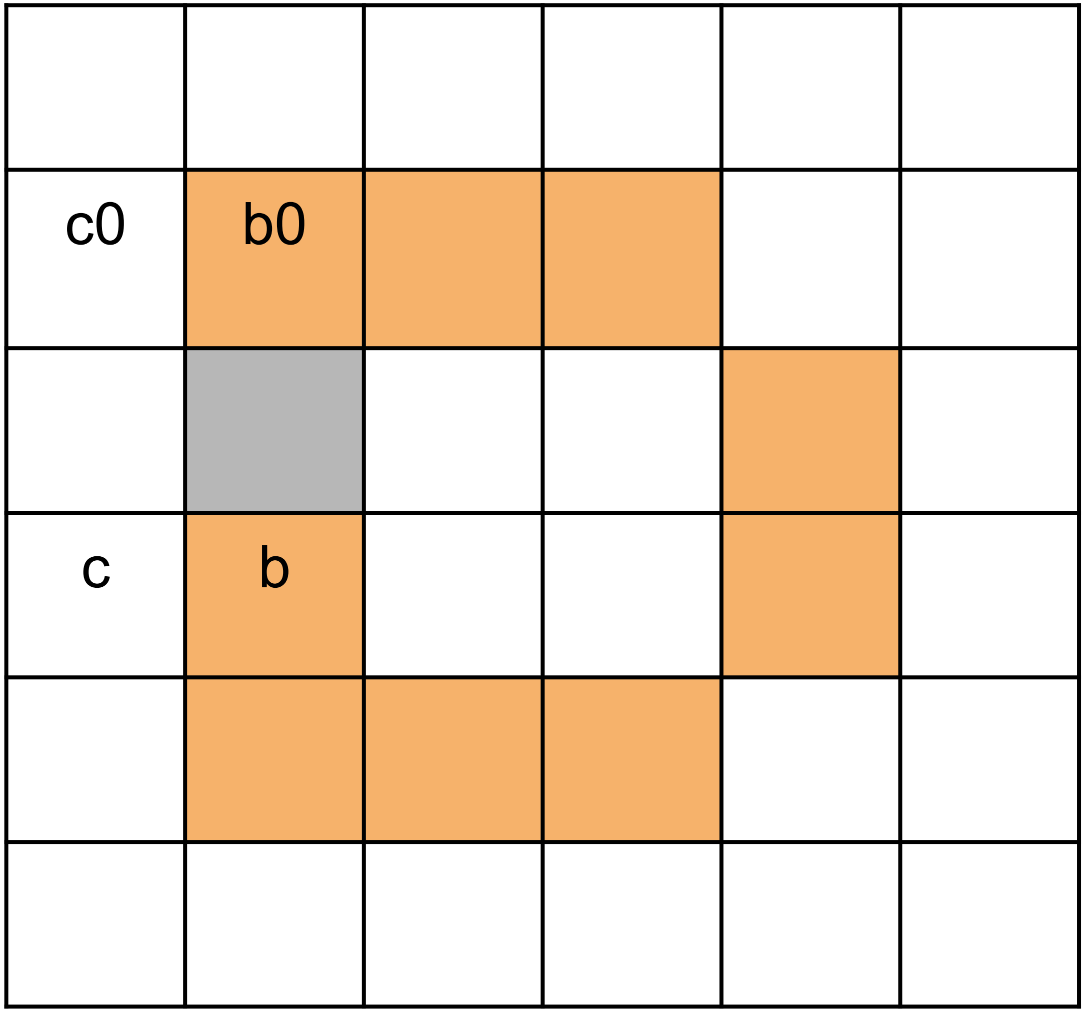


Moore boundary tracking



Moore boundary tracking



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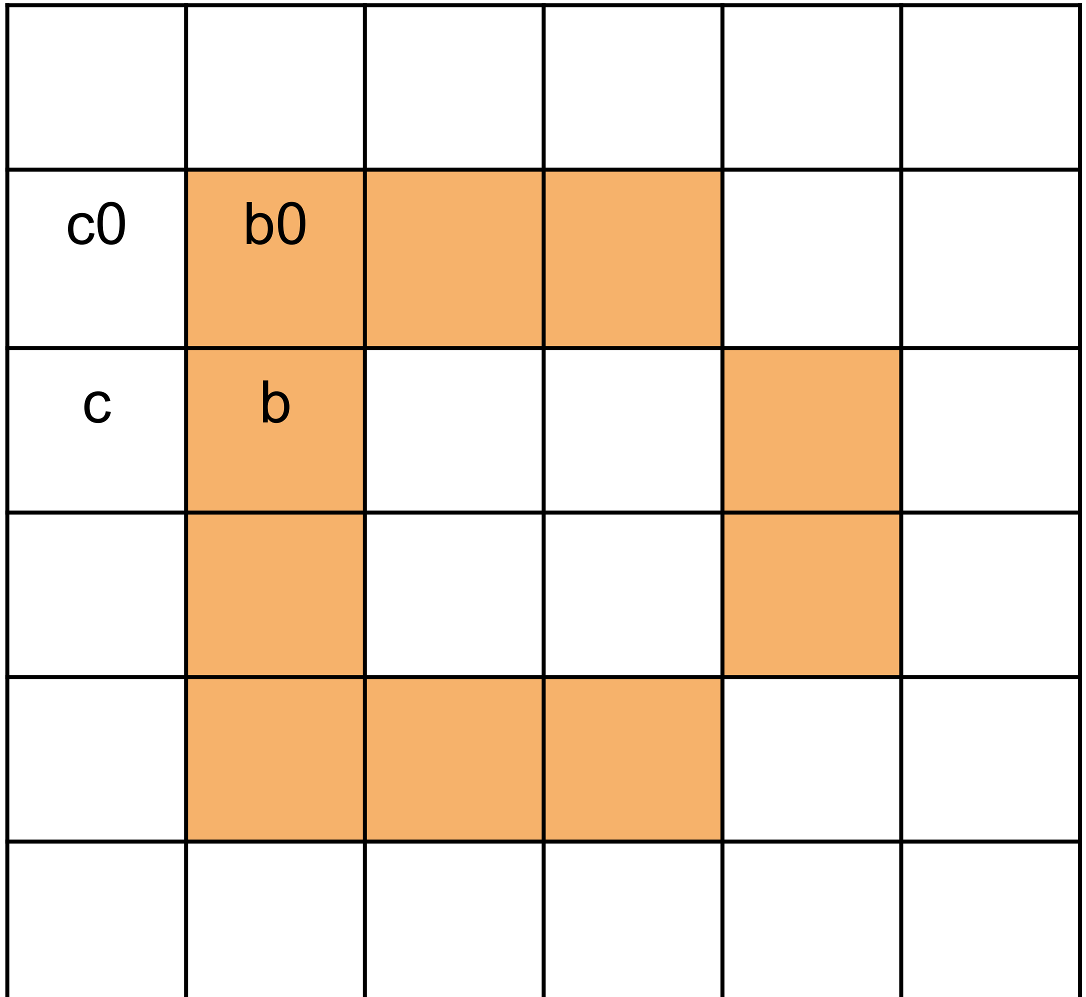

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Moore boundary tracking