1. Consider the universe of people and the following language:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>“Anne” (constant)</td>
</tr>
<tr>
<td>$b$</td>
<td>“John” (constant)</td>
</tr>
<tr>
<td>$C$</td>
<td>“to be a singer” (unary relation)</td>
</tr>
<tr>
<td>$I$</td>
<td>“to be italian” (unary relation)</td>
</tr>
<tr>
<td>$A$</td>
<td>“loves” (binary relation)</td>
</tr>
<tr>
<td>$D$</td>
<td>“hates” (binary relation)</td>
</tr>
</tbody>
</table>

Formalize the following sentences

(i) John loves all Italian singers
(ii) John loves some Italian singer
(iii) Anne hates some singer loved by John
(iv) There are people who hate all singers loved by John

Solution:

(i) $\forall x (I(x) \land C(x) \rightarrow bAx)$;
(ii) $\exists x (I(x) \land C(x) \land bAx)$;
(iii) $\exists x (aDx \land C(x) \land bAx)$;
(iv) $\exists x \forall y (C(y) \land bAy \rightarrow xDy)$
2. (i) Provide a proof in natural deduction of Peirce law \(((A \rightarrow B) \rightarrow A) \rightarrow A\).

(ii) Is the formula \(A \lor \neg A\) derivable in Intuitionistic Logic? If not, why?

Solution:

(i) See the notes of the course.

(ii) IL satisfies the following condition: \(\vdash_{IL} A \lor B\) iff either \(\vdash_{IL} A\) or \(\vdash_{IL} B\). It follows that \(\not\vdash_{IL} A \lor \neg A\).

3. Give a proof of the formula \(A \rightarrow (B \rightarrow (A \rightarrow A))\) in typed lambda calculus.

Solution:

\[
\begin{align*}
\lambda x.x : A & \rightarrow A \\
\lambda y.\lambda x.x : B & \rightarrow (A \rightarrow) \\
\lambda x.\lambda y.\lambda x.x : A & \rightarrow (B \rightarrow (A \rightarrow A))
\end{align*}
\]