Modelling retrial-upon-conflict systems with product-form stochastic Petri nets

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Aim of the paper

Systems with a retrial-upon-conflict behaviour are common

- Concurrent activities which may lead to a conflict
- After a recovery phase the activity is tried again.
- Examples: computer networks, transactional DBs, memory buses ...

We consider a simple class of SPNs which can be used to model this behaviour

- We show that it has a Product-Form solution according to [Balsamo et al., 2012]
- We show that, under stability, we haven't any other rate constraint
- We give numerical examples

Context: building blocks

Definition (Building block (BB) [Balsamo et al., 2012])

Given an ordinary (connected) SPN S with set of transitions $\mathcal{T}$ and set of N places $\mathcal{P}$, then S is a *building block* if it satisfies the following conditions:

1. For all $T \in \mathcal{T}$ then either $\mathbf{O}(T) = \mathbf{0}$ or $\mathbf{I}(T) = \mathbf{0}$. In the former case we say that $T \in \mathcal{T}_O$ is an *output transition* while in the latter we say that $T \in \mathcal{T}_I$ is an *input transition*. Note that $\mathcal{T} = \mathcal{T}_I \cup \mathcal{T}_O$ and $\mathcal{T}_I \cap \mathcal{T}_O = \emptyset$, where $\mathcal{T}_I$ is the set of input transitions and $\mathcal{T}_O$ is the set of output transitions.
2. For each $T \in \mathcal{T}_I$, there exists $T' \in \mathcal{T}_O$ such that $\mathbf{O}(T) = \mathbf{I}(T')$ and vice versa.
3. Two places $P_i, P_j \in \mathcal{P}$, $1 \leq i, j \leq N$, are connected, written $P_i \sim P_j$, if there exists a transition $T \in \mathcal{T}$ such that the components i and j of $\mathbf{I}(T)$ or of $\mathbf{O}(T)$ are non-zero. For all places $P_i, P_j \in \mathcal{P}$ in a BB, $P_i \sim^* P_j$, where $\sim^*$ is the transitive closure of $\sim$.

Context: Product-Form SPNs

Theorem (Theorem 2 of [Balsamo et al., 2012])

Consider a BB S with N places. Let $\rho_y = \lambda_y/\mu_y$, where λ_y, μ_y are the firing rates for $T_y, T'_y \in \mathcal{T}, |y| \geq 1$, respectively. If the following system of equations has a unique solution ρ_i, $(1 \leq i \leq N)$:

$$\begin{cases} \rho_y = \prod_{i \in y} \rho_i & \forall y : \; T_y, T'_y \in \mathcal{T} \wedge |y| > 1 \\ \rho_i = \frac{\lambda_i}{\mu_i} & \forall i : \; T_i, T'_i \in \mathcal{T}, 1 \leq i \leq N \end{cases} \tag{1}$$

then the net's balance equations – and hence stationary probabilities when they exist – have product-form solution:

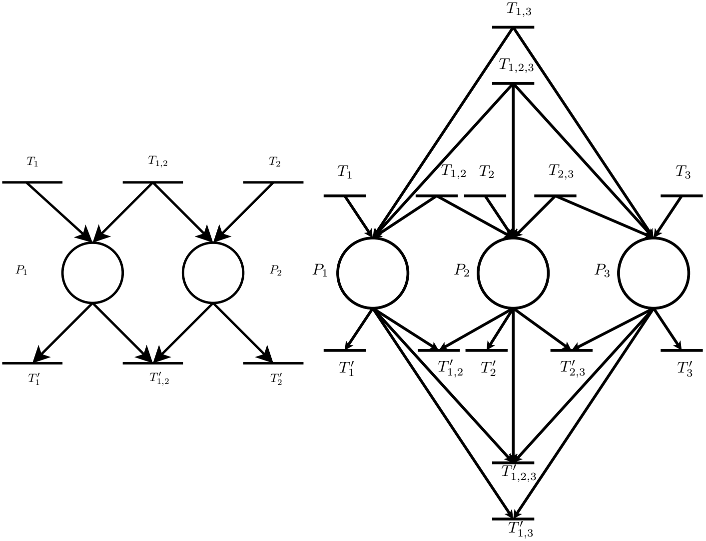
$$\pi(m_1, \ldots, m_N) \propto \prod_{i=1}^{N} \rho_i^{m_i}. \tag{2}$$

The conflict model

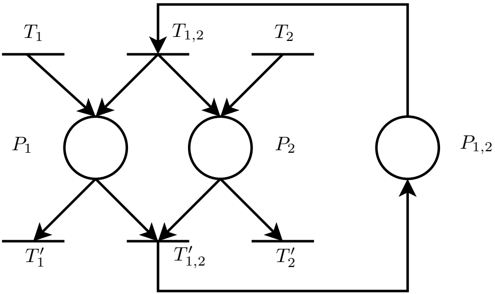
A set of interconnecting building blocks

- a Main Building Block (MBB)
  - a set L of l places $L = \{P_1, \dots, P_l\}$
  - for each place P_i, an incoming transition T_i (rate λ_{P_i}) and an outgoing transition T_i' (rate μ_{P_i})
  - for each $C \subseteq L, |C| \geq 2$, an incoming transition T_C (rate λ_C) and an outgoing transition T_C' (rate μ_C).
- a set of Conflicting Building Blocks (CBBs)
  - single place
  - one for each pair of transitions T_C' (input), T_C (output).
  - total number of CBBs is $\sum_{k=2}^{l} \binom{l}{k} = 2^l - l - 1$
  - firing semantics of transitions T_C, with $|C| \geq 2$, can be single server or infinite servers
- Total number of places: $|\mathcal{P}| = 2^l - 1$
- Total number of transitions: $|\mathcal{T}| = 2|\mathcal{P}| = 2^{l+1} - 2$.

Conflict model: Main Building Block



Conflict model: the complete picture



Product form of conflict model

Proposition (Product-form of the conflict model)

The conflict model consists of building blocks satisfying the structural conditions of [Balsamo et al., 2012]. Moreover, in stability, it yields **without any rate-constraint** *the following product-form solution:*

$$\pi(\mathbf{m}) = \prod_{C \in 2^L \setminus \emptyset} g_C(m_C)$$

where m_C *is the component of the joint state associated with place* P_C *and*

$$g_C(m_C) =$$
$$\begin{cases} (1 - \frac{\lambda_P}{\mu_P})(\frac{\lambda_P}{\mu_P})^{m_P} & \text{if } C = \{P\} \\ (1 - \frac{\mu_C}{\lambda_C} \prod_{P \in C} \frac{\lambda_P}{\mu_P})(\frac{\mu_C}{\lambda_C} \prod_{P \in C} \frac{\lambda_P}{\mu_P})^{m_C} & \text{if } |C| \geq 2 \text{ and } T_C \text{ is single server} \\ (\frac{\mu_C}{\lambda_C} \prod_{P \in C} \frac{\lambda_P}{\mu_P})^{m_C} \exp(-\frac{\mu_C}{\lambda_C} \prod_{P \in C} \frac{\lambda_P}{\mu_P}) \frac{1}{m_C!} & \text{if } |C| \geq 2 \text{ and } T_C \text{ is } \infty \text{ servers} \end{cases}$$

Stability of conflict model

Proposition

The conflict model is stable if the following conditions hold:

$$\forall i \in \{1, \ldots, l\} \quad \lambda_i < \mu_i, \tag{3}$$

for the places of the main building block, while for the places of conflict building blocks P_C whose corresponding T_C is single server, we have that

$$\forall C \subseteq L \quad \overline{\mu}_C = \mu_C \prod_{P_i \in C} \rho_{P_i} < \lambda_C, \tag{4}$$

where $\overline{\mu}_C$ identifies the throughput (reversed rate) of transition T'_C.

Applications: network collisions

Consider a computer network with a set L of l transmitting stations $L = \{s_1, \ldots, s_l\}$.

- packets become ready to be sent from each station s_i according to an homogeneous Poisson process (param. λ_i)
- time to transmit from s_i is exponentially distributed with parameter μ_i^*
- the channel is capable of transmitting with a global rate M
- a collision can occur between any combination of k stations, $2 \leq k \leq L$, with probability $p_k(L)$
- after a collision, an exponentially-distributed recovery time, with parameter μ_C is performed. After that time, a new transmission is retried.
- we assume $\mu_{s_i} = \mu_1$, $\lambda_{s_i} = \lambda_1$, $\forall s_i \in L$, $\lambda_C = \lambda_{|C|}$ and $\mu_C = \mu_{|C|}$, $\forall C \subseteq L, |C| \geq 2$

Network collisions: parameters derivation

We can abstract the system with a Conflict Model with an infinite server firing semantics.

- $q = \frac{\lambda_1}{M}$ is the probability, for a station, to be in transmitting phase
- for $C \subset L, |C| = k \geq 2$, the service rate is

$$\mu_k = \mu^* q^k (1-q)^{L-k}$$

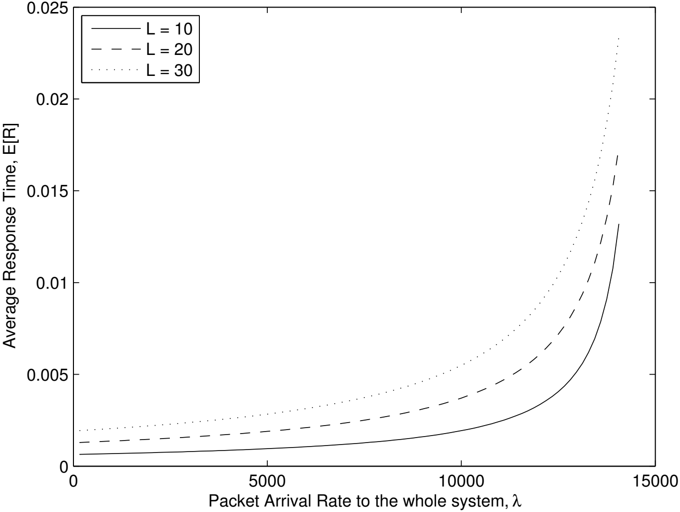
- for $\mu_{s_i} = \mu_1$ we have

$$\mu_1 = \mu^* \left(1 - \sum_{k=2}^{L} \binom{L-1}{k-1} q^k (1-q)^{L-k} \right)$$

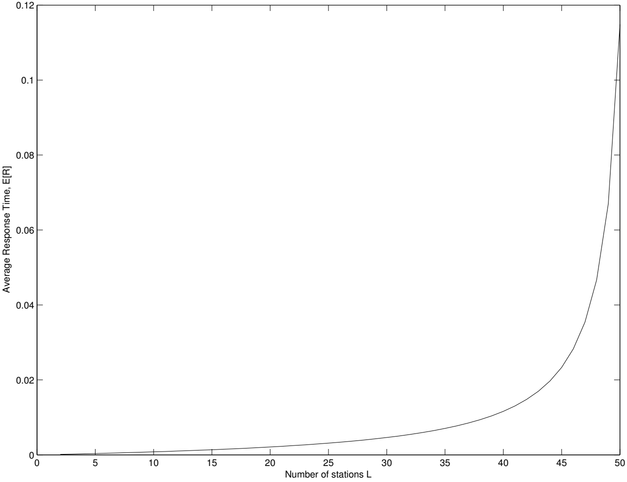
- The average response time is

$$E[N] = l \frac{\rho_1}{1-\rho_1} + \sum_{k=2}^{l} \binom{l}{k} k \rho_k$$

Network collisions: numerical example (1)



Network collisions: numerical example (2)

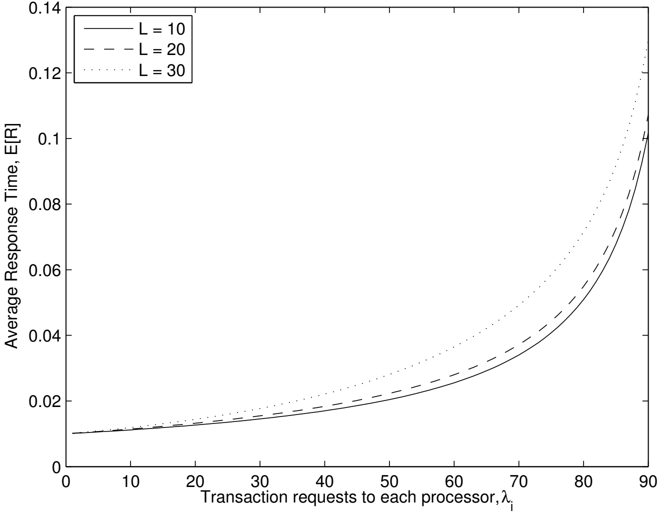


Applications: transactional databases

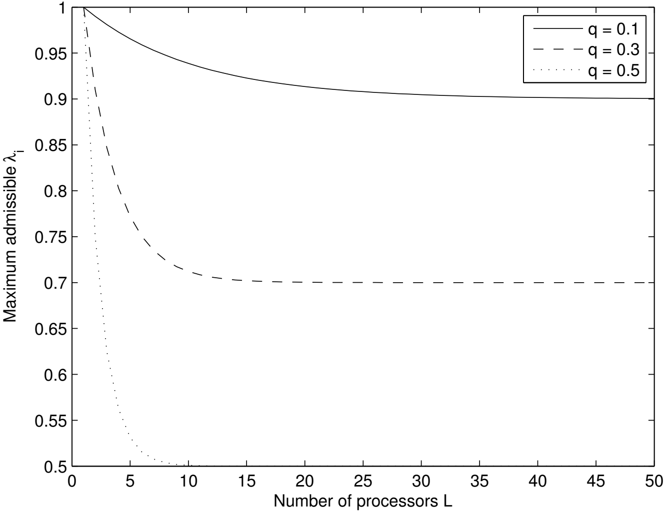
- a set L of l processors $L = \{s_1, \ldots, s_l\}$
- transactions request to be processed by s_i according to an homogeneous Poisson process with parameter λ_i
- time for a transaction to be processed is exponentially distributed with parameter μ_i^*
- conflicts can occur during parallel transaction executions between any subset C of k processors, $2 \leq k \leq L$, with probability $p_k(L)$.
- after a conflict, all the participating transactions are started again, after a exponentially distributed recovery time.
- we can model the system using a conflict model
- since recovery requests are enqueued, conflict building blocks have the ordinary firing semantics of SPNs.
- computation of μ_i and ρ_i from μ_i^* is analogous to the previous example

$$E[N] = \sum_{k=1}^{l} \binom{L}{k} k \frac{\rho_k}{1 - \rho_k}$$

Numerical example: transactional databases (1)



Numerical example: transactional databases (2)



Conclusions

- We have shown how retrial-upon-conflict systems can be modelled by product-form SPNs
- we have shown how this class of SPNs does not require assumptions on rates, except for what is due stability, to be in product-form
- we described two examples of possible applications of this class of models, and we have derived some performance indices for them.

Future works:

- consider also closed SPNs (normalisation issues)
- further explore the parametrisation issue.

References

[Balsamo et al., 2012] Balsamo, S., Harrison, P. G., and Marin, A. (2012). Methodological construction of product-form stochastic Petri nets for performance evaluation. *Journal of Systems and Software*, 85(7):1520–1539.

[Marin et al., 2012] Marin, A., Balsamo, S., and Harrison, P. G. (2012). Analysis of stochastic Petri nets with signals. *Perform. Eval.*, 69(11):551–572.

Thanks!

Thank you for your attention
any question?