Graph transformation systems, Petri nets and Semilinear Sets: Checking for the Absence of Forbidden Paths in Graphs

Barbara König

Institut für Informatik und interaktive Systeme, Universität Duisburg-Essen, Germany

Abstract. We introduce an analysis method that checks for the absence of (Euler) paths in the set of graphs reachable from a start graph via graph transformation rules. This technique is based on the approximation of graph transformation systems by Petri nets and on semilinear sets of markings.

Introduction. Given a graph transformation system (GTS) and a regular expression r, we want to show that no reachable graph admits a (Euler) path such that the sequence of labels on this path is described by r. That is, the regular expression specifies a set of forbidden paths. There are several conceivable applications of this technique to the analysis of mobile and distributed systems specified by graph transformation. One such application (to an infinite-state version of the dining philosophers) is sketched below.

As a starting point for our method we assume that the given (hyper-)graph transformation system $\mathcal{G}$ has already been approximated by a Petri graph [1, 2], i.e., a Petri net N with a hypergraph structure G over it (where the places of the net are to the hyperedges of the graph). The Petri graph approximates the graph transformation system in the following sense: let m be a marking (i.e., a mapping of places resp. edges to natural numbers), then we denote by $\mathit{graph}(m)$ the graph which is obtained by copying in G every edge according to its multiplicity in m (and dropping the edges to which 0 is assigned). Then it must hold that for every graph R reachable in $\mathcal{G}$ there exists a reachable marking m of N such that R can be mapped to $\mathit{graph}(m)$ via a morphism which is bijective on edges.

Euler Cycles and Semilinear Sets. Now, given a regular expression r, the idea is to specify a set of markings S such that a marking m is contained in S if and only if $\mathit{graph}(m)$ has an Euler path corresponding to r, i.e., a path that traverses every edge of the graph exactly once and whose sequence of labels is contained in the language of r. Since furthermore the property of having an Euler path is preserved by edge-bijective graph morphisms, it is sufficient to check that no such marking of S is reachable on the approximating net.

It turns out that the set S is always semilinar [4], i.e., it is the finite union of sets of the form $\{m_0 + k_1 \cdot m_1 + \dots k_n \cdot m_n \mid k_i \in \mathbb{N}_0\}$, where $m_0, m_1, \dots, m_n$ are markings. Furthermore S can be computed in a simple way via products of finite automata.

Analysis of Graph Transformation Systems. The problem of whether a marking of a semilinear set S is reachable in a net N is decidable, since it is reducible to the reachability problem for Petri nets [4]. However, since this problem is not known to be primitive recursive and all known algorithms are very complex, it will usually be necessary to resort to approximative reachability methods based on the marking equation and traps [3, 5, 6].

Similar to checking for the absence of Euler paths, one can instead also check for the absence of Euler cycles. Using such a technique it can be shown automatically that the infinite-state system of dining philosophers used as an example in [1] will never deadlock, where deadlocks manifest themselves as cycles.

Instead of checking for the absence of paths or cycles where every edge is traversed exactly once, it is often useful to be able to verify the absence of paths where very edge is traversed at most once. The corresponding set S' of markings can be obtained from S as follows: replace every set of the form $\{m_0 + k_1 \cdot m_1 + \cdots + k_n \cdot m_n \mid k_i \in \mathbb{N}_0\}$ by $\{m \mid m \geq m_0\}$. To check whether a marking of S' is reachable is a simpler problem called the coverability problem for Petri nets and can be checked via so-called coverability graphs, although for efficiency reasons it can also be useful to employ the approximative methods described above.

Conclusion. We have sketched how to use the concept of semilinear sets to verify the absence of certain Euler paths in all reachable graphs of a GTS. In the future we plan to generalize this technique by using hyperedge replacement grammars instead of regular expressions. This should also lead to a broader applicability of the presented method.

Acknowledgements: I would like to thank Volker Diekert, Javier Esparza, Paolo Baldan and Arwed von Merkatz for interesting discussions on the topics of this paper.

References

1. Paolo Baldan, Andrea Corradini, and Barbara König. A static analysis technique for graph transformation systems. In *Proc. of CONCUR '01*, pages 381–395. Springer-Verlag, 2001. LNCS 2154.
2. Paolo Baldan and Barbara König. Approximating the behaviour of graph transformation systems. In *Proc. of ICGT '02 (International Conference on Graph Transformation)*, pages 14–29. Springer-Verlag, 2002. LNCS 2505.
3. J. Esparza and S. Melzer. Verification of safety properties using integer programming: Beyond the state equation. *Formal Methods in System Design*, 16:159–189, 2000.
4. Christophe Reutenauer. *Aspects mathématiques des réseaux de Pétri.* Masson, 1989.
5. Claus Schröter. *Halbordnungs- und Reduktionstechniken für die automatische Verifikation von verteilten Systemen.* PhD thesis, Universität Stuttgart, 2005.
6. Arwed von Merkatz. Analyse von Graphtransformationssystemen mit Hilfe von Petrinetzen und Logiken. Master's thesis, Universität Stuttgart, July 2006. No. 2442.