Algebraic High-Level Nets as Weak Adhesive HLR Categories

Ulrike Prange

Technical University of Berlin, Germany
uprange@cs.tu-berlin.de

Abstract. Adhesive high-level replacement (HLR) system have been recently introduced as a new categorical framework for graph transformation in the double pushout approach [1, 2]. They combine the well-known framework of HLR systems with the framework of adhesive categories introduced by Lack and Sobociński [3, 4]. The main concept behind adhesive categories are the so-called van Kampen squares, which ensure that pushouts along monomorphisms are stable under pullbacks and, vice versa, that pullbacks are stable under combined pushouts and pullbacks. In the case of adhesive HLR categories the class of all monomorphisms is replaced by a subclass $\mathcal{M}$ of monomorphisms closed under composition and decomposition.

Algebraic high-level nets combine algebraic specifications with Petri nets [5] to allow the modelling of data, data flow and data changes within the net. While elementary Petri nets are adhesive HLR categories, the categories of ordinary place/transition nets and of algebraic high-level nets with fixed signature and algebra only satisfy a weaker version of adhesive HLR categories [6] called weak adhesive HLR categories. The reason is that the category **PTNets** of place/transition nets has general pullbacks, but pullbacks in general cannot be constructed componentwise in **Sets**. However, pullbacks along monomorphisms in **PTNets** can be constructed componentwise in **Sets**. This is the key idea to weaken the concept of adhesive HLR categories using weak VK squares. In this case, van Kampen squares ensure the corresponding properties only under stricter requirements on the morphisms. Nevertheless, the framework of weak adhesive HLR categories is still sufficient to show under some additional assumptions (which are necessary also in the non-weak case) the following main results:

1. Local Church-Rosser Theorem,
2. Parallelism Theorem,
3. Concurrency Theorem,
4. Embedding and Extension Theorem,
5. Local Confluence Theorem - Critical Pair Lemma.

Thus, underlying an adhesive HLR systems we consider either a weak or a non-weak adhesive HLR category.

Since this concept of adhesive HLR systems includes all kinds of graphs mentioned above, and also elementary nets, place/transition nets and algebraic high-level nets with fixed algebra, adhesive HLR systems can be seen as a suitable unifying framework for graph and Petri net transformations.

The question arises, if and how different types of algebraic high-level nets, where we do not fix the algebra or the signature, fit into the framework of adhesive HLR systems.

In case that only the algebra may change, but the signature SP is fixed, this category of algebraic high-level nets can be shown to be a weak adhesive HLR category, if the underlying category of algebras $\mathbf{Alg(SP)}$, together with a suitable morphism class $\mathcal{M}$, is a weak adhesive HLR category.

In case both signature and algebra may change, the corresponding category of algebraic high-level nets is isomorphic to a categorical construction using Grothendieck categories. Up to now it is not clear if it is possible to obtain a suitable morphism class $\mathcal{M}$ such that Grothendieck categories become weak adhesive HLR categories. Our recent work on distributed categories and distributed graphs, presented at ICGT'06 [7], is a first step in this direction.

In this talk, we introduce (weak) adhesive HLR categories, give proof ideas for the facts that elementary Petri nets, place/transition nets and algebraic high-level nets with fixed signature are weak adhesive HLR categories and discuss the problems concerning categories of algebraic high-level nets, where the signature is not fixed.

References

[1] Ehrig, H., Habel, A., Padberg, J., Prange, U.: Adhesive High-Level Replacement Categories and Systems. In Ehrig, H., Engels, G., Parisi-Presicce, F., Rozenberg, G., eds.: Proceedings of ICGT 2004. Volume 3256 of LNCS., Springer (2004) 144–160

[2] Ehrig, H., Ehrig, K., Prange, U., Taentzer, G.: Fundamentals of Algebraic Graph Transformation. Springer (2006)

[3] Lack, S., Sobociński, P.: Adhesive Categories. In: Proc. of FOSSACS '04. Volume 2987 of LNCS. Springer (2004) 273–288

[4] Lack, S., Sobociński, P.: Adhesive and Quasiadhesive Categories. Theoretical Informatics and Applications **39**(3) (2005) 511–546

[5] Padberg, J., Ehrig, H., Ribeiro, L.: Algebraic High-Level Net Transformation Systems. MSCS **2** (1995) 217–256

[6] Ehrig, H., Prange, U.: Weak Adhesive High-Level Replacement Categories and Systems: A Unifying Framework for Graph and Petri Net Transformations. In Futatsugi, K., Jouannaud, J.P., Meseguer, J., eds.: Algebra, Meaning and Computation. Volume 4060 of LNCS., Springer (2006) 235–251

[7] Ehrig, H., Orejas, F., Prange, U.: Categorical Foundations of Distributed Graph Transformation. In Corradini, A., Ehrig, H., Montanari, U., Ribeiro, L., Rozenberg, G., eds.: Proceedings of ICGT 2006. Volume 4178 of LNCS., Springer (2006) 215–229