Hierarchical automata and the cospan-span model

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Abstract. In the context of the cospan-span model [8] we define a notion of hierarchical automaton, and show how the P-systems of Gh. Păun [9] may be described in terms of these automata. A distributive law for cospans of spans is proved which relates two different views of hierarchical systems: the first view is that hierarchical systems consist of layers, with communication between adjacent layers; the second is that a hierarchical system is an evolving hierarchy or tree structure.

There are a wide variety of models of hierarchical systems, from the statecharts of Harel [4], to the mobile ambients of Cardelli and Gordon [2], to the recently defined P-systems of Gh. Păun [9]. In this talk we study the model of Păun. The aim of the talk is to define a notion of *hierarchical automaton*, in terms of a subalgebra of the algebra of cospans of spans of graphs introduced in [8], and to show how P-systems can be described in this algebra.

This comparison suggests two new aspects of the algebra of cospans of spans. Firstly, in this talk the parallel operation is used to model the interaction of two adjacent layers of a hierarchy, whereas in previous papers ([5], [6], [8]) it has been used to model interaction of processes in a single layer. As a result, in this talk a system (an expression in the cospan-span algebra) has a layer structure, or equivalently, in the paradigm of P-systems a membrane structure. Secondly we prove a new distributive law for cospan-spans which provides a precise relation between two fundamentally different ways of looking at hierarchical systems. Such a system may be thought of (i) as a vertical composition of layers, each layer having its own evolution in communication with adjacent layers, or alternatively (ii) as an evolving tree structure - evolving snapshots of the hierarchical structure.

P-systems were introduced as a class of distributed parallel computing devices of a biochemical type. The basic model consists of a membrane structure composed by several cell-membranes, hierarchically embedded in a main membrane called the skin membrane. The membranes delimit regions and can contain objects, which evolve according to given evolution rules associated with the regions. Such rules are applied in a maximally parallel manner: at each step, all the objects which can evolve should evolve. Membranes themselves may be dissolved. A computation device is obtained: we start from an initial configuration and we let the system evolve. A computation halts when no further rule can be

applied. The objects in a specified output membrane (or expelled through the skin membrane) are the result of the computation. Many variants are considered in [9], [10], [11] and [1]. A survey and an up-to-date bibliography can be found at the web address http://bioinformatics.bio.disco.unimib.it/psystems.

The theory of P-systems as it stands is non-compositional. A connection has been made with automata theory in [3] though their automata are also non-compositional. Connection between P-systems and the ambient calculus of Cardelli and Gordon (which arose as a calculus to describe secure and mobile administrative domains in the internet) have been made in [12].

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