Analysis and Verification of Software
Homework 2
due by February 20, 2015
Exercise 1

- Prove that if $L$ is a lattice, then for each $x,y$ in $L$ it is true that: $\text{lub}(x, \text{glb}(x,y)) = x$
Exercise 2

• Let $A$ be any set, and consider the set $B$ whose elements are all the subsets of $A$, partially ordered by subset inclusion.
• Prove that $B$ is a complete lattice.
• What are the lub and glb operators?
Exercise 3

• Design (at your choice) a (finite) complete lattice $A$, with at least 15 elements.
  Provide four (non-trivial) examples of $f:A\to A$ such that $f$ is, respectively ...
  1. non monotone
  2. monotone but not continuous
  3. a function with no fixpoints
  4. continuous

In the last case, list the set of fixpoints of $f$. 
Exercise 4

• Prove the following theorem:

If \((P, \prec)\) is a complete lattice, and \(f\) is a monotone (increasing) function from \(P\) to \(P\), then \(f\) has exactly one minimal fixed point.
Exercise 5

- Compute the reaching definitions for the nodes in the following flowgraph.