

Computer Vision

The Pinhole Camera Model

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Imaging device

Let's try to build a simple imaging device



Film/Sensor

We are unable to get a reasonable image.

Can you guess why?



Università Ca' Foscari

Venezia

Camera obscura

Key idea: Put a barrier with a small hole (aperture) between the object and the sensor



Blurring is reduced! ... but the aperture should be as small as possible. This is also known as **pinhole camera**



Camera obscura

Leonardo da Vinci (1452–1519), after an extensive study of optics and human vision, wrote the oldest known clear description of the camera obscura in mirror writing in a notebook in 1502





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Pinhole camera

How small must the pinhole be?







Pinhole camera

How small must the pinhole be?

Large pinhole: Rays are mixed up -> Blurring!

Small pinhole:

We gain focus, but

- Less light passes through (long exposure time)
- Diffraction effect (we lost focus again!)



Pinhole camera





Cameras and Lenses

Solution: Use lenses!



A lens focuses light onto the film/sensor



Cameras and Lenses

All parallel rays converge to one point on a plane located at the focal length **f**





Cameras and Lenses

Unlike the ideal pinhole camera, there is a specific distance at which the objects are in focus





Cameras and Lenses

Rays are deflected when passing through the lens according to the Snell's law





Cameras and Lenses

Problem: Lens has different refractive indices for different wavelengths.



Color fringing



Cameras and Lenses

Problem: Imperfect lenses may cause radial distortion (deviations are most noticeable for rays passing through the edge of the lens)



No Distortion

Barrel Distortion

Pincushion Distortion



Cameras and Lenses

Problem: Imperfect lenses may cause radial distortion (deviations are most noticeable for rays passing through the edge of the lens)





Pinhole camera model







Pinhole camera model

With the image plane behind the optical center (like in the real camera obscura) the image appears upside-down.

It is common to consider a virtual image plane in front of the center of projection.





Pinhole camera model



Considering similar triangles, we can derive the following:

$$p = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \qquad p' = \begin{bmatrix} \frac{x_p}{z_p} f \\ \frac{y_p}{z_p} f \end{bmatrix}$$



Projection

When we capture a scene with a pinhole camera, we are mapping 3D points to 2D points (onto the image plane) according to the following function:

$$E : \mathbb{R}^3 \to \mathbb{R}^2$$
$$(x, y, z) \to \left(\frac{x}{z}f, \frac{y}{z}f\right)$$

The function is not linear due to the division by z. How we can make it linear?



Projection

By using homogeneous coordinates, we can express the projection with a linear mapping

$$E: \mathbb{P}^3 \to \mathbb{P}^2$$



The division by z occurs only when we transform P' from homogeneous coordinates to eucliean



Projection

In most cameras, pixels are arranged in a grid in which the pixel (0,0) is at the top left and not at the center.

To project a 3D point in 2D pixel coordinates we need:

- The focal length be expressed in pixels (conversion from metric to pixels)
- To translate the projected point wrt. The pixel coordinates of the principal point (cx,cy)



Projection matrix





Intrinsic parameters

Projection matrix

$$p' = \begin{bmatrix} fx_p + c_x z_p \\ fy_p + c_y z_p \\ z_p \end{bmatrix} =$$

Matrix of the intrinsic parameters



Camera pose

What we have seen so far was under the assumption that object points were expressed in the camera reference system

When dealing with multiple cameras it is common to represent points in a common world reference





Camera pose

A rotation matrix **R** and a translation vector **T** indicate the mapping from world reference system to the camera reference system (Camera pose)

6 degrees of freedom, R and T define the extrinsic parameters





World to Camera

To be projected, a point p_w in the world reference system must first be transformed into the camera coordinate system

In 4D homogeneous coordinates we got
p = [RT]p_w





Complete projection

$$p' = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} RT \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$
$$p' = \mathbf{K}[\mathbf{RT}]p_w$$



Lens Distortion

The pinhole camera model describes the image projection as a linear operator when working in projective spaces.

Lens distortion produces a non-linear displacement of points after their projection

> Lines does not project to lines anymore!





Radial Distortion

Usually it is a good approximation to model the lens distortion with the polynomial radial distortion model:

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = (1+\mathbf{k_1}r^2+\mathbf{k_2}r^4+\mathbf{k_3}r^6) \begin{bmatrix} x_d \\ y_d \end{bmatrix} + \begin{bmatrix} 2\mathbf{p_1}x_dy_d + \mathbf{p_2}(r^2+2x_d^2) \\ \mathbf{p_1}(r^2+2y_d^2) + 2p_2x_dy_d \end{bmatrix}$$

point location in retina
plane (unitary f) if the
pinhole camera were
perfect

$$r^2 = x_d^2 + y_d^2$$

(k₁, k₂, k₃, p₁, p₂) are distortion parameters



Radial Distortion

