Classical Logic as Limit
An intuitionistic model of Δ^0_2-maps using Parallel Computations

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Abstract

Let $\mathcal{N}$ be the structure of natural numbers and recursive maps. We propose an purely intuitionistic model $\mathcal{N}$ of Classical Logic, an extension of N including parallel computations and "indetermined" integers. Integers in are interpreted in $\mathcal{N}$ by learning maps, a kind of convergent sequences of integers, which, nevertheless, may have more than one limit value. We claim that $\mathcal{N}$ may be used in real programming, and enable us to "program with non-recursive maps".

Learning maps are used to interpret intuitionistically, in recursive way, the non-recursive objects defined by Classical Logic. Non-determinism, instead, is used to obtain a more natural interpretation. Take, for instance, any recursive property P. Consider the non-recursive and non-deterministic definition: "pick any c such that $P(c)$, with $c = 0$ if none exists". c is the so-called Heyting constant for P. In our model $\mathcal{N}$, c is interpreted, as expected, by some convergent succession L, whose set of limits values consists exactly of all $x \in \mathbb{N}$ such that $P(x)$, if P is inhabited, while it is $\{0\}$, if P is empty. We cannot decide which is the case, though.

In this talk, for sake of simplicity, we define the model $\mathcal{N}$ only for Δ^0_2-maps, enough to interpret *Excluded Middle over 1-quantifier formulas*. In a forthcoming paper we will extend $\mathcal{N}$ to some model $\mathcal{N}'$ for the whole Peano Arithmetic.