An iterative characterization of computable functions on hyperwords of natural numbers

G. Germano, Dipartimento di Informatica - Università di Pisa

S. Mazzanti, Dipartimento di Arti e Disegno Industriale - Istituto Universitario di Architettura Venezia

The set of words on a given alphabet is a well know data type which occurs in most of programming languages. The notion of word can be generalized to the notion of *hyperword*, by allowing that a word can be used as a new letter to construct new words. Hyperwords arise in various ways in computer science: e.g., consider the list structures of LISP-like languages or the operand stack of a PostScript interpreter [1].

This work considers the set of all computable functions on the set $\mathbb{N}^H$ of hyperwords on $\mathbb{N}$ and characterizes it inductively as the closure H of a small set of basic functions with respect to composition and iteration.

So, class H joins a taxonomy of classes of computable functions over data structures like single natural numbers, sequences (of fixed length) of natural numbers, sequences of integers and stacks of natural numbers [2-6, 8,9] .

Hyperwords can be profitably used to construct interpreters for languages based on iteration; indeed the syntax of H can be easily represented by means of the hyperwords generated by a set of letters corresponding to the basic functions of H.

First, in this work, we define the sets of words and hyperwords on a given alphabet and introduce hyperword algebras.

Then, we introduce a set of basic functions on $\mathbb{N}^H$, a set of functional operators comprehending an iteration operator and we define the class H .

Eventually, we extend the Church Thesis concerning $\mathbb{N}$ to the Church Thesis concerning $\mathbb{N}^H$. We do this by using the method of anticonjugation, sketched in [7,10] and widely used e. g. in [5,6,8]. So, we have to use a coding of $\mathbb{N}^H$ into $\mathbb{N}$, defined as the unique isomorphism from the hyperword algebra of hyperwords on $\mathbb{N}$ onto an hyperword algebra with $\mathbb{N}$ as universe. A technical lemma concerning the computability of this coding is proved in the appendix.

References

1. Adobe Systems Inc., *PostScript Language Reference Manual, third edition*, Addison-Wesley, Reading, MA 2000 .

2. S. Eilemberg, C.C. Elgot, *Iteration and recursion*, Proc. Nat. Acad. Sci. USA 61 (1968) 378-379.

3. S. Eilemberg, C.C. Elgot, *Recursiveness*, Academic Press, New York, 1970.

4. G. Germano, A. Maggiolo Schettini, *Sequence to sequence functions*, Information Processing Letters, vol. 4, (1975), p. 1-6

5. G. Germano, A. Maggiolo Schettini, *Computable stack functions for the semantics of stack programs*, Journal of Computer and System Sciences, vol. , (1976), p. 133-144.

6. G. Germano, S. Mazzanti, *Partial closures and semantics of while: towards an iteration-based theory of data types*, Proceedings of the Logic Colloquium, Aachen 1983, Lecture Notes in Mathematics n. 1104, Springer Verlag, Berlin, 1984, p. 163-174.

7. G. Germano, S. Mazzanti, *A setting for generalized computability*, Rödding Memorial Volume, Lecture Notes in Computer Science n. 270, Springer Verlag, Berlin, 1987, p. 154-165.

8. G. Germano, S. Mazzanti, *General iteration and unary functions*, Annals of Pure and Applied Logic, vol. 54, (1991) p. 137-178.

9. S. Mazzanti, *Iterative characterizations of computable unary functions: a general method*, Mathematical Logic Quarterly, vol. 43, (1997), p. 29-38.

10. H. Rogers, *The theory of Recursive Functions and Effective Computability*, The MIT Press, 1987.