APPLYING BCMP MULTI-CLASS QUEUEING NETWORKS FOR THE PERFORMANCE EVALUATION OF HIERARCHICAL AND MODULAR SOFTWARE SYSTEMS

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- Software Modularity and Performance Evaluation
- BCMP Queueing Networks
- An approach to modular and hierarchical software design
- The unfolding algorithm
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- Conclusion

Modular and Hierarchical Software Design

- Performance analysis of modular and hierarchical systems always an important topic in research (see Smith (1990)).
- Software system: interaction between black-box components, made of other black-box components ...

An example of a black-box component





Main problem

Defining efficient algorithms to derive performance indexes.

BCMP Networks

BCMP Networks (Baskett et al. (1975)) are one of the most useful models for performance evaluation.

- A set of queueing centers and a (possibly infinite) set of customers.
- Classes, that determine:
 - routing probabilities;
 - service time distributions.
- Each class belongs to a chain
 - open (external arrivals) or closed.
- Class switching only within the same chain.
- Queueing station of one of the following types:

 - **1** FCFS Queueing discipline. Exponential and class-independent service time distribution.

 - PS Queueing discipline,
 - **3** The station has infinite servers (*Delay Station*),
 - LCFSPR service discipline. (4)

With coxian service time distribution

BCMP Theorem (salient results)

Let us consider a multiple-class and multiple-chain QN, open, closed or mixed, whose queueing stations are of type 1, 2, 3 or 4. If the underlying stochastic process is ergodic, then:

$$\pi(\mathbf{n}) = \frac{1}{G} \prod_{i=1}^{M} g_i(n_i)$$

where

- $\mathbf{n} = (n_1, \dots, n_M)$ is the state of the network, n_i is the state of station S_i ;
- π is the steady-state distribution of the QN;
- $g_i(n_i)$ is the steady-state distribution of station S_i considered in isolation;
- *G* is a normalising constant.

- BCMP Networks are widely used. Various solution algorithms.
 - for *open* networks, G = 1.
 - for *closed* or *mixed* network, algorithms to compute G or directly derive the average performance indices.
- BCMP Networks are inherently *flat*.
 - no modular and/or hierarchical design.

We propose an algorithm to compute a BCMP from an modular and hierarchical high-level model.

The design framework

A framework for the specification of hardware and software architectures.

- A set of interacting components $d_1, d_2, \ldots, d_{\ell_1}$. Each component can be
 - A BCMP queueing station
 - A sub-model consisting of components $d_{(i)1}, \ldots, d_{(i)\ell_2}$.
- Components interact as stations of an open multiple-class and multiple-chain QN.
- Sub-models as black boxes
 - access points with some labels, i.e., input and output classes.

We require that

- the set of input classes and the set of output classes must be equal,
- the model must be *well formed*.

More on the design framework

- A higher level class could be connected to any lower level class.
- Multiple classes of the higher level submodel could be connected to the same lower level class.
- The same component could be reused in different submodels.



Class connections: $A_{d_i,d_j}: \mathcal{R}_i \to \mathcal{R}_j$

How to keep routing information in case of component or class reuse?

The Algorithm

Algorithm UnfoldComponents

```
Input: routing matrix \mathbf{P}_d of component d, component counter array Dc, functions A_i;
Output: unfolded routing matrix \mathbf{P'}_d of component d
if d is a station then
       foreach class r of d do
               insert in \mathbf{P}' rows and a columns for EI_d, r and EO_d, r of \mathbf{P}
       end
else
       foreach d_i | d \succ d_i do
               Dc_i \leftarrow Dc_i + 1
               Let Rc_i be a class counter array
               foreach class rk of di as named in d do
                       r_{i,i} \leftarrow \mathbb{A}_{d,d}(r_k)
                       Rc_{i,i} \leftarrow Rc_{i,i} + 1
                       if \tilde{Rc}_{i,j} > \max Rc_i then

U = UnfoldComponents(P_{d_i})
                               rename each class r_{i,j} of d_i in U as r_{i,j,Dc_i,Rc_i}
                               insert in \mathbf{P}' rows and columns of \mathbf{U}
                       end
                       replace column El_{d_i}, r_{i,j,Dc_i,Rc_{i,j}} in \mathbf{P}' with column d_i, r_k of \mathbf{P}
                       replace row EO_{d_i}, r_{i,j,Dc_i,Rc_i} in P' with row d_i, r_k of P
               end
       end
end
return P'
```

Some notes on the algorithm

- Recursive and top-down.
- It adds classes whenever it is needed in order to not loose customer routing information.
- Base cases: BCMP queueing stations.
- Output: multiple-class, multiple-chain BCMP network.
- Computational complexity $\mathcal{O}(rd^n)$ if, on average:
 - every component has the same number of sub-components d,
 - every component has the same number of classes r,
 - the depth of the model is *n*.

A small example

Database-indexed file archive for a CMS.

- A class of customer for read operations.
- A class of customers for write operations.



How many classes should station d_4 have at the end of the algorithm run?

 d_4 should have at least 4 classes.

- The algorithm doubles the number of classes for d_2 .
- The classes of d_2 translates directly in classes of d_4 .
- We cannot know if d_4 or d_2 are used by other components.
- More complex model topology may lead to a rapid increase of the number of classes.

Conclusion and future works

- A modeling technique for hierarchical systems.
- An algorithm that transforms such models in a multi-class and multi-chain BCMP.
- Main advantages:
 - a modular and hierarchical modelling technique,
 - an efficient and exact analysis method.
- More expressive formalisms, like LQNs in Woodside et al. (1995) may require approximate algorithms.
- Future works:
 - extension of the tractable class of models,
 - integration of the framework with web mining and log analysis.

Any question?

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