1. Mohamed Abdelhamied Vote INS
2. Fatma Metwally Vote 23
3. Paola Urbani Vote 24
4. Aliye Kuerban Vote 25

1. Explain what formula represents the sentence: “I come either tomorrow morning or tomorrow night. In the first case, I eat with you”.

\[ P \equiv \text{I come tomorrow morning} \]
\[ Q \equiv \text{I come tomorrow night} \]
\[ R \equiv \text{I eat with you} \]

(a) \( \neg P \rightarrow Q \)
(b) \( P \rightarrow (Q \lor R) \)
(c) \( (P \rightarrow Q) \lor \neg R \)
(d) \( P \rightarrow (Q \lor \neg R) \)
(e) \( (P \lor Q) \rightarrow \neg R \)
(f) \( (P \lor Q) \land (P \rightarrow R) \)

\text{Solution: The right sentence is} (P \lor Q) \land (P \rightarrow R).

2. Consider the universe of people and the following language:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>“Anne”</td>
</tr>
<tr>
<td>C</td>
<td>“to be a student”</td>
</tr>
<tr>
<td>I</td>
<td>“to be a professor”</td>
</tr>
<tr>
<td>A</td>
<td>“likes”</td>
</tr>
<tr>
<td>D</td>
<td>“hates”</td>
</tr>
<tr>
<td></td>
<td>(constant)</td>
</tr>
<tr>
<td></td>
<td>(unary relation)</td>
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<td></td>
<td>(unary relation)</td>
</tr>
<tr>
<td></td>
<td>(binary relation)</td>
</tr>
</tbody>
</table>

Formalize the following sentences
(i) Anne is a professor who likes all students
(ii) Every professor likes some student
(iii) Some student hates all professor

Solution: (i) $I(a) \land \forall y(C(y) \rightarrow aAy)$.  
(ii) $\forall x(I(x) \rightarrow \exists y(C(y) \land xAy))$  
(iii) $\exists x(C(x) \land \forall y(I(y) \rightarrow xHy))$

3. (i) Provide a proof in natural deduction of the formula

$$(A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$$

(ii) Find a $\lambda$-term which represents a proof of the above formula.

Solution:

\[
\begin{array}{ccc}
[x : A \rightarrow B]^* & [y : A]^* & [z : A \rightarrow (B \rightarrow C)]^* \quad [y : A]^* \\
\hline
xy : B & zy : B \rightarrow C \\
\hline
(zy)(xy) : C \\
\hline
\lambda y. (zy)(xy) : A \rightarrow C \\
\hline
\lambda x. \lambda y. (zy)(xy) : (A \rightarrow B) \rightarrow A \rightarrow C \\
\hline
\lambda z. \lambda x. \lambda y. (zy)(xy) : (A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C \\
\end{array}
\]