Segmentation

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Graph theoretic clustering

- Represent tokens (which are associated with each pixel) using a weighted graph.
  - affinity matrix (pi same as pj => affinity of 1)
- Cut up this graph to get subgraphs with strong interior links and weaker exterior links

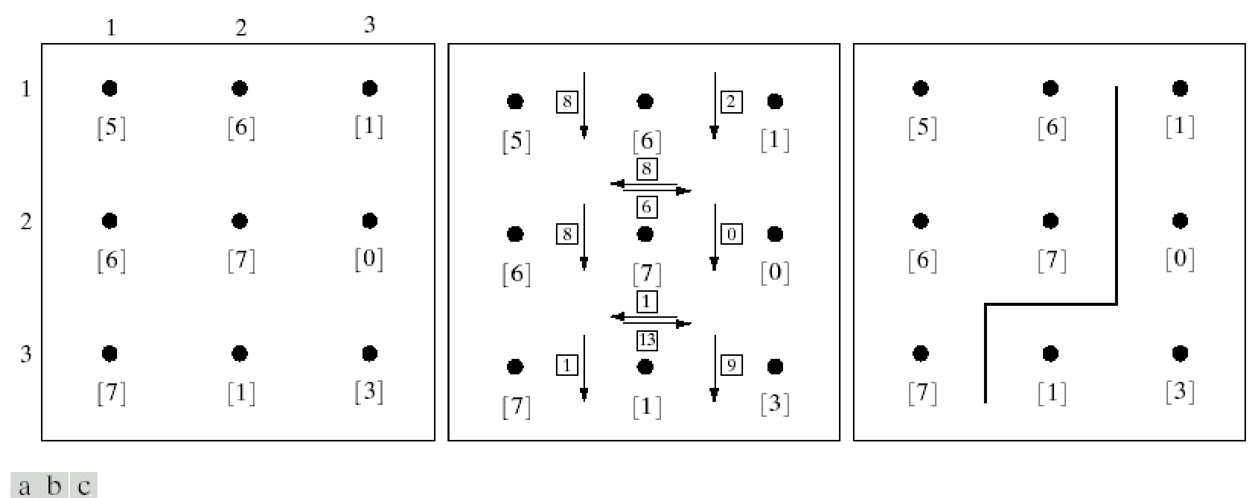


FIGURE 10.23 (a) A 3 × 3 image region. (b) Edge segments and their costs. (c) Edge corresponding to the lowest-cost path in the graph shown in Fig. 10.24.

Measuring Affinity

Intensity

$$\text{aff}(x, y) = exp\left(-\frac{1}{2\sigma_i^2}(I(x) - I(y))^2\right)$$

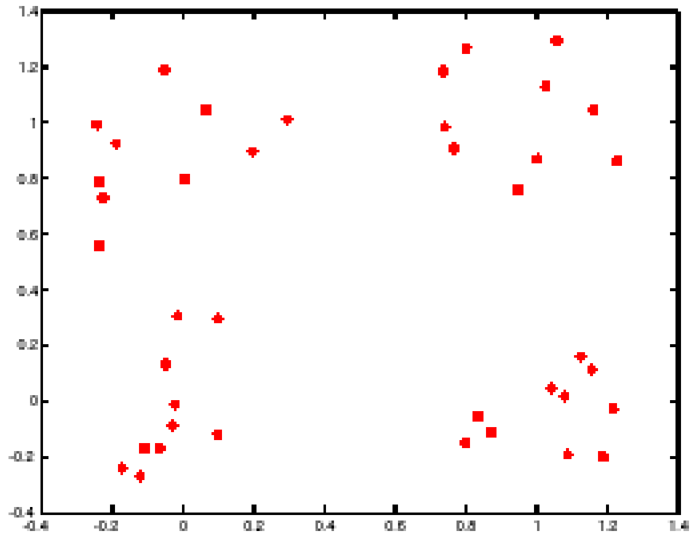
Distance

$$\text{aff}(x, y) = exp\left(-\frac{1}{2\sigma_i^2}||x - y||^2\right)$$

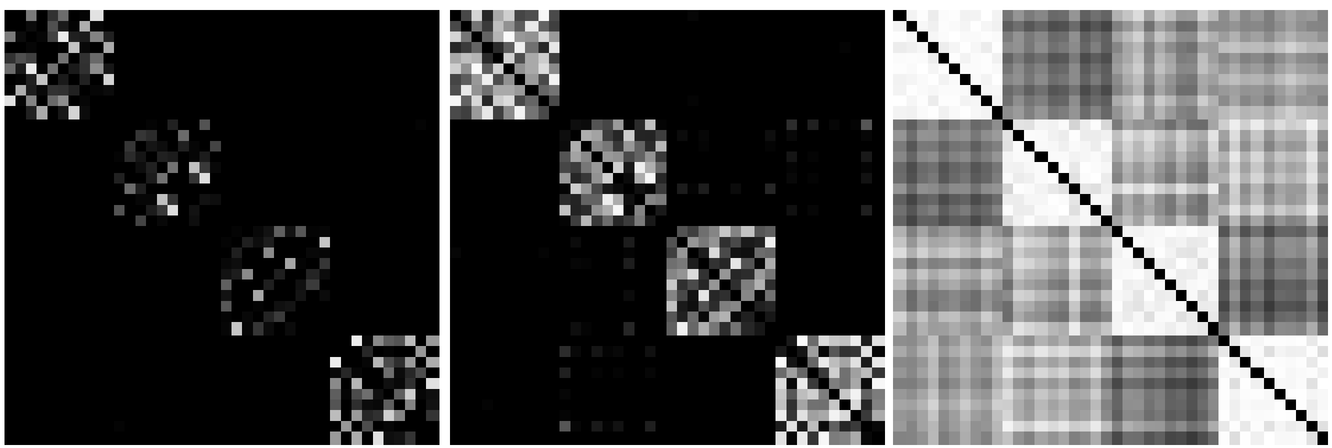
Texture

$$\text{aff}(x, y) = exp\left(-\frac{1}{2\sigma_i^2}(c(x) - c(y))^2\right)$$

Scale affects affinity

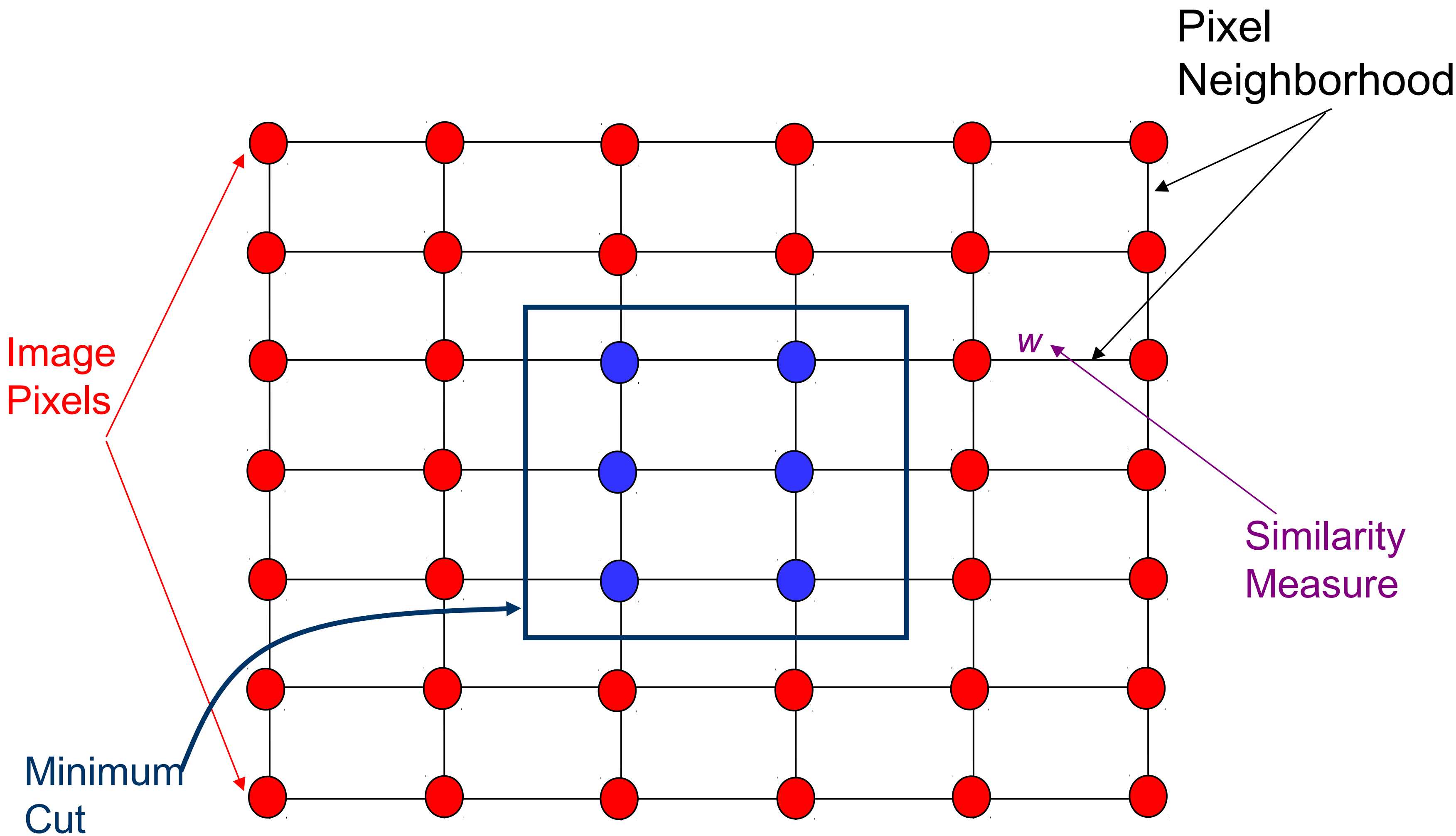


σ=.2



σ=.1 σ=.2 σ=1

Image Segmentation & Minimum Cut



- There can be more than one minimum cut in a given graph
- All minimum cuts of a graph can be found in polynomial time

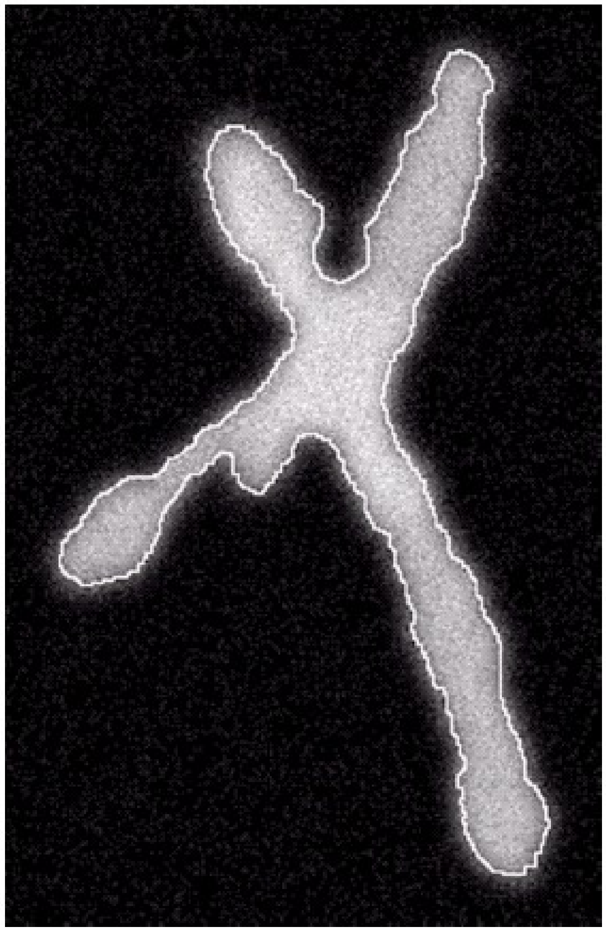
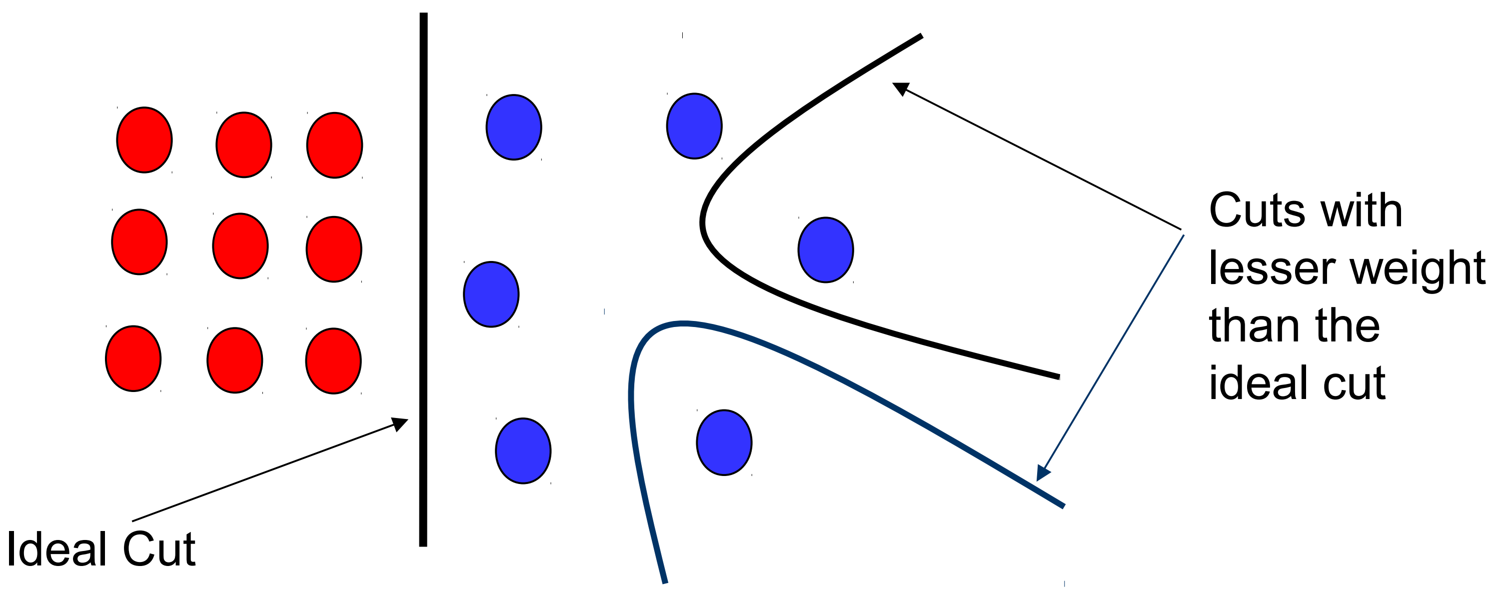


FIGURE 10.25 Image of noisy chromosome silhouette and edge boundary (in white) determined by graph search.

Drawbacks of Minimum Cut

- Weight of cut is directly proportional to the number of edges in the cut.



Normalized Cuts

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- Normalized cut is defined as

$$Ncut(A) = \frac{cut(A)}{asso(A)} + \frac{cut(\bar{A})}{asso(\bar{A})} = \frac{\sum_{i \in A} \sum_{j \in \bar{A}} w(i,j)}{\sum_{i \in A} \sum_{j} w(i,j)} + \frac{\sum_{i \in \bar{A}} \sum_{j \in A} w(i,j)}{\sum_{i \in \bar{A}} \sum_{j} w(i,j)}$$

- Ncut(A,B) is the measure of dissimilarity of sets A and B.
- Small if
  - Weights between clusters small
  - Weights within a cluster large
- Minimizing Ncut(A,B) maximizes a measure of similarity within the sets A and B

Finding Minimum Normalized-Cut

- Finding the Minimum Normalized-Cut is NP-Hard.
- Polynomial Approximations are generally used for segmentation

- It can be shown that $$\min N_{cut} = \min_{\mathbf{y}} \frac{\mathbf{y}^{\mathrm{T}}(\mathbf{D}-\mathbf{W})\mathbf{y}}{\mathbf{y}^{\mathrm{T}}\mathbf{D}\mathbf{y}}$$

  with $$y(i) \in \{1, -b\},\ 0 < b \le 1, \text{ and } \mathbf{y}^{T}\mathbf{D}\mathbf{1} = 0$$

- If y is allowed to take real values then the minimization can be done by solving the generalized eigenvalue system

$$(\mathbf{D}-\mathbf{W})\mathbf{y} = \lambda \mathbf{D}\mathbf{y}$$

Normalized cuts

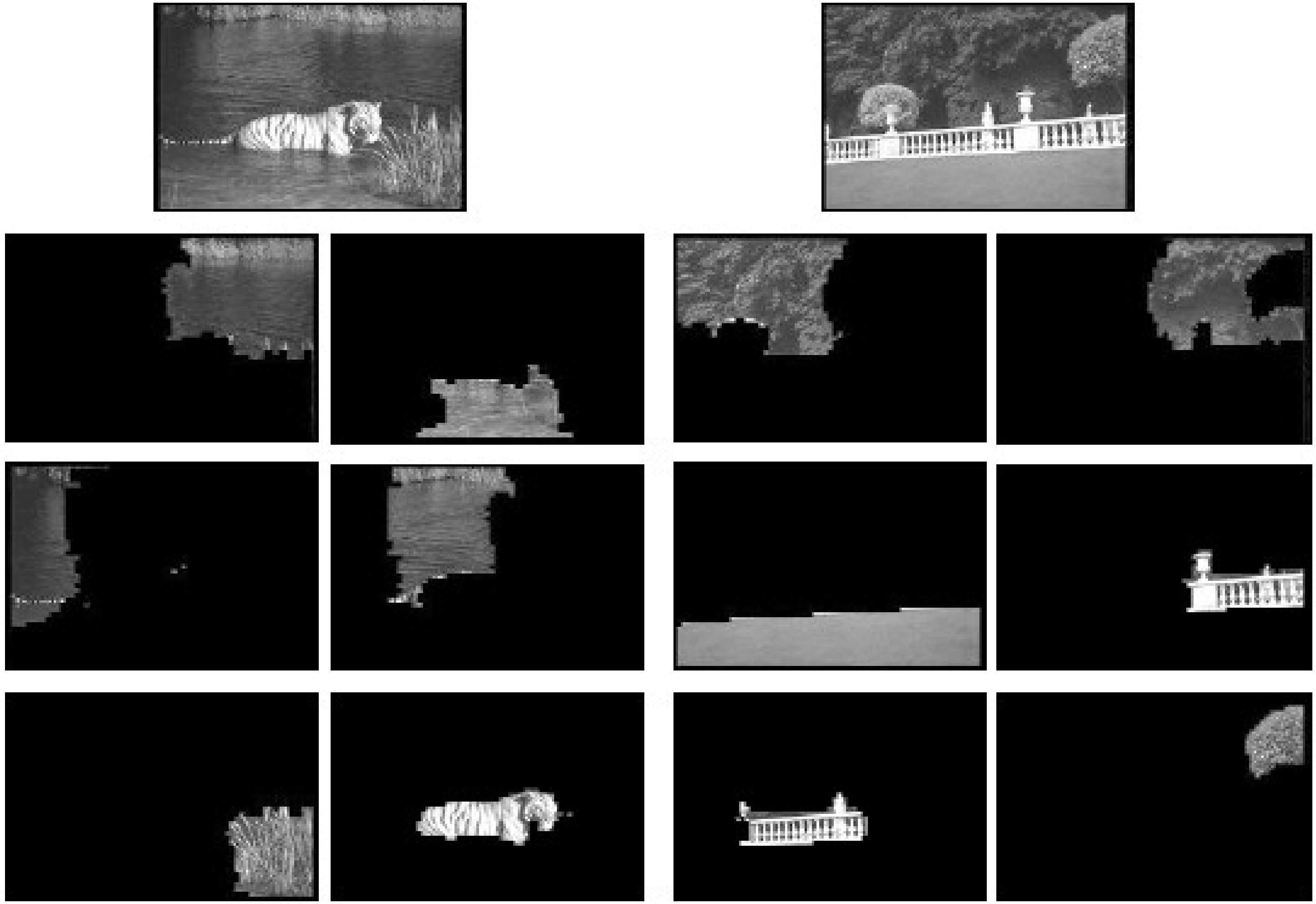
- Compute affinity matrix W and the degree matrix D

$$D = (d_{ij}) \text{ with } d_{ij} = \begin{cases} \sum_k w_{ik} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

- Find the eigenvectors of

$$D^{-1/2}(D - W)D^{-1/2}$$

- Use the sign of the eigenvector with second smallest eigenvalue to bipartition the graph
- Recursively partition the segmented parts if necessary.



Region Growing

Region Growing

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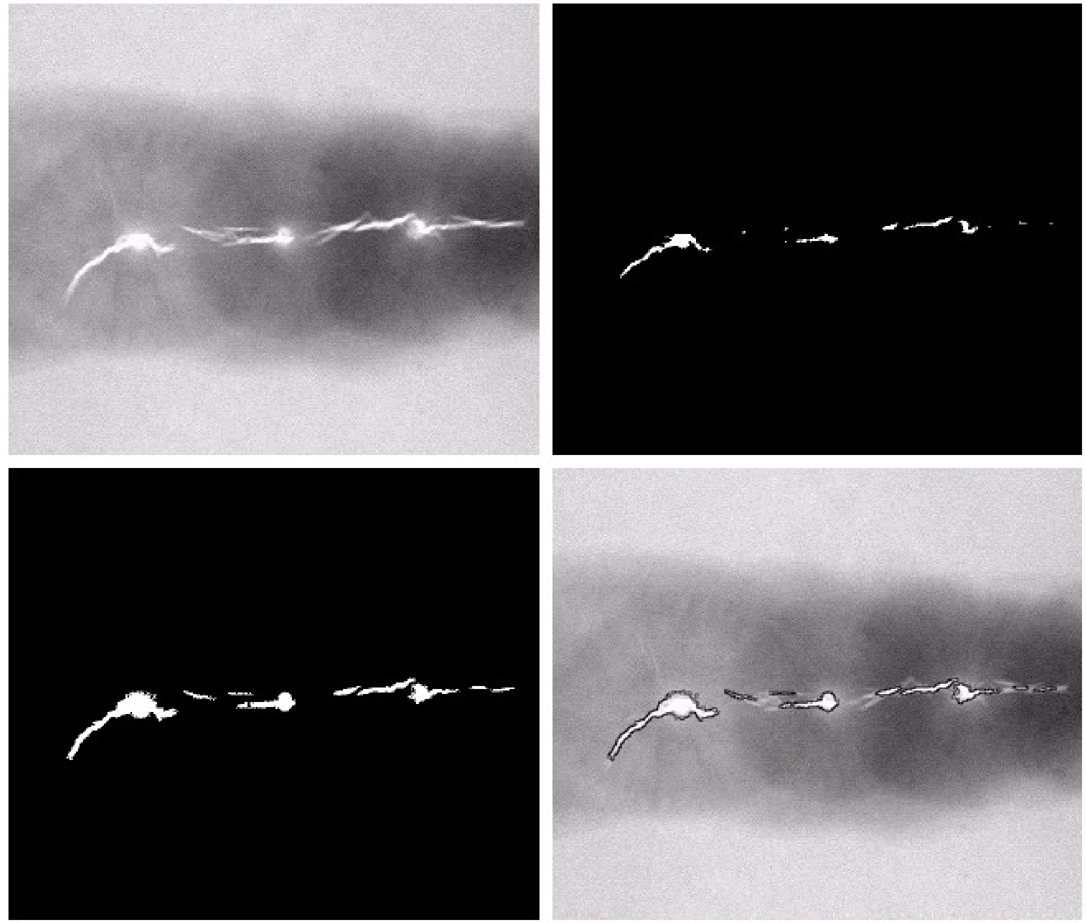


a b
c d

FIGURE 10.40 (a) Image showing defective welds. (b) Seed points. (c) Result of region growing. (d) Boundaries of segmented defective welds (in black). (Original image courtesy of X-TEK Systems, Ltd.).

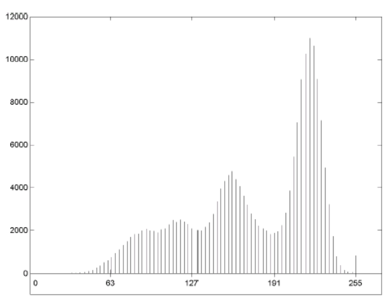


FIGURE 10.41 Histogram of Fig. 10.40(a).

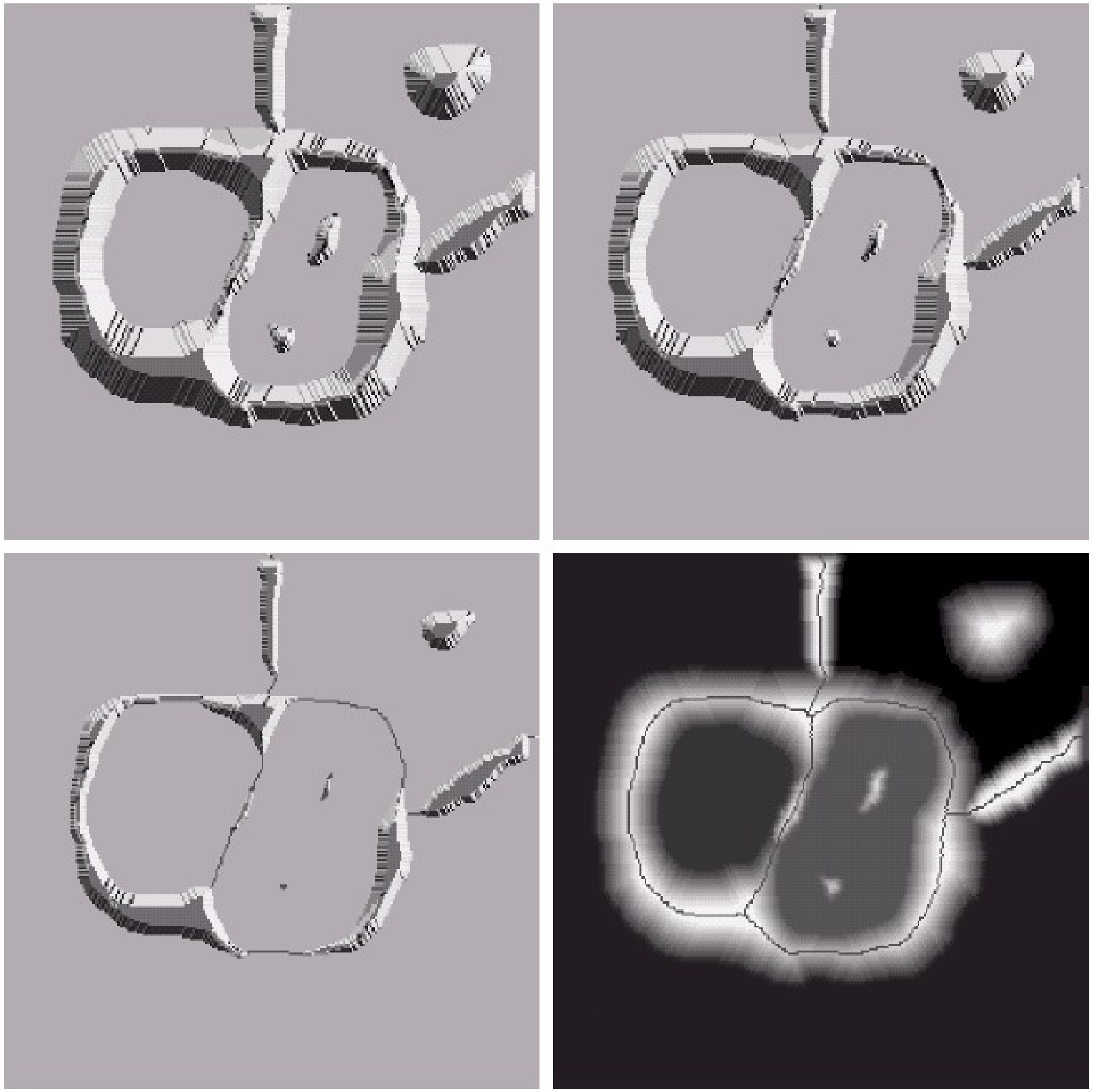
Morphological Watersheds

Image I(x,y) interpreted as a 3D surface

Water raises from minima flooding the basins

When distinct basins join a damis creted (separation)

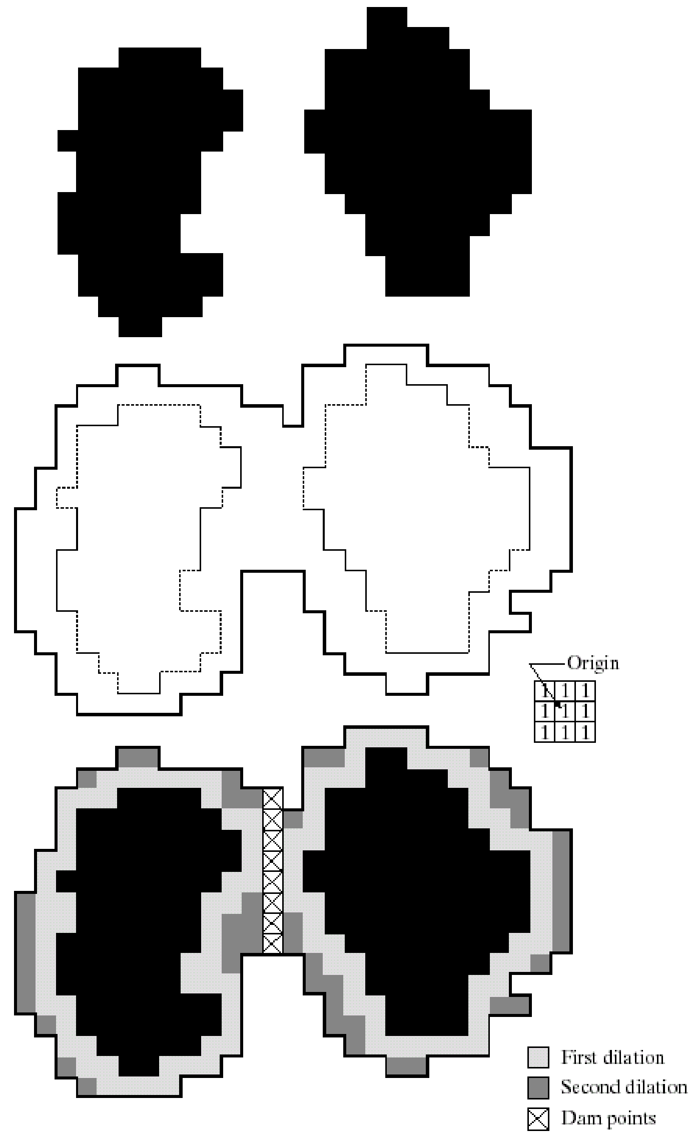
Morphological Watersheds



e f
g h

FIGURE 10.44
(Continued)
(e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Dam building



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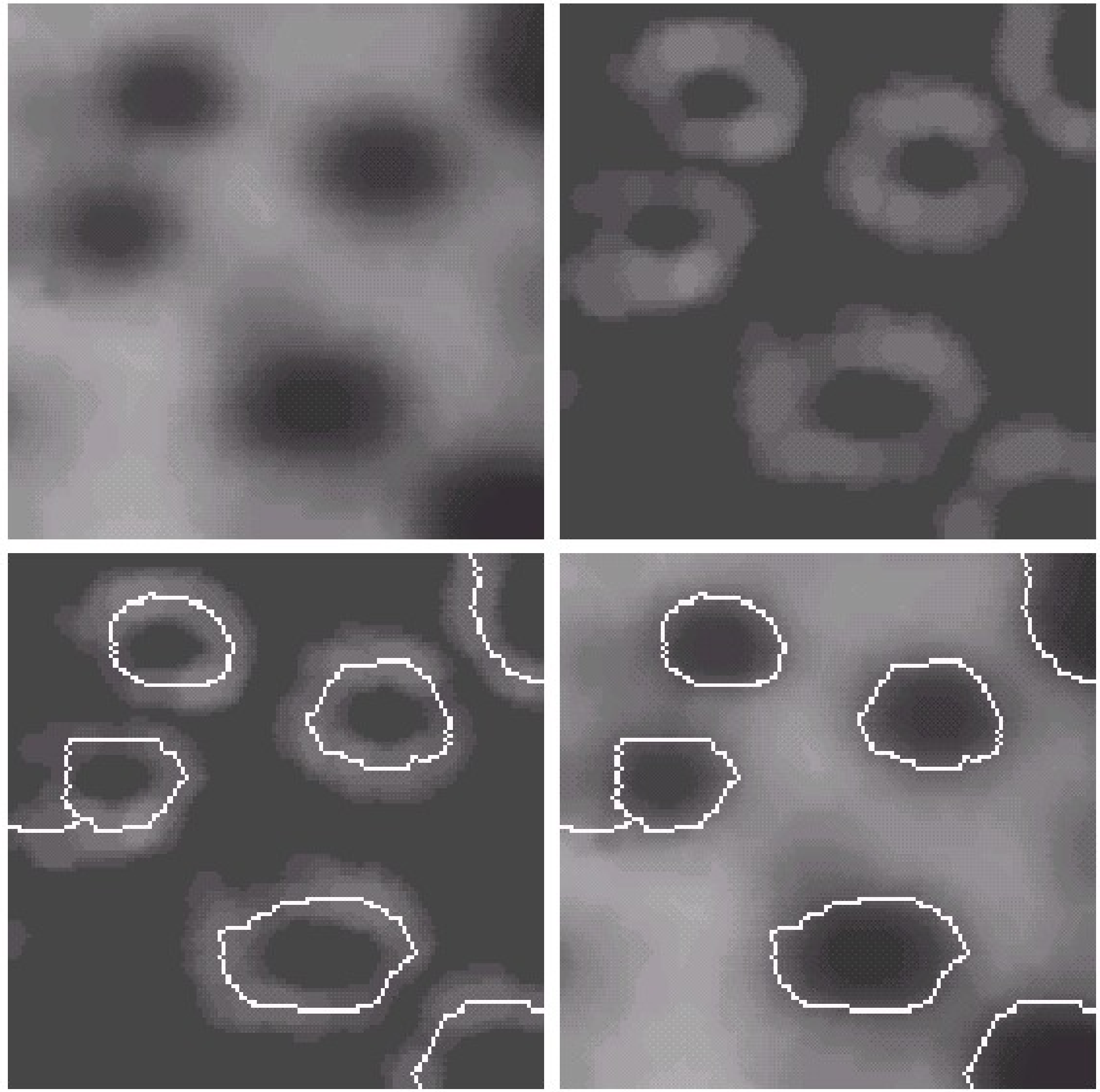
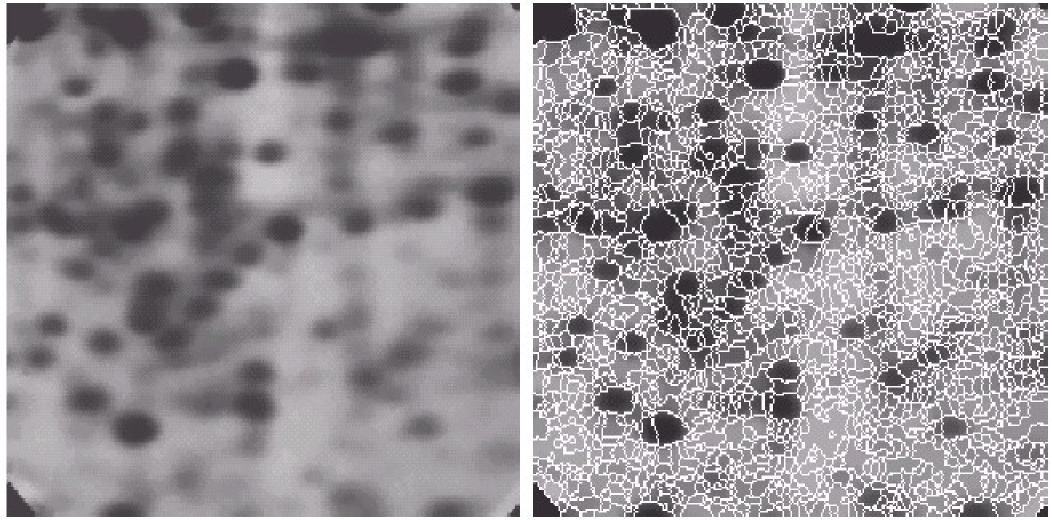


a b
c d

FIGURE 10.46
(a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Oversegmentation



a b

FIGURE 10.47 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

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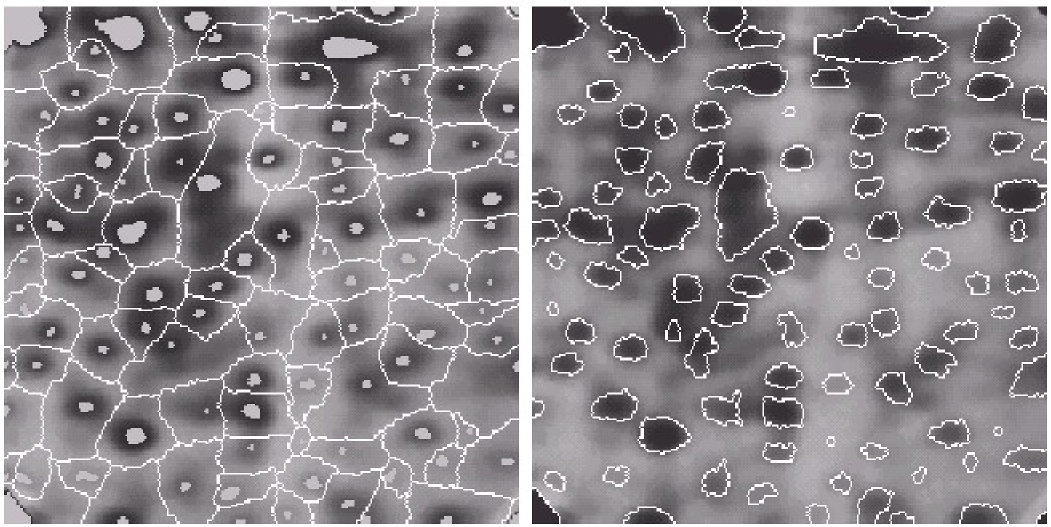
- Oversegmentation is common
  - Every localminima creates a distinct basin
  - Noise creates local minima

Markers

- Soluzion: external information (markers)
  1. Internal Markers: interesting minima
  2. External Markers: interesting watershed points



a b

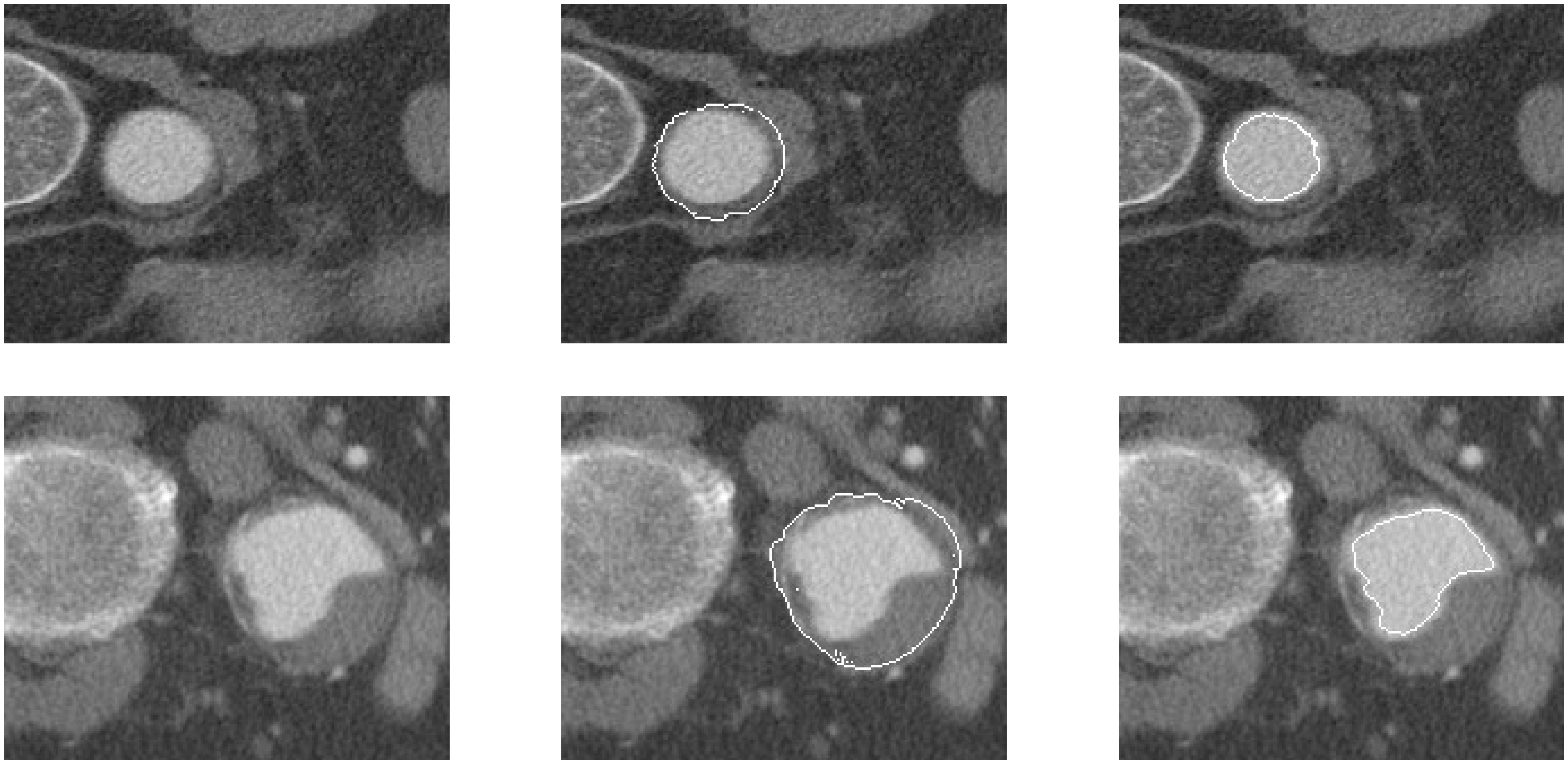
FIGURE 10.48 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Boundary Extraction

Snakes: Introduction

- The active contour model, or snake, is defined as an energy-minimizing spline.
- Active contours results from work of Kass et.al. in 1987.
- Active contour models may be used in image segmentation and understanding.
- The snake's energy depends on its shape and location within the image.
- Snakes can be closed or open

Example



Aorta segmentation using active contours

Snakes

- First an initial spline (snake) is placed on the image, and then its energy is minimized.
- Local minima of this energy correspond to desired image properties.
- Unlike most other image models, the snake is active, always minimizing its energy functional, therefore exhibiting dynamic behavior.
- Also suitable for analysis of dynamic data or 3D image data.

Kass Algorithm

- The snake is defined parametrically as v(s)=[x(s),y(s)], where s□[0,1] is the normalized arc length along the contour.
- The energy functional to be minimized may be written as

$$
\begin{aligned}
E^*_{snake} &= \int_0^1 E_{snake}(v(s))ds \\
&= \alpha \int_0^1 E_{cont}(v(s))ds + \beta \int_0^1 E_{curv}(v(s))ds \\
&\quad + \gamma \int_0^1 E_{image}(v(s))ds
\end{aligned}
$$

- E_{cont} = snake continuity
- E_{curv} = snake curvature
- E_{image} = image forces (e.g., edge attraction)

Internal Energy - Smoothness

Continuity:

$$E_{cont}(v(s)) = \left|\frac{dv}{ds}\right|^2$$

Curvature:

$$E_{curv}(v(s)) = \left|\frac{d^2v}{ds^2}\right|^2$$

External Energy - Data Fidelity

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Dark/Bright Lines

$$E_{image}(v(s)) = I(v(s))$$

Edges

$$E_{image}(v(s)) = -\left|\nabla I(v(s))\right|$$

Tradeoff

$$E^*_{snake} = \int_0^1 E_{snake}(v(s))ds$$
$$= \alpha\int_0^1 E_{cont}(v(s))ds + \beta\int_0^1 E_{curv}(v(s))ds$$
$$+ \gamma\int_0^1 E_{image}(v(s))ds$$

α, β, γ determine trade-off

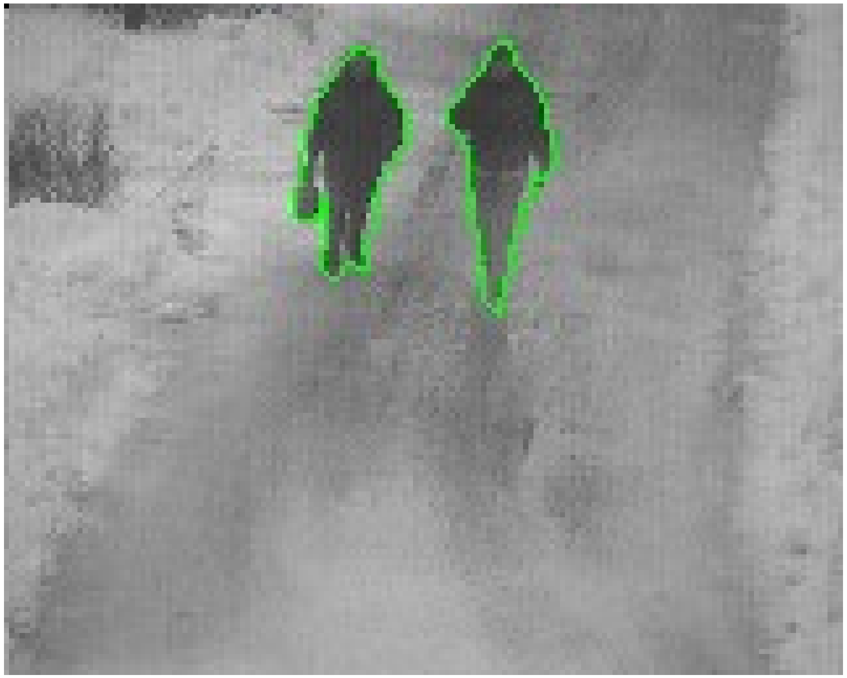
Algorithm

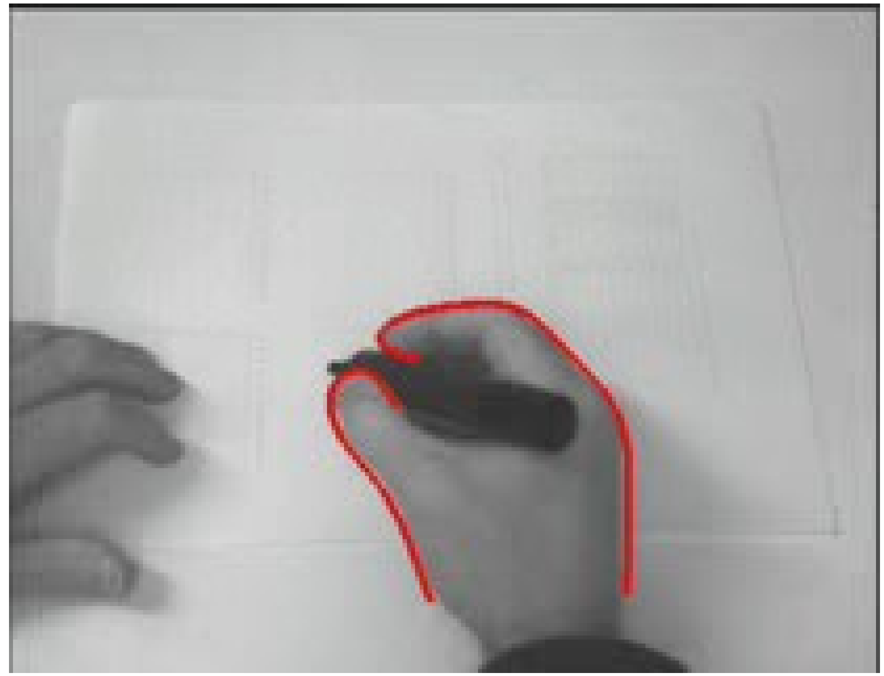
$$E^*_{snake} \approx \sum_k \alpha |p_k - p_{k-1}|^2 + \beta |p_{k+1} - 2p_k + p_{k-1}|^2 - \gamma |\nabla I(p_k)|$$

- Select N initial locations $p_1, \ldots, p_N$
- Update until convergence

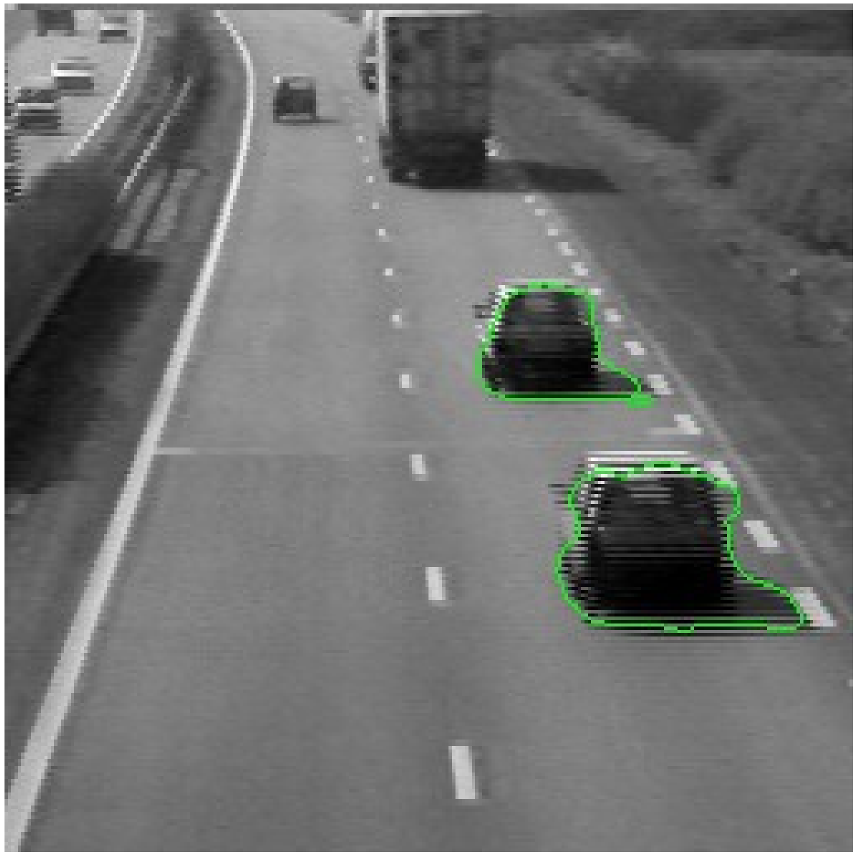
$$p_i \leftarrow p_i - 0.1 \frac{\partial}{\partial p_i} \sum_k \begin{bmatrix} \alpha |p_k - p_{k-1}|^2 \\ + \beta |p_{k+1} - 2p_k + p_{k-1}|^2 \\ + \gamma |\nabla I(p_k)| \end{bmatrix}$$

Examples

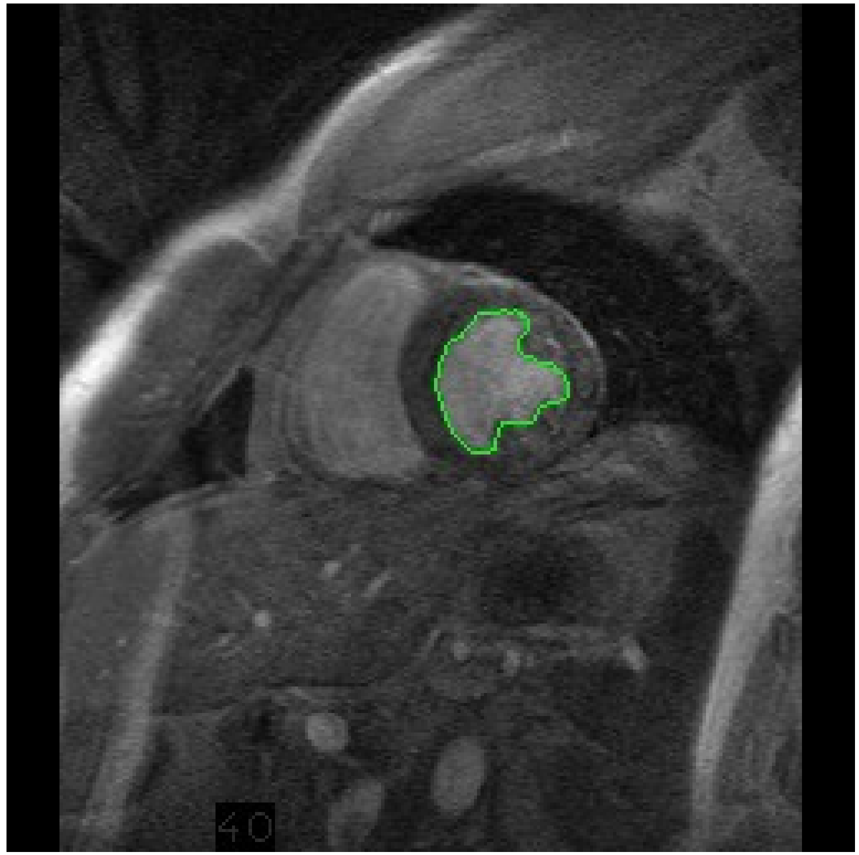




Examples



Example



Heart

Example

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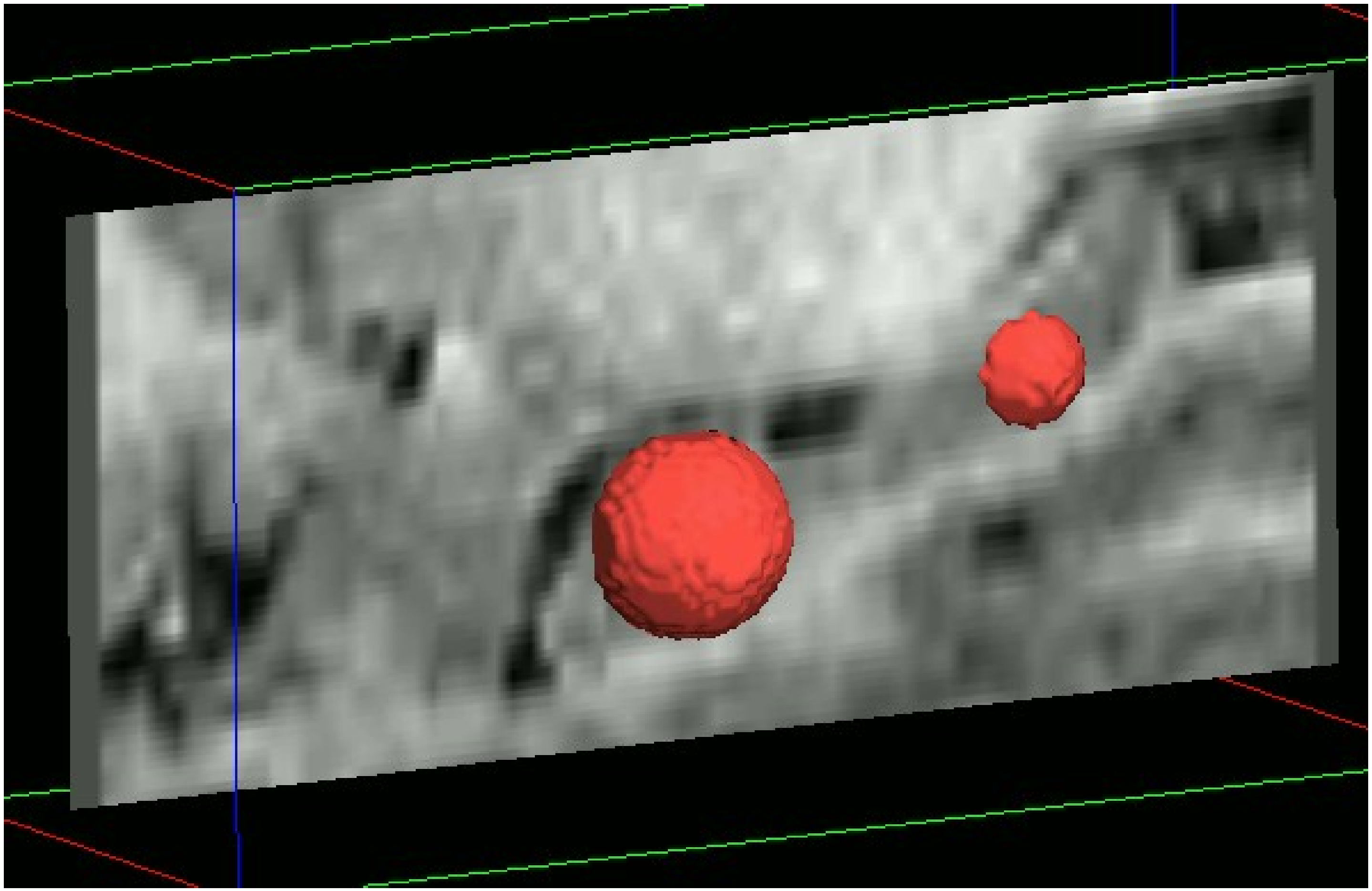
3D Segmentation of the Hippocampus



Problems with Snakes

- Snakes sometimes degenerate in shape by shrinking and flattening.
- Stability and convergence of the contour deformation process unpredictable.

- Solution: Add some constraints

- Initialization is not straightforward.

- Solution: Manual, Learned, Exhaustive