Primi ausili al calcolo

## In principio fu l'abaco

## 500 B.C.

500
1000
1500
Present

Salamis tablet calculi hand-abacus
(300 B.C.)
apices

## Counting Board



One of the earliest counting board found in Salamis Island, dating about 300 BC .

Counting board was used in Europe until about 1500 AD.

Hindu-Arabic vs Counting Board

## La tavola di Salamina (300 a.C.)




A page from the first widely used printed book on arithmetic in the English language. This book, by Robert Recorde, was in print from 1542 right up to the start of the 1700 s.

## Roman Numerals

- I
- V
- X
- L
- C
- D
- M

1
5
10
50
100
500
1000

Early Roman numerals are purely additive.

III 3
IIII
4
VII 7
VIIII 9
DCCCCI 901
MMDLXIII 2563

## Counting Board

A board with lines indicating 1, 10, 100, and 1000.

In-between lines stand for 5, 50, and 500 .

M (1000)
D
C (100)
L
X (10)

I (1)

## Counting Board Number

MMMDCCCLXXIIII
(3874)

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII



## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII



## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Neaten it up by the | MMMMMDDCCCCCLXXXXXVVI |
| :--- | :--- | :--- | :--- |
| following rules: |  |

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Neaten it up by the |
| :--- | :--- |
| following rules: | MMMMMDDCCCCCLXXXXXXI

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII



## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Neaten it up by the |
| :--- | :--- |
| following rules: | MMMMMDDCCCCCCXI

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Neaten it up by the |
| :--- | :--- |
| following rules: |$|$ MMMMMDDDCXI

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Neaten it up by the | MMMMMMDCXI |  |
| :--- | :--- | :--- | :--- |
| following rules: |  |  |

## Counting Board MMDCCXXXVII + MMMDCCCLXXIIII

| Final answer: | IכJMDCXI | (6611) |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Counting Board Subtraction



## Counting Board Subtraction



## Counting Board Subtraction



## Counting Board Subtraction



## Counting Board Subtraction



## Counting Board Multiply: 83x26



## Counting Board Multiply: 83x26



## Counting Board Multiply: 83x26



Neaten up the doubling result

## Counting Board Multiply: 83x26



Save a copy of doubled number for later use

Multiply by 10 by shifting up 1 line

## Counting Board Multiply: 83x26



Save $83 \times 20$, copy $83 \times 2$

## Counting Board Multiply: 83x26



## Counting Board Multiply: 83x26



## Counting Board Multiply: 83x26

| xxvi (26) | $83 \times(20+2+4)$ |
| :---: | :---: |
|  | - |
|  | - |
|  | -0.000 |
|  | - 0 |
| -0 | $00000{ }^{-0}$ |
| $\bullet$ | - |
| - | - 00- |
|  | Push the saved copies over |

## Counting Board Multiply: 83x26

| XXVI (26) | The product is MMCLVIII (2158) |
| :--- | :--- |
|  |  |
|  | Clean up the result |

## L’abaco



## L'abaco in Oriente

The oriental wire and bead abacus appears to have its origin in the Middle East some time during the early Middle Ages.

A type of abacus was developed that had several wires, each of which was strung with ten beads. The Turks called this a coulba, the Armenians a choreb, and the Russians, where it can still be seen in use today, referred to it as a stchoty.

This device almost certainly entered the Far East through the standard trade routes of the day, the merchant classbeing the first to adopt its use and then it slowly spread to the upper levels of society .

Its introduction may well have been helped by international traders, such as Marco Polo, who had to travel through several different countries on their way to China and thus had ample opportunity to pick up different techniques along the way.

## L'abaco russo (Schoty)



## L'abaco cinese (Suan pan)

By the time it was firmly entrenched in Chinese society, about the year A.D. 1300, the abacus consisted of an oblong frame of wood with a bar running down its length, dividing the frame into two compartments.

## L’abaco giapponese (Soroban)

From China the concept of a wire and bead abacus spread to Japan. Again it was likely the merchant class who actually spread the idea, for there was a great deal of trade going on between the two countries during the period A.D. 1400-1600.


# ABACUS: Mystery of the Bead 

## Abacus: Mystery of the Bead

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http://webhome.idirect.com/~totton/abacus

## Using the Soroban



## Setting numbers on a Soroban

Use only the thumb and index fingers to manipulate beads on a soroban. The thumb moves the earth beads up toward the beam. The index finger moves everything else (all earth beads down away from the beam and all heaven beads up \& down).


## Examples

In Fig. 6 from left to right, the numbers on single rods show 1, 3, 5, 7 \& 9. Designating rod F as the unit rod, the soroban on the right shows the number 42,386 on rods B, C, D, E and F.


## Clearing a Soroban

Place the soroban flat on the table in front of you, then tilt the frame toward you. Gravity pulls all the beads down. At this point only the earth beads have been cleared away from the beam. Place the soroban back onto the table and hold it with the left hand. Then, using the back of the right index finger, make a sweeping motion from left to right between the top of the beam and the bottom of the heaven beads (Fig.7). This forces the heaven beads up away from the beam. When none of the rods shows any value, this is what is known as a "cleared" frame.


## Complementary Numbers: The Process of Thoughtlessness

In competent hands, a soroban is a very powerful and efficient calculating tool. Much of its speed is attributed to the concept of mechanization. The idea is to minimize mental work as much as possible and to perform the physical task of adding and subtracting beads without hesitation: in a sense, to develop a process of thoughtlessness. With this in mind, one of the skills employed by the operator is the use of complementary numbers with respect to 5 and 10.

- In the case of 5, the operator uses two groups of complementary numbers: 4 \& 1 and 3 \& 2.
- In the case of 10 , the operator uses five groups of complementary numbers: $9 \& 1,8 \& 2,7 \& 3,6 \& 4,5 \& 5$.

With time and practice using complementary numbers becomes effortless and mechanical.

## Example 1: $4+8=12$

In this example, set 4 on rod $B$. Add 8 to rod $B$. It's evident that rod $B$ doesn't have a value of 8 available. Instead, use complementary numbers.

The complementary number for 8 with respect to 10 is 2 . Therefore, subtract 2 from 4 on rod $B$ and add 1 to tens rod $A$. This leaves the answer 12. (Fig.8)
$4+8=12$ becomes $4-2+10=12$


Fig. 8

## Example 2: <br> $6+7=13$

In this example, set 6 on rod $B$. Add 7 to rod $B$. It is evident that rod $B$ does not have a value of 7 available. Instead, use complementary numbers,

The complementary number for 7 with respect to 10 is 3 . Therefore, subtract 3 from 6 on rod $B$ and add 1 to the tens rod immediately on the left to equal 13. (Fig.9)
$6+7=13$ becomes $6-5+2+10=13$


Fig. 9

## Example 3: 11-7

In this example, set 11 on rods $A B$. Subtract 7 from rod B. Because there aren't enough beads available use complementary numbers.

The complementary number for 7 with respect to 10 is 3 . **In subtraction the order of working the rods is slightly different from that of addition.** Begin by subtracting 1 from the tens rod on A , then add the complementary 3 to rod B to equal 4. (Fig.10)

$$
11-7=4 \text { becomes } 11-10+3=4
$$



Fig. 10

## Example 4: 13-6

In this example, set 13 on rods $A B$. Because there are not enough beads available to subtract 6 from rod $B$ use complementary numbers.

The complementary number for 6 with respect to 10 is 4 . In subtraction the order of working the rods is slightly different from that of addition. Begin by subtracting 1 from the tens rod on A , then add the complementary 4 to rod $B$ to equal 7. (Fig.11)

$$
13-6=7 \text { becomes } 13-10+5-1=7
$$



Fig. 11

## The power of the Abacus

In 1947 Kiyoshi Matsuzake of the Japanese Ministry of Communications used a soroban (the Japanese version of the abacus) to best Private Tom Wood of the United States Army of Occupation, who used the most modern electrically driven mechanical calculating machine, in a contest of speed and accuracy in calculation.

The contest consisted of simple addition and subtraction problems, adding up long columns of many-digit numbers, and multiplication of integers.

Matsuzake clearly won in four out of the five contests held, being only just beaten out by the electrically driven calculator when doing the multiplication problems.

## Il metodo della "gelosia" (o del reticolo)

This method is known to be very old, it likely developed in India and there are records of its use in Arabic, Persian, and Chinese societies from the late Middle Ages.

The method was introduced into Italy sometime in the fourteenth century, where it obtained its name from its similarity to a common form of Italian window grating.


## John Napier (1550-1617)

## Cannon of Logarithms

(1614)

- Numbers in an arithmetic series are the logarithms of other numbers in a geometric series, to a suitable base.

Rabdologia (1617)

- aka Napier's "Bones"
- Multiplicationis
 Promptuarium


## Napier's Bones

| $\times$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | \% | $0 / 1$ | $0 / 2$ | 0/3 | $0 / 4$ | $4 \%$ | 0/6 | $0 / 7$ | \% 8 | 8 |
| 2 | 0 | 0 | $0 / 4$ | 0 | 0 | $81 / 0$ | $1 / 2$ | $1 / 4$ | $1 / 6$ |  |
| 3 | 0 | 0 | $0 / 6$ | 0 | $1 / 2$ | $21 / 5$ | $1 / 8$ | $2 / 1$ | $2 / 4$ |  |
| 4 | \% | 0 | 0 | $1 / 2$ | $1 / 6$ | 62 | $2 / 4$ | $2 / 8$ | $3 / 2$ |  |
| 5 | 0 | $0 / 5$ | 1/0 | $1 / 5$ | $2 / 0$ | $02 / 5$ | $3 / 0$ | $3 / 5$ | $5 / 0$ | 0 |
| 6 | 0 | 0 | 1/2 | $1 / 8$ | $2 / 4$ | $43 / 0$ | 3/6 | $4 / 2$ | 248 | 8 |
| 7 | 0 | $0 / 7$ | $1 / 4$ | $2 / 1$ | $2 / 8$ | $83 / 5$ | $4 / 2$ | $4 / 9$ | $5 / 6$ | $6$ |
| 8 | \% | 0 | 1/6 | $2 / 4$ | $3 / 2$ | 240 | $4 / 8$ | $5 / 6$ | $6 / 4$ |  |
| 9 | 0 | 0 | $1 / 8$ | $2 / 7$ | $3 / 6$ | $64 / 5$ | $5 / 4$ | $6 / 3$ | 372 | 281 |

The Napier's bones consist of vertical strips of the table.

Each entry is the product of index number and strip number, e.g., $7 \times 8=56$, with 5 at the upper left half of square and 6 on lower right.

## The "Bones"

A box of Napier's "bones", one of the oldest calculating "machine" invented by the Scotsman Napier in 1617. The strips with 4, 7, 9 give partial products of any digit from 1 to 9 times 479.
.



## Napier's "Bones"



- Made out of animal bones (ivory)
- About the size of a cigarette and in a leather pouch
- Multiples, multiples, multiples!
- Arrange multiplicand and read multiplier row


## Napier's "Bones"



- Multiplicand is: 423
- Align bones for 4,2 \& 3
- Add values on the diagonal
- Be careful of carries!
- Write down resultant


## Example of Use of Napier's Bones



Result for $7 \times 47526$

## Square and Cube Roots

|  |  |  |
| :--- | :--- | :--- |
| $0 / 1$ | 2 | 1 |
| $0 / 4$ | 4 | 2 |
| $0 / 9$ | 6 | 3 |
| $1 / 6$ | 8 | 4 |
| $2 / 5$ | 10 | 5 |
| $3 / 6$ | 12 | 6 |
| $4 / 9$ | 14 | 7 |
| $6 / 4$ | 16 | 8 |
| $8 / 18$ | 18 | 9 |


| $3 / 2$ |  |  |  |
| :--- | :--- | :--- | :--- |
| $0 / 0$ | 1 | 1 | 1 |
| $0 / 0$ | 8 | 4 | 2 |
| $0 / 2$ | 7 | 9 | 3 |
| $0 / 6$ | 4 | 16 | 4 |
| $1 / 2$ | 5 | 25 | 5 |
| $2 / 1$ | 6 | 36 | 6 |
| $3 / 4$ | 3 | 49 | 7 |
| $5 / 1$ | 2 | 64 | 8 |
| $7 / 2$ | 9 | 81 | 9 |



Gaspard Schott's version of Napier's bones.

## Genaille-Lucas Rulers



Similar to Napier's bones, but without the need to mentally calculate the partial sum. Just follow the arrows and read off the answer backwards (least significant to most significant digits).
The device was invented by Genaille in 1885.

## Use of the Genaille-Lucas Ruler



Must start from the topmost number.
We read 2 -> 8 -> 6 -
> 2 -> 3 -> 3
Or
$7 \times 47526=332682$.

This is part of the strip.

## Genaille-Lucas Ruler, 2207×47



Use the strip 2 twice, strip 0, and 7 to form 2207. Read from the $4^{\text {th }}$ row, we get 08828 (starting from topmost, then following the arrows), and $7^{\text {th }}$ row, 15449. Then add

$$
\begin{array}{r}
88280 \\
+\quad 15449 \\
\hline 103729
\end{array}
$$

## Genaille-Lucas Division Rulers



## Inventor of Logarithm

"Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances. And having thought upon many things to this purpose, I found at length some excellent brief rules to be treated of (perhaps) hereafter. But amongst all, none more profitable than this which together with the hard and tedious multiplications, divisions, and extractions of roots, doth also cast away from the work itself even the very numbers themselves that are to be multiplied, divided and resolved into roots, and putteth other numbers in their place which perform as much as they can do, only by addition and subtraction, division by two or division by three."


About twenty-five years before Napier published his description of logarithms, the problem of easing the workload when multiplying two sines together was solved by the method of prosthaphaeresis, which corresponds to the formula:

$$
\sin a \times \sin b=[\cos (a-b)-\cos (a+b)] / 2
$$

Once it had been shown that a rather nasty multiplication could be replaced by a few simple additions, subtractions, and an elementary division by 2 , it is entirely likely that this formula spurred scientifically oriented individuals, including Napier, to search for other methods to simplify the harder arithmetical operations.

Indeed, We know that Napier knew of, and used, the method of prosthaphaeresis, and it may well have influenced his thinking because the first logarithms were not of numbers but werelogarithms of sines.

Another factor in the development of logarithms at this time was that the properties of arithmetic and geometric series had been studied extensively in the previous century.

We now know that any numbers in an arithmetic series are the logarithms of other numbers in a geometric series, in some suitable base.

For example, the following series of numbers is geometric, with each number being two times the previous one:

```
natural numbers 12481632641282565121024.
```

And the series below is an arithmetic one whose values are the corresponding base 2 logarithms :
logarithms 012345678910 .

## A Property of Logarithm

- Let U and V be some positive real numbers, let W
$=\mathrm{U} \times \mathrm{V}$
- Then Log $\mathrm{W}=\log \mathrm{U}+\log \mathrm{V}$
- E.g.: $8 \times 64=512$ $\lg 8=3, \lg 64=6, \lg 512=9$
Of course, $3+6=9$, or $2^{3} \times 2^{6}=2^{9}$
- Thus, multiplication can be changed into addition if we use logarithm.


## Square Root with Logarithm

- To compute $y=\sqrt{x}$
we take

$$
\log y=\log \sqrt{x}=\log x^{1 / 2}=\frac{1}{2} \log x
$$

- Thus the square root is found by taking the log of $x$, divide by 2 , and taking the inverse of log (that is exponentiation).


## II regolo (slide rule)



Fu inventato dal matematico inglese Edmond Gunter (ca. 1620). Serviva ad eseguire moltiplicazioni e divisioni (ma anche quadrati, radici e tante altre operazioni) attraverso la somma o la differenza su scale logaritmiche. Ebbe una diffusione vastissima e fu usato da tecnici e ingegneri fino a quando non comparvero le calcolatrici tascabili (anni '60 circa), che ne decretarono la fine.

Nella seconda metà del XVI secolo, dopo gli studi di John Napier sui logaritmi, vennero costruiti i primi REGOLI.

L'utilizzo di una scala a base logaritmica consente di sfruttare le proprietà dei logaritmi e cioè di calcolare un prodotto come se fosse una somma. La divisione viene ricondotta alla differenza.

Questo fatto ha permesso un grosso successo ed una larghissima diffusione del regolo come potente strumento di calcolo. Esso è utilizzato fino ai nostri giorni, soprattutto in ambienti tecnici di precisione.

## History of the slide rule

- The slide rule was invented around 1620-1630, shortly after John Napier's publication of the concept of the logarithm.
- Edmund Gunter of Oxford developed a calculating device with a single logarithmic scale, which, with additional measuring tools, could be used to multiply and divide.
- In 1630, William Oughtred of Cambridge invented a circular slide rule, and in 1632 he combined two Gunter rules, held together with the hands, to make a device that is recognizably the modern slide rule.
- In 1722, Warner introduced the two- and three-decade scales, and in 1755 Everard included an inverted scale; a slide rule containing all of these scales is usually known as a "polyphase" rule.
- In 1815, Peter Roget invented the log log slide rule, which included a scale displaying the logarithm of the logarithm. This allowed the user to directly perform calculations involving roots and exponents. This was especially useful for fractional powers.


## Principi del regolo



Per illustrare il funzionamento del regolo calcolatore vediamo come si può fare la somma utilizzando i due righelli graduati seguenti.

Se si vuole sommare ad esempio 2 e 3 basta mettere lo 0 del righello B in corrispondenza del 2 del righello A. Abbiamo così impostato 2+.

| A 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | B | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

## Principi del regolo

Per sommare 3 basta leggere sul righello A il numero in corrispondenza del 3 del righello $B$ ottenendo così 5. Se invece si vuole fare $2+6$ non occorre spostare il righello (già impostato su 2+) ma è sufficiente leggere il risultato sul righello A in corrispondenza del 6 del righello $B$ (ottenenedo ovviamente 8).


Per fare le sottrazioni si utilizza il procedimento inverso. Nella figura a fianco è illustrata l'operazione 8-5

## Principi del regolo



Come esempio ulteriore se vogliamo fare $\mathbf{3 x 2}$ impostiamo l'1 della scala B sul 3 della scala A e poi leggiamo il risultato sulla scala A in corrispondenza del 2 della scala $B$.


Ex. $12 \times 3=6$


Per le divisioni nulla di più semplice che invertire la sequenza delle operazioni come visto per la sottrazione.

L'immagine seguente illustra l'operazione 8/4.


Basta mettere il 4 della scala B in corrispondenza dell' 8 della scala A e leggere il risultato sulla scala $A$ in corrispondenza dell' 1 della scala $B$.

## Il compasso di Galileo



## Demo IMSS (Firenze)

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