Interpolation

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Spatial transforms

I(x,y) image

s: $R^2 \to R^2$

Spatial transformations

$$I \to f(I) \quad f(I)(s(x,y))=I(x,y)$$

$$f(I)(x,y)=I(s^{-1}(x,y))$$

f transforms the geometry of the image plane

Values ouside the samples

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- In the continuous domain you only need to get the intensity/color at the new location

$$I' = f(I) \Rightarrow I'(x,y) = I(f^{-1}(x,y))$$

- After sampling the information is limited and the transformed point an fall outside of the samples!

- Example: translate the image by $(0.5,0)^T$

  $f(x,y)=(x-0.5,y)^T$

  $I'(x,y)=I(x+0.5,y)$

  but samples exist only for whole numbers!

Nearest Neighbour

- Need to estimate the values using the information from nearby samples (interpolation)

- 1^a possibility: Nearest Neighbour

  use the image value at the closest integer location to $f^{-1}(x,y)$ [Round($f^{-1}(x,y)$)]

$$I'(x,y)=I(Round(f^{-1}(x,y)))$$

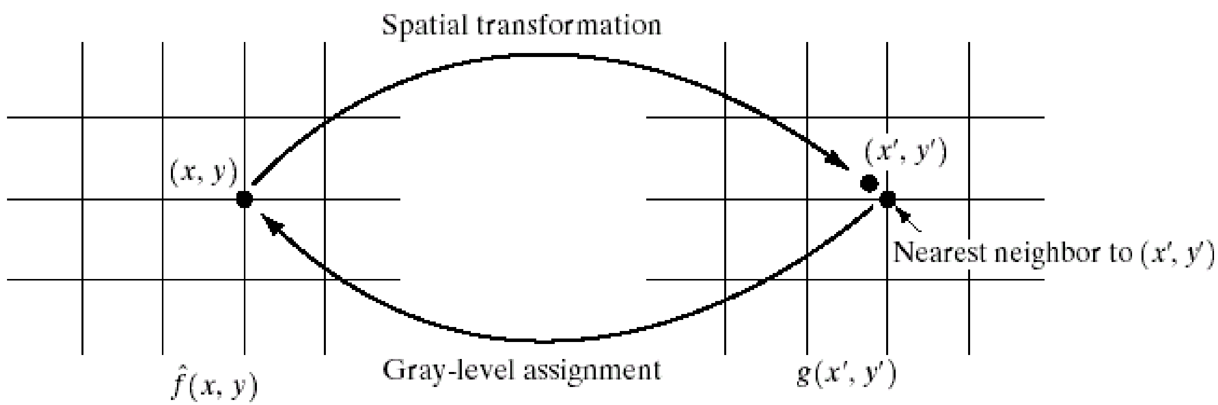


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

Nearest Neighbour

- When translating by $(a,0)^T$

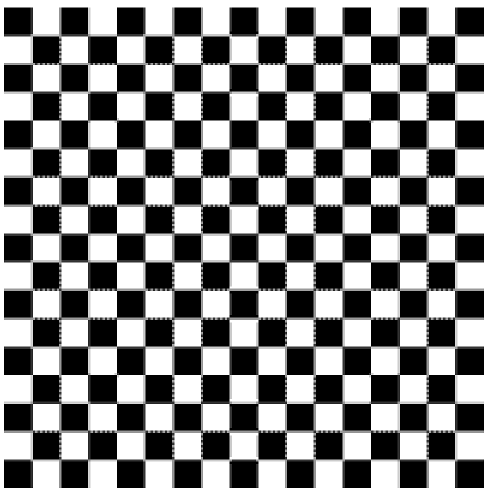
  $$I'(x,y)=I(Round(x+a),y))$$

- What happens when you zoom?

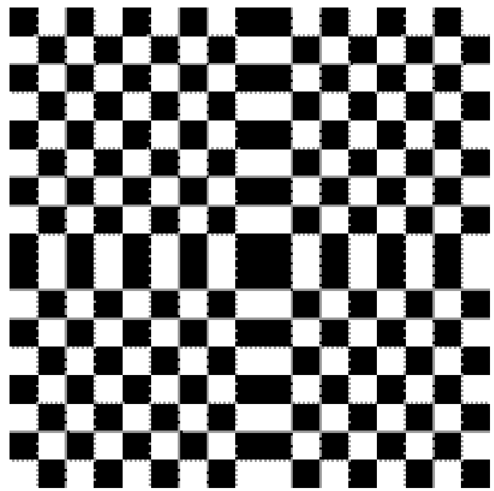
  artifacts! (blocks)

Nearest Neighbour

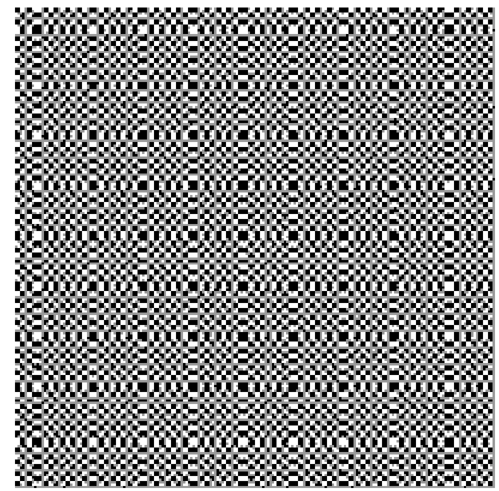
Nearest neighbour interpolation suffers from artifacts in presence of scale changes



The original texture



The same texture reduced by 90% using point sampling



The errors repeat throughout the point sampled texture

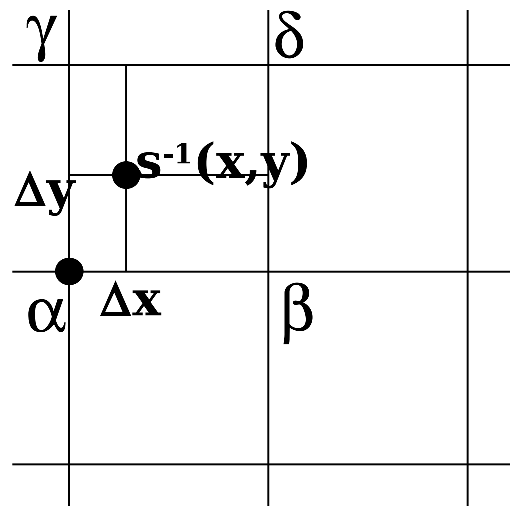
Bilinear interpolation

- 2ª possibility: bilinear interpolation
  Use the 4 points around $f^{-1}(x,y)$ (linear combination)

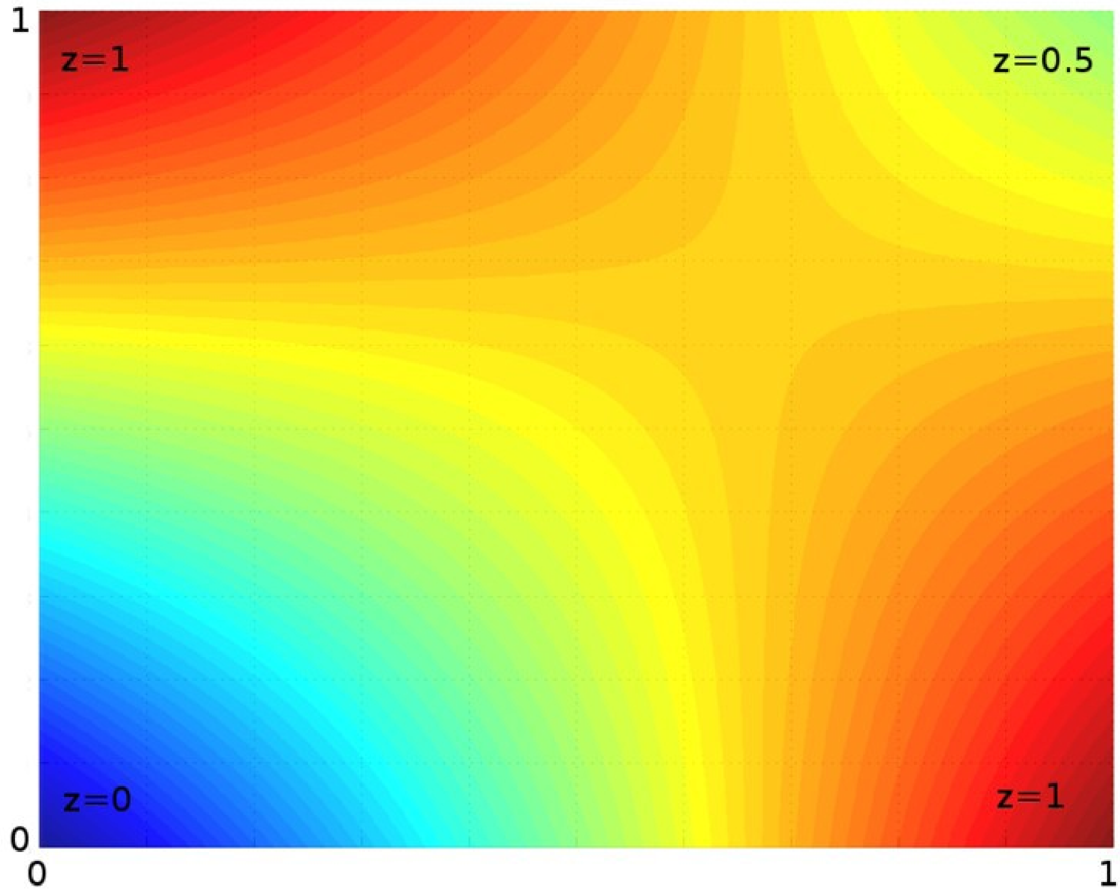
  $I'(x,y)=\alpha I\ (x',y')+\beta I(x'+1,y') +\gamma I(x',y'+1)+\delta I(x'+1,y'+1)$

  where $x'<=s_x^{-1}(x,y)<=x'+1$ and $y'<=s_y^{-1}(x,y)<=y'+1$

  $\Delta x = s_x^{-1}(x,y)-x'$
  $\Delta y = s_y^{-1}(x,y)-y'$
  $\alpha=(1-\Delta x)(1-\Delta y)$
  $\beta=\Delta x(1-\Delta y)$
  $\gamma=(1-\Delta x)\Delta y$
  $\delta=\Delta x\Delta y$



Bilinear interpolation



NN Vs bilinear interpolation

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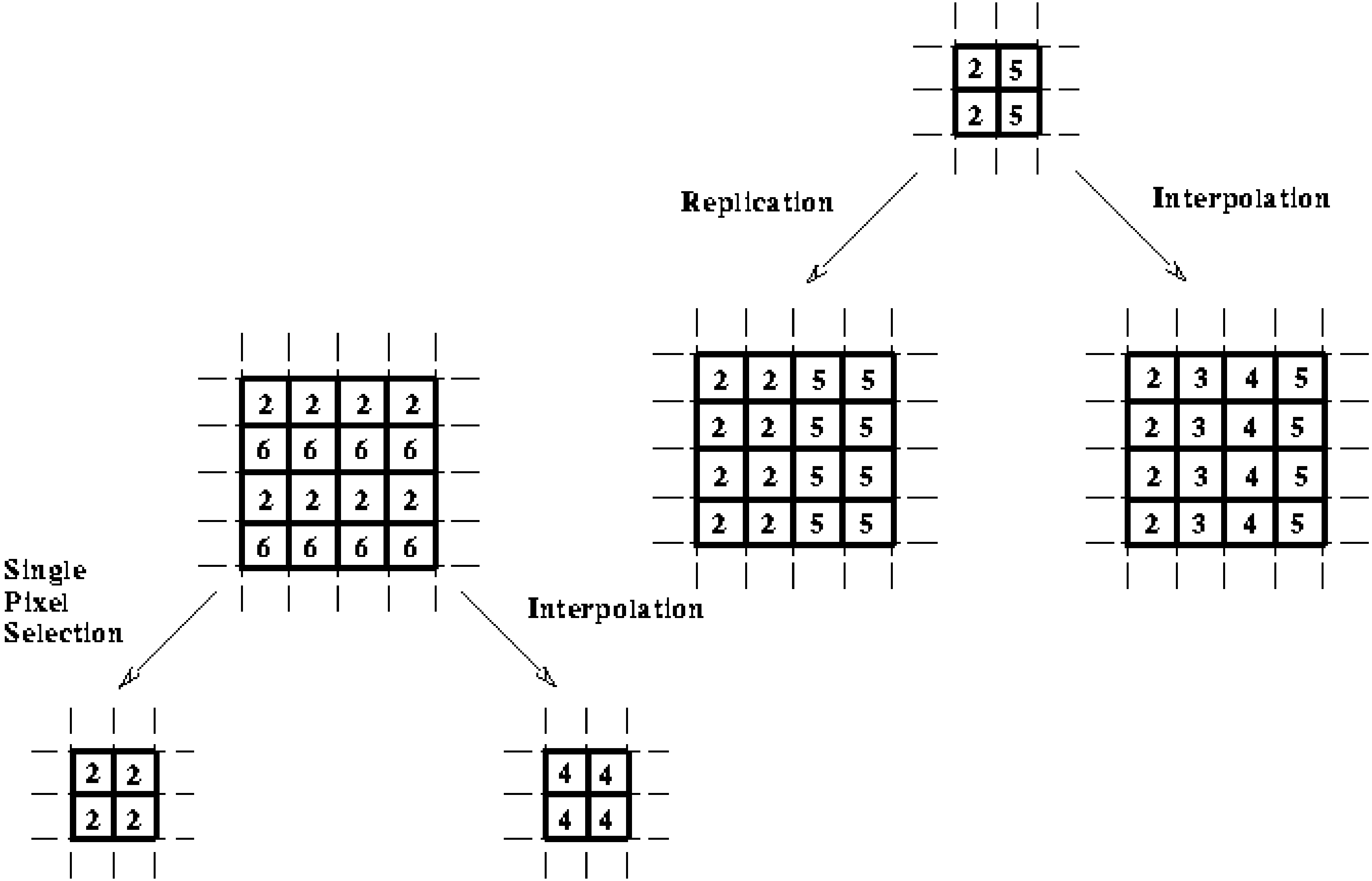
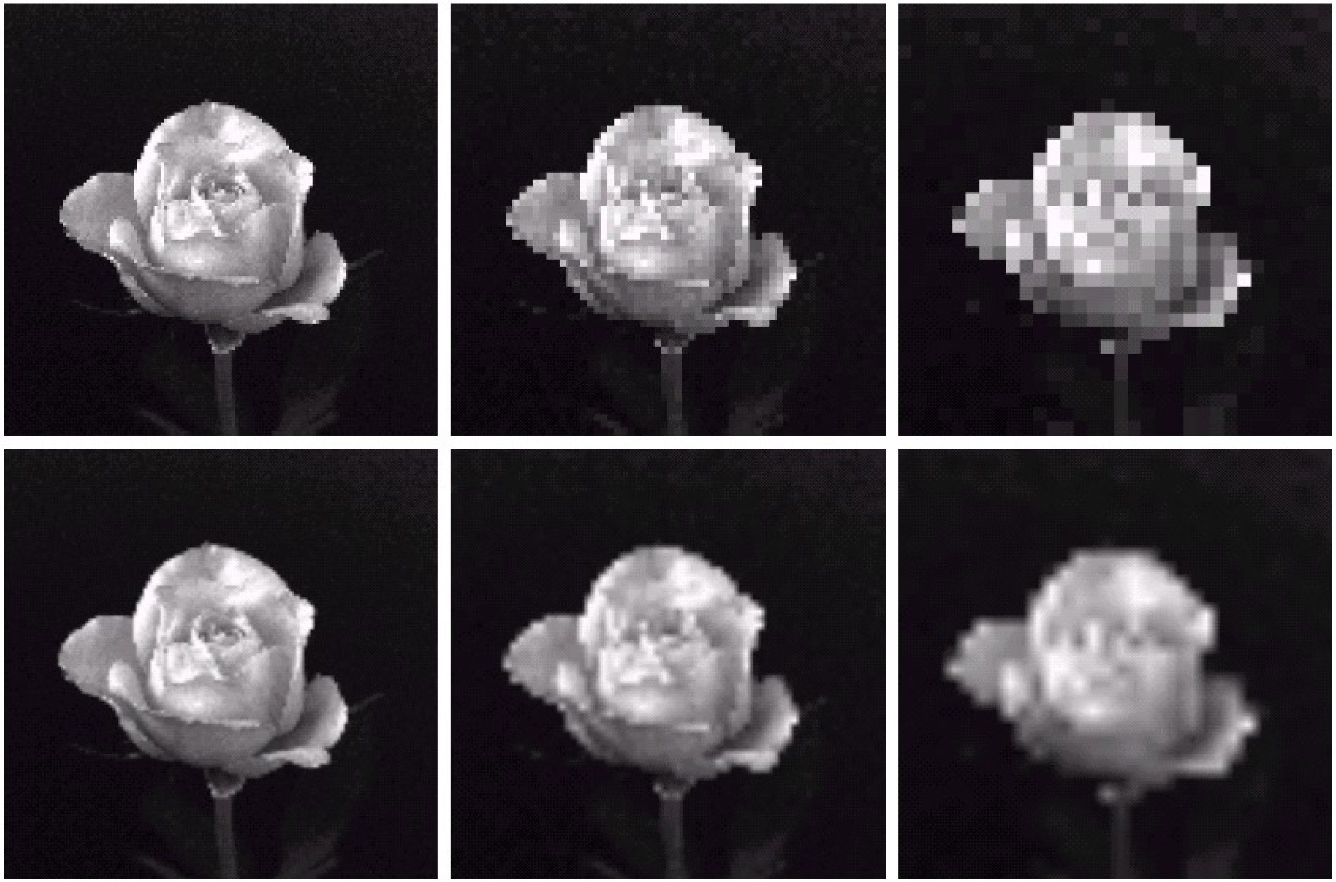
NN Vs bilinear interpolation



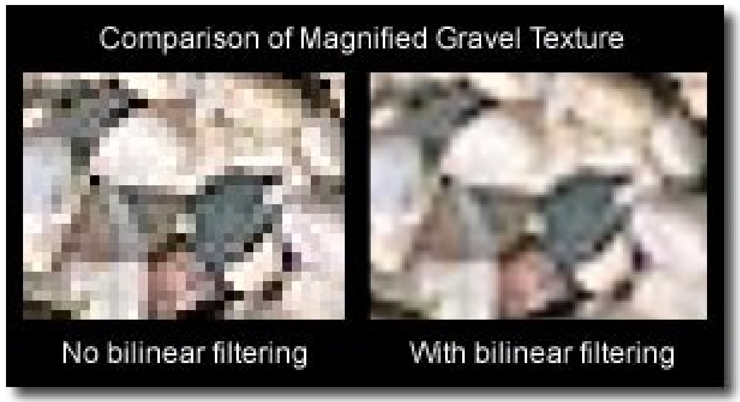
Zoom NN vs bilinear



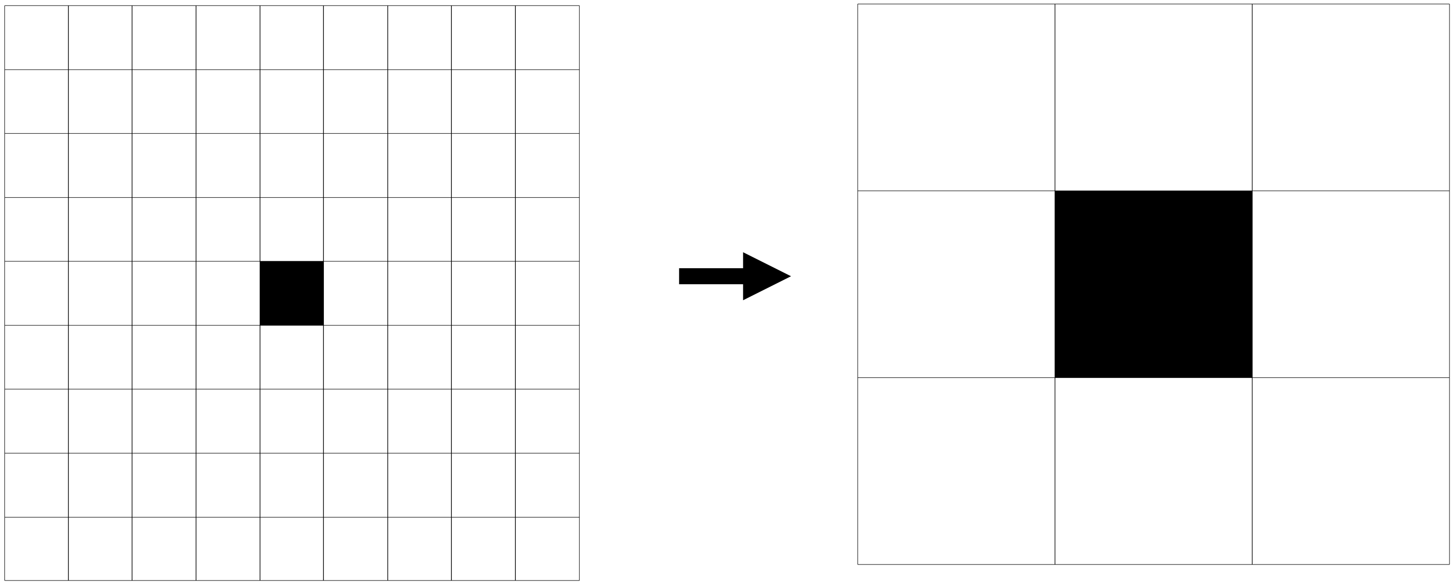
a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Zoom NN vs bilinear



Zoom out

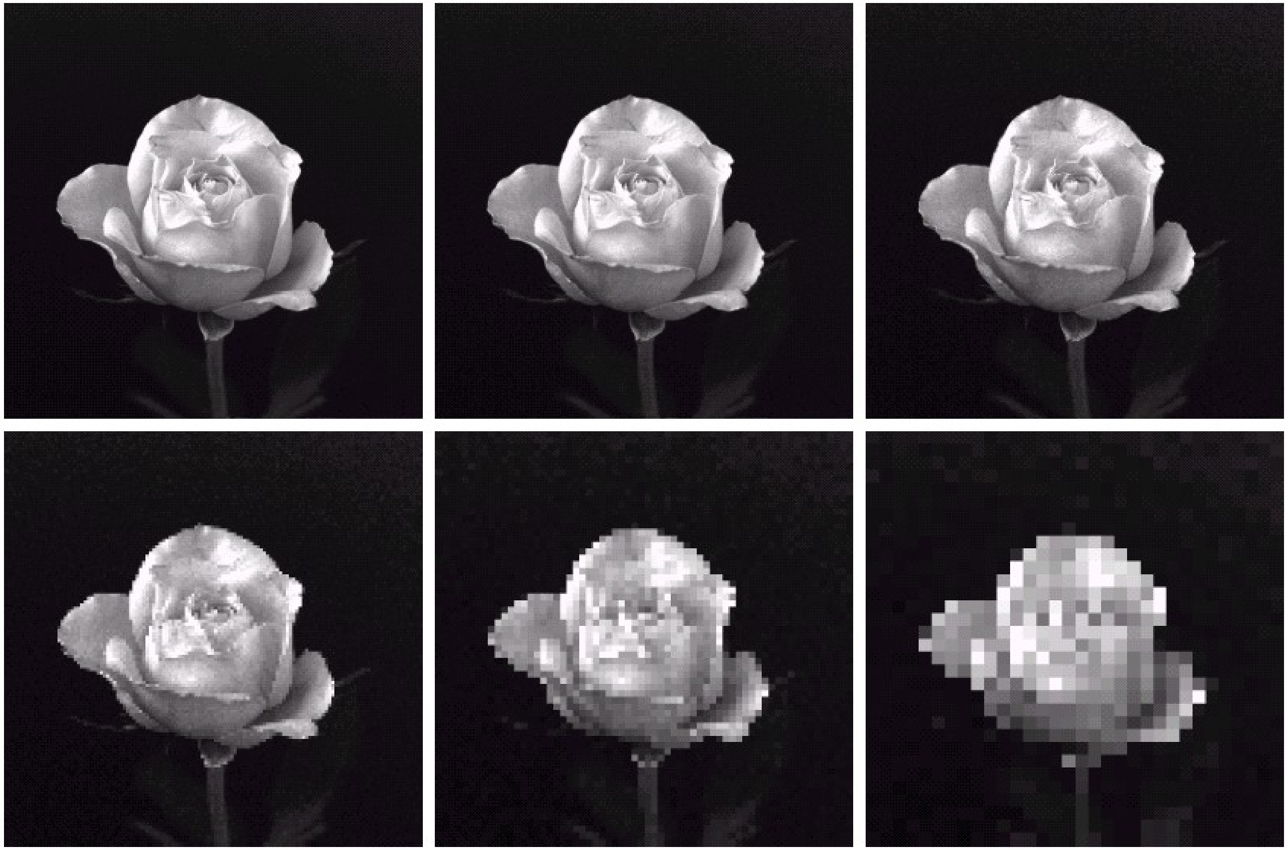


- In the left image the black pixel is 1/81 the size of the whole image
  after the scale change it weight 1/9
- Recall that a sensor integrates the energy throughout its domain
  - the intensity should be lower!

Sampling effects

FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Sampling effects



a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

Quantization effects